

AIRBORNE ACOUSTIC VELOCITY MEASUREMENT UTILISING LASER DOPPLER ANEMOMETRY COMBINED WITH PHOTON CORRELATION IN LOW SEEDED CONDITIONS

T Koukoulas Quality of Life Division, Acoustics Group, National Physical Laboratory, UK
P Theobald Quality of Life Division, Acoustics Group, National Physical Laboratory, UK
T Schlicke Fluid Dynamics and Acoustics Group, University of Edinburgh, UK
R Barham Quality of Life Division, Acoustics Group, National Physical Laboratory, UK

1 INTRODUCTION

The calibration of sound-in-air microphones is traditionally achieved using the internationally accepted reciprocity method that establishes the acoustic pressure¹ and free-field sensitivities². This technique is used to provide the sensitivity of microphones typically at discrete frequency points in the range 63 Hz to 20 kHz and is currently the primary standard for microphone calibration. The uncertainty of this method is 0.03 dB, increasing up to 0.18 dB (re 1V per Pa) for the frequency range stated above at the 95% confidence level.

The pressure sensitivity of the microphone to be calibrated is established from electrical measurements without requiring knowledge of the actual acoustic pressure. For a primary standard, this feature might be considered as a weakness of the methodology as the technique is indirect. Furthermore, the reciprocity method is applicable to standard 1" and ½" laboratory and working microphones only. New emerging microphone technologies such as MEMS might ultimately require the development of radically different calibration methodologies. Sources of error such as electrical cross-talk and low signal-to-noise ratio (SNR) at low pressure levels in addition to increasing uncertainty with frequency are accounted for, well understood and are part of the standard uncertainty budget for the reciprocity method. However, as microphones reduce in size into the micro range, these issues become more dominant will therefore demand an alternative calibration methodology.

Optical methods offer an alternative approach and a possible solution to the above issues. Laser Doppler anemometry (LDA) in particular has been used traditionally for the measurement of fluid flow. The obvious advantages of this technique are high spatial resolution, no perturbing effects, as well as its direct nature: the measured acoustic particle velocity being directly proportional to the acoustic pressure. Therefore, a technique that realizes the acoustic Pascal in this direct way would be suitable as the future primary standard.

This paper describes an optical system combining LDA and photon correlation for the measurement of the particle velocity of airborne acoustic fields.

2 THE PRINCIPLE OF LASER DOPPLER ANEMOMETRY AND PHOTON CORRELATION

The principle behind LDA is that the frequency shift of light due to moving particles in a medium is proportional to the particle velocity: the Doppler effect. As an acoustic field propagates through a medium, the particles naturally follow the oscillation to some degree; therefore by measuring the Doppler shift that the photons experience, one can calculate particle velocity and therefore directly calculate the acoustic pressure of the wave-front.

In practice, however, this shift is very small and difficult to measure accurately. Therefore, a suitable and robust system is required in order to facilitate the measurement with the required reproducibility.

Such a system is the classical dual-beam laser Doppler anemometer which uses two intersecting laser beams (with half-angle θ), forming an interference volume consisting essentially of dark and light regions. A graphical illustration of the interference region is shown in figure 1.

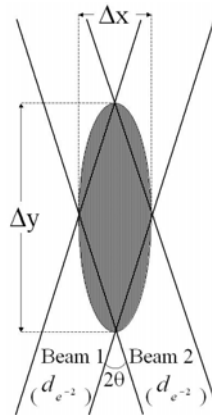


Figure 1: Two intersecting beams forming an interfering volume in a dual beam LDA system

It should be noted that both intersecting beams must have a high degree of similarity; having the same beam waist, polarization and alignment. The volume itself, is essentially an ellipsoid, with dimensions given as follows ($d_{e^{-2}}$ is the focal waist):

$$\Delta x = \frac{d_{e^{-2}}}{\cos \theta} \quad (1)$$

$$\Delta y = \frac{d_{e^{-2}}}{\sin \theta} \quad (2)$$

$$\Delta z = d_{e^{-2}} \quad (3)$$

The fringe spacing Λ and the number of fringes N_f , are related to the wavelength of the source λ , the half-angle θ and the focal waist as follows:

$$\Lambda = \frac{\lambda}{2 \sin \theta} \quad (4)$$

$$N_f = \frac{2d_{e^{-2}}}{\lambda} \tan \theta \quad (5)$$

As particles oscillate due to the presence of an acoustic field, they cross the dark and light regions of the interference volume and by interacting with the laser light, scatter photons whose frequency shift is proportional to the velocity of the particle. A photo-multiplier tube (PMT) can be used to

detect these photon events, producing an electrical TTL-compatible output signal (the LDA signal) that can be processed to provide acoustic particle velocity.

Two classical techniques can be combined with LDA. The first is to introduce a frequency shift in one of the laser beams, and by analyzing the frequency modulated spectrum of the LDA signal (the difference in the main and side peaks), the particle velocity can be extracted. Taylor^{3,4} first reported such a system and others have performed modifications and analyzed similar configurations with different details^{5,6}. The second is to count individual photon events at specific sample rates and then calculate the auto-correlation function from which the particle velocity can be extracted. This technique is termed photon correlation⁷⁻¹¹.

The post-signal processing technique one may utilize also strongly depends on the density of the medium. Due to the fact that air in a standard laboratory does not have enough airborne particles to produce enough light scattering, artificial seeding is required. Frequency based analysis requires more seeding, as this enhances the SNR of the received signal thus improving the calculation of the difference between main and side peaks in the spectrum that yields the particle velocity. Photon correlation based analysis, on the other hand, relaxes the requirements for heavy seeding allowing a closer approximation to air and hence is the technique selected for this study.

By utilizing the photo-multiplier, it is possible to count discrete photon events as long as the sampling is relatively high. The BrookHaven PMT and correlator hardware used for this study, performs a standard auto-correlation of the following form:

$$R(\tau_j) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N n_i n_{i-j} \quad \text{for } j=1, 2, 3, \dots, M \quad (6)$$

where τ_j is the j^{th} delay time, n_i is the photon count number during Δt centered at time t , n_{i-j} is the photon count number during Δt centered at time $t - \tau_j$ and M is the number of correlator channels (each holds one point for the calculated ACF). In its theoretical form, the ACF is given by the following relation¹²⁻¹⁴:

$$R(\tau) = E[x(t)x(t+\tau)] = (kC_0 \int_{-\infty}^{+\infty} W(\beta y)(1 + \cos Dy)dy)^2 + \frac{k^2 C_1 g_0}{2} \int_{-\infty}^{+\infty} \rho_n(y; \tau) R_w(\beta y)(1 + \cos Dy)dy \quad (7)$$

Equation (7) relates the ACF with a number of parameters, namely the expectation operator of the PMT output at different time intervals ($E[\]$ related to $x(t)$ and $x(t+\tau)$), the optical power and detector sensitivity (k), the spatial weighting function representing the envelope on the fringes due to the Gasussian cross section of the laser beams ($W(\beta y)$), frequency-to-velocity conversion factor (D), average number of particles per unit length in the measuring volume (g_0), the probability density function of the particle displacement variable ($\rho_n(y; \tau)$), the auto-correlation of the previously named spatial weighting function ($R_w(\beta y)$) and the particle scattering cross section.

The probability density function of the particle displacement variable itself further includes the velocity component that is required to be calculated. The mathematical proof of this is clearly beyond the scope of this paper and may be obtained in literature¹²⁻¹³. However, after certain transformations and simplifications of Bessel function operators, as well as implementing small angle approximations, it may be shown¹²⁻¹³ that once an ACF has been acquired, the particle velocity within an acoustic plane is given by the following simple expression:

$$u_m = \frac{3.832}{Dt_{\min}} \quad (8)$$

where t_{min} is the time for the ACF to reach its first minimum and D is the previously mentioned frequency-to-velocity conversion factor related to the half-angle θ and the wavelength of the laser source λ :

$$D = \frac{4\pi \sin \theta}{\lambda} \quad (9)$$

3 EXPERIMENTAL RESULTS

For this study, a 70 mW Nd:YAG laser source with $\lambda=532$ nm was utilized. The main reason for choosing this particular source was the capability of operating at high power in order to maximise photon scattering. Using a suitable convex-concave double lens configuration, the primary beam waist was reduced to approximately 0.5 mm. A beam-splitter and subsequent polarizer produced two identical secondary beams. The focal waist (d_{e-2}) of the secondary beams was further reduced to 0.11 mm using a 75 mm plano-convex lens which also converged the secondary beams to intersect, creating the required ellipsoid interference region. The dimensions of this measurement volume were $\Delta x=0.11$ mm, $\Delta y=0.83$ mm, $\Delta z=0.11$ mm, $\Lambda=2.01$ μ m and $N_f=55$.

Different standing wave tubes (SWT) with lengths matching a range of acoustic frequencies for experimentation were used. The SWT was rigidly terminated on one end, with a small loudspeaker connected to a function generator attached to the other end. The ellipsoid measurement volume was located such that it was on a velocity anti-node within the tube, as well as aligned with the rotational axis of the tube itself.

An aperture was placed outside the SWT so that the exit beams were completely blocked and only the forward scattered light from the measurement volume was allowed to pass. A BrookHaven Instruments PMT was placed behind the aperture; the photo-multiplier employed a telescopic arrangement allowing manual focus of the PMT on the interference volume. The TTL level analog output of the PMT was processed using a BrookHaven Instruments BI-9000AT PC-based correlator board that produced the resulting ACFs, operating in a manner described by equation (6).

The alignment is naturally of paramount significance in any optical system, an example being the reduction of the effects of astigmatism. However in this case, an additional parameter is the half-angle θ itself. Distortion of the fringes could occur if the half-angle is too sharp. A graphical illustration of the system is shown in figure 2.

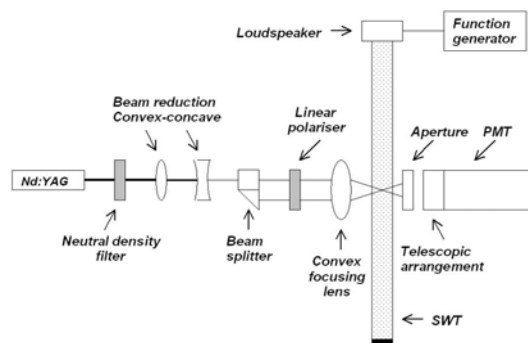


Figure 2: the acousto-optic LDA system

Having established an optimal alignment and fringe interaction region, it was necessary to fine-tune the amount of optical power and artificial seeding. In the literature, it has been reported that

approximately 20 mW are sufficient to produce ACFs in smaller standing wave tubes than used in the current arrangement with substantial smoke seeding levels^{4,11}.

As a starting point, the laser power was set at around 15 mW for the primary beam (utilizing suitable neutral density filters), and a pure tone with a frequency of 1 kHz was introduced in the SWT with no artificial seeding. The power of the secondary beams was in the region of 4 mW (due to introduction of the beamsplitter and the attenuation of the polarizer). Figure 3 shows the obtained ACF; clearly random noise. Artificial seeding was then introduced for durations reported in the literature^{4,11}, introducing again the same pure tone. Figure 4 shows clearly an ACF with the form of a damped zeroth order Bessel function^{12,13}.

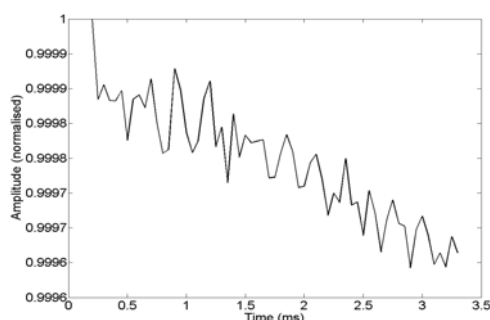


Figure 3: ACF resulting from a 1 kHz pure tone with no seeding

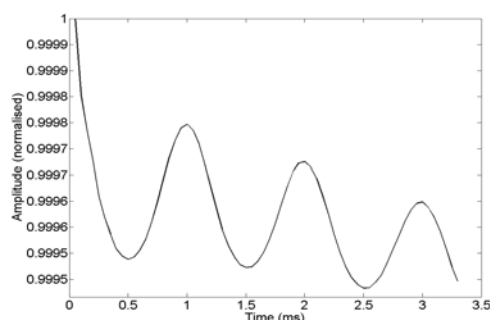


Figure 4: ACF resulting from a 1 kHz pure tone with seeding

For the particle velocity calculation, equation (8) is utilised, by first extracting the time required for the ACF to reach its first minimum, t_{min} . The y-axis scaling of figure 4 provides an indication that the ACF could potentially oscillate in a very narrow dynamic range, therefore reducing the repeatability and increasing the uncertainty of the measurement. Clearly, the ACF is very smooth in shape but it would be favourable to increase the dynamic range and therefore the resolution.

After experimentation with different tubes at different frequencies, with different optical powers and seeding duration, the conditions were set to give an optical power of just 1 mW in the secondary beams and with a seeding insertion duration of 0.5 second, proving an extremely low seeded environment. The SWT had a length of 1 m and an internal diameter of 40 mm, resulting in a volume of $1.26 \times 10^{-3} \text{ m}^3$. Smoke seeding was used from an incense stick with a short time allowed following the 0.5 second insertion for the particles to occupy the entire tube and also to minimise thermal flows that would potentially cause deviation on the measured result. The humidity and temperature were measured prior to seeding insertion to be 46% and 20°C respectively, resulting in a figure of 343.94 ms^{-1} for the speed of sound. It was assumed that the extremely low seeding level would have only a small influence on the speed of sound measurement.

For illustration, the result for a 170 Hz pure tone is presented in figure 5 for a relatively low loudspeaker drive level (50 mV drive voltage).

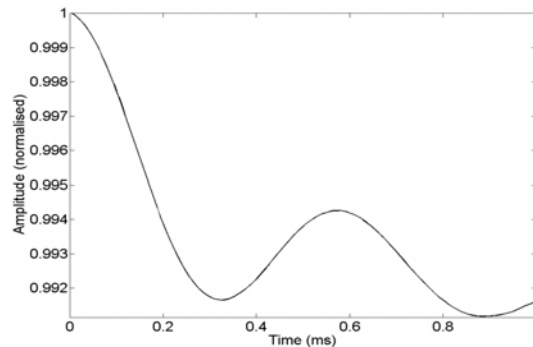


Figure 5: ACF obtained for a pure tone of 170 Hz with low seeding and low acoustic drive level

Different drive levels resulted into different ACFs and by using equation (8) the resulting acoustic velocities were calculated. The next step is to directly calculate the acoustic pressure in RMS, given by:

$$P_{rms} = \frac{u \cdot \rho \cdot c}{\sqrt{2} \cdot \sin\left(\frac{2 \cdot \pi \cdot f \cdot (l - x)}{c}\right)} \quad (10)$$

where u is the acoustic velocity, ρ is the density of the medium, c is the speed of sound, f is the acoustic frequency, l is the length of the tube and x is the distance of the probing region from the rigid end. Table 1 presents the results regarding the acoustic particle velocity measurements and pressure calculations.

| Function generator (V) | Particle velocity (mm s ⁻¹) | Pressure (Pa) | Pressure (dB re:20 µPa) |
|------------------------|---|---------------|-------------------------|
| 0.05 | 3.9 | 1.1 | 94.8 |
| 0.08 | 6.1 | 1.8 | 99.1 |
| 0.10 | 7.9 | 2.2 | 100.8 |
| 0.12 | 9.1 | 2.6 | 102.3 |
| 0.14 | 10.7 | 3.0 | 103.5 |
| 0.16 | 12.3 | 3.2 | 104.1 |
| 0.18 | 13.6 | 3.7 | 105.3 |
| 0.20 | 15.4 | 4.1 | 106.2 |

Table 1: Particle velocity measurements and resulting pressure levels in Pa and dB using the LDA system

In order to validate the results presented in Table 1, a microphone comparison was made. A ½ inch B&K 4134 working standard microphone calibrated using the reciprocity method was utilised and placed on the rigid end of the tube; as mentioned previously, the actual interference region was placed on a velocity anti-node which in a standing wave tube is a pressure node. For this reason, the pressure microphone was placed on the rigid end of the tube forming part of the termination (pressure anti-node). Table 2 presents the measured results.

By comparing tables 1 and 2, the agreement between LDA and reciprocity gives a figure of 0.2 dB, a figure that appears to be pressure independent.

| Function generator (V) | Pressure (Pa) | Pressure (dB re:20 μ Pa) |
|------------------------|---------------|------------------------------|
| 0.05 | 1.1 | 94.8 |
| 0.08 | 1.7 | 98.6 |
| 0.10 | 2.1 | 100.4 |
| 0.12 | 2.5 | 101.9 |
| 0.14 | 2.9 | 103.2 |
| 0.16 | 3.3 | 104.3 |
| 0.18 | 3.7 | 105.3 |
| 0.20 | 4.1 | 106.2 |

Table 2: Drive level for the loudspeaker: output from function generator and measured anti-nodal pressure in the SWT using laboratory pressure microphone

4 DISCUSSION AND FURTHER WORK

There are clearly a number of factors that contribute to the 0.2 dB figure presented above. Equation (7) provides an insight into possible causes.

It is assumed that all particles are of the same size, but in reality this is not the case. Not only different sizes will result in different photon counts but also particles larger than the fringe spacing will deteriorate the result. Also, although sufficient time was allowed for the particles to occupy the entire volume of the SWT, the combination of a minor mean flow component resulting in a inhomogeneities of the medium and an assumption inherent to the LDA technique that the particles faithfully follow the acoustic wave are more likely to be the dominant source of error in the measurement. The amplitude stability of the laser source can also be a source for error given the relatively long sampling times.

The generated ACFs correspond to the entire acoustic cycle, so contributions from different phases (therefore velocities) of the acoustic cycle were effectively averaged together. Even though one end of the SWT was rigidly terminated, the use of a small loudspeaker at its other end will most certainly introduce a minor phase shift on the standing wave cycle. In addition, the inclusion of a microphone for pressure comparison will also contribute. Effectively, this means that the velocity anti-node will be moving slightly with respect to the position of the interference volume. The acoustic frequency was matched to the actual length of the SWT, but by including given tolerances a minor mismatch could also be present. The purity of the acoustic tone introduced is another factor; the presence of minor harmonics produced by the loudspeaker also contribute. Although small, there is also an uncertainty associated with the calibrated sensitivity of the pressure microphone used for comparison.

Despite the above factors, very good agreement is observed. Naturally, the next step will be to extend the range of frequencies for comparison with reciprocity. A gating method which allows the velocity at different phases of the acoustic cycle should also be considered, so that the effect of the minor flow component can be further investigated and if necessary, separated from the acoustic velocity component.

Ideally, these measurements would be performed in a free-field environment and this is ultimately required if the technique is to be implemented as a future primary standard for free-field calibration of microphones. This poses significant challenges due to thermal flows and low scattering arising from natural seeding. The NPL system is currently be adapted for use in a pseudo-free field to investigate some of these issues. The relaxation in optical power requirements achieved through optimisation of seeding levels and optical alignment in this investigation are encouraging for use in free-field environments where seeding levels will be lower and forward scattering distances increased.

5 CONCLUSION

This paper has considered the development of an LDA system utilising the photon correlation method for the measurement of the acoustic particle velocity (and hence acoustic pressure) of airborne acoustic fields under minimal seeding conditions. The conditions of the propagation medium used for the current study more closely approximates to air than previous work and this allows a more realistic comparison with conventional methods. The acoustic pressure obtained from the LDA velocity measurement in a 1 m standing wave tube was compared to measurements made using a microphone, which was previously calibrated for its pressure response sensitivity using reciprocity. Encouraging agreement of around 0.2 dB is demonstrated between the two calibration methods.

The major challenge for the use of such an LDA technique as a future primary standard for the calibration of microphones arises in transferring to a free-field environment and reducing required seeding to natural levels. Photon correlation is better suited to these conditions as it relies less on particle concentration of the medium.

The authors gratefully acknowledge the financial support of the National Measurement System Programmes Unit of the UK Department for Innovation, Universities and Skills.

© Crown copyright 2008. Reproduced by permission of the Controller of HMSO.

6 REFERENCES

1. IEC 61094-2: 1992, "Measurement microphones - Part 2: Primary method for the pressure calibration of laboratory standard microphones by the reciprocity technique".
2. IEC 61094-3: 1995, "Measurement microphones - Part 3: Primary method for the free-field calibration of laboratory standard microphones by the reciprocity technique".
3. J. Taylor, "Absolute measurement of acoustic particle velocity", *Journal of the Acoustical society of America*, Vol 59, No. 3, 1976.
4. J. Taylor, "Absolute calibration of microphones by a laser-Doppler technique", *Journal of the Acoustical society of America*, Vol 70, No. 4, 1981.
5. C. Valiere, P. Herzog, V. Valeu, G. Tournois, "Acoustic velocity measurements in the air by means of laser Doppler velocimetry: dynamic and frequency range limitations and signal processing improvements", *Journal of Sound and Vibration*, 229 (3), 2000.
6. T. MacGillivray, D. Campbell, C. Greated, "The development of a microphone calibration technique using laser Doppler anemometry", *Acustica - Acta Acustica*, Vol. 88, No. 1, 2002.
7. P. Sharpe and C. A. Greated, "The measurement of periodic acoustic fields using photon correlation spectroscopy", *Journal of Physics D: Applied Physics*, 20, 1986.
8. P. Sharpe, C. A. Greated, D. M. Campbell, "The measurement of complex acoustic impedance using photon correlation spectroscopy", *Acustica*, Vol. 66, 1988.
9. P. Sharpe and C. A. Greated, "A stochastic model for photon correlation measurements in sound fields", *J. Phys. D: Appl. Physics*, 22, pp 1429-1433, 1989.
10. D. Hann and C. A. Greated, "Acoustic measurements in flows using photon correlation spectroscopy", *Meas. Sci. Technol.*, 4, pp 157-164, 1993.
11. T. MacGillivray, D. Campbell, C. Greated, R. Barham, "The development of a microphone calibration technique using photon correlation spectroscopy", *Acta Acustica*, Vol. 89, 2003.
12. J. P. Sharpe and C. A. Greated, "A stochastic model for photon correlation measurements in sound fields", *J. Phys. D: Appl. Physics*, 22, pp 1429-1433, 1989.
13. Durrani and Greated, "Laser systems in flow measurement", Plenum Press, 1977.
14. D. Hann and C. A. Greated, "Acoustic measurements in flows using photon correlation spectroscopy", *Meas. Sci. Technol.*, 4, pp 157-164, 1993.