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DERIVATION OF FOUR-POLE PARAMETERS FOR INCLUDING HIGHER-ORDER MODE EFFECTS ON ELLIPTICAL EXPANSION CHAMBER WITH MEAN FLOW

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1. INTRODUCTION

The elliptic cylindrical chamber of mufflers are generally used in the silencing systems for exhaust noise of automobiles. Its characteristics however have not yet been throughly investigated theoretically. A method to derive the four-pole parameters of the elliptical cylindrical chamber including the higher-order mode effects with mean flow is presented. Furthermore, the method is developed in the case of the circular cylindrical chamber.

2. ANALYTICAL APPROACH

The theory has been developed for the elliptic cylindrical chamber of eccentricity e_w and length ℓ which has two elliptical piston-driven of eccentricity e_0 and e_ℓ fitted at the center of the input and the output side as shown in Fig. 1. Assume that the boundaries of cylindrical chamber are rigid walls and the sound propagates without loss at the walls.

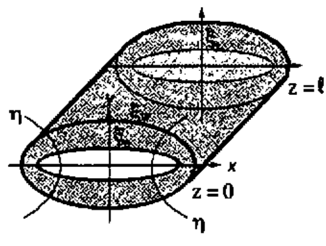

Fig. 1 Model of analysis

The wave-equation with flow velocity $\boldsymbol{V}$ is expressed as[1]

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi=\frac{1}{c}\left(\frac{\partial^2}{\partial t^2}+2\boldsymbol{V}\cdot\nabla\frac{\partial}{\partial t}+(\boldsymbol{V}\cdot\nabla)^2\right)\phi \quad (1)$$

where ϕ is the velocity potential and c is the sound velocity in the medium, respectively. Assume that the flow is a one-dimensional uniform flow in the chamber. The general solution for axially symmetric waves in the cylinder is given as follows.

$$\phi(\xi,\eta,z)=(A_0 \exp[(\beta+\gamma)z]+B_0 \exp[(\beta-\gamma)z]) \sum_{m=0}^{\infty} C_m Ce_m(\xi,s)ce_m(\eta,s) \quad (2)$$

$$\text{where} \quad \gamma=\frac{\sqrt{\mu^2(1-M^2)-k^2M^2}}{(1-M^2)} \;;\; \beta=-\frac{jkM}{(1-M^2)} \;;\; s=q^2\frac{(k^2+\mu^2)}{4} \quad (3)$$

A_0, B_0, C_m and μ are arbitrary constants, $ce_m(\eta,s)$ and $Ce_m(\xi,s)$ are the Mathieu function and the modified Mathieu function of mth-order, respectively. q is the distance between the foci and the origin, k is the wave number and M is the Mach number.

Let V_0 and V_ℓ be the components of particle velocity which propagates in the ξ and z direction, respectively. The boundary conditions are as follows

[1] $-\partial\phi/\partial\xi=0$ at $\xi=\xi_W$ (4)

[2] $-\partial\phi/\partial z=V_0 F_0(\xi,\eta)$ at $z=0$ (5)

[3] $-\partial\phi/\partial z=V_\ell F_\ell(\xi,\eta)$ at $z=\ell$ (6)

where

$$F_0(\xi,\eta)=\begin{cases}1 & (0\le\xi\le\xi_0, 0\le\eta\le2\pi)\\ 0 & (\xi_0<\xi\le\xi_W, 0\le\eta\le2\pi)\end{cases} \;;\; F_\ell(\xi,\eta)=\begin{cases}1 & (0\le\xi\le\xi_\ell, 0\le\eta\le2\pi)\\ 0 & (\xi_\ell<\xi\le\xi_W, 0\le\eta\le2\pi)\end{cases} \quad (7)$$

According to these above boundary conditions, the sound pressure distribution on the input and output piston are given by

$$\overline{P_0}=jZ_W\left[\frac{1}{\sin\alpha\ell}\{U_0(\cos\alpha\ell+jM\sin\alpha\ell)-\exp(-\beta\ell)U_\ell\}+\sum{}'\Delta_{2n,l}\left\{U_0Q_{0,0}\left(\frac{1}{\tanh\gamma_{2n,l}\ell}+\frac{\beta}{\gamma_{2n,l}}\right)-\frac{\exp(\beta\ell)}{\sinh\gamma_{2n,l}\ell}U_\ell Q_{0,\ell}\right\}\right] \quad (8)$$

$$\overline{P_\ell}=jZ_W\left[\frac{1}{\sin\alpha\ell}\{U_0\exp(-\beta\ell)-(\cos\alpha\ell+jM\sin\alpha\ell)\}+\sum{}'\Delta_{2n,l}\left\{U_0Q_{0,\ell}\times\frac{\exp(\beta\ell)}{\sinh\gamma_{2n,l}\ell}-\left(\frac{1}{\tanh\gamma_{2n,l}\ell}-\frac{\beta}{\gamma_{2n,l}}\right)U_\ell Q_{0,\ell}\right\}\right] \quad (9)$$

where

$$\alpha=\frac{k}{(1-M^2)} \qquad \Delta_{2n,l}=\frac{k\,\gamma_{2n,l}}{\beta^2-\gamma_{2n,l}^2} \quad (10)$$

$$\gamma_{2n,l}=\frac{1}{(1-M^2)\,a_W}\sqrt{(1-M^2)\,\lambda_{2n,l}^2-(k\,a_W)^2} \quad (11)$$

$$Q_{l,l} = \frac{\pi q^2}{2}\frac{S_W}{S_l S_l}\int_0^{\xi_l} Ce_{2n}(\xi, s_{2n,l})\left[2A_0^{(2n)}\cosh 2\xi - A_2^{(2n)}\right] d\xi \times \frac{\int_0^{\xi_l} Ce_{2n}(\xi, s_{2n,l})\left[2A_0^{(2n)}\cosh 2\xi - A_2^{(2n)}\right] d\xi}{\int_0^{\xi_W} Ce_{2n}^2(\xi, s_{2n,l})\left[\cosh 2\xi - \Theta_{2n}\right] d\xi} \tag{12}$$

U_0, S_0 are the volume velocity and the cross-sectional area at the input piston, U_ℓ, S_ℓ are those of the output piston, $Z_w=\rho c/S_w$, S_W is the cross-sectional area of the chamber, $A_l^{(2n)}$ and Θ_{2n} are constants. The symbolic Σ' means $\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}$ without $n=l=0$. The relationship between $\lambda_{2n,l}$ in Eq. (11) and e_W is illustrated in Fig. 2.

From Eqs.(8) and (9), the four-pole parametes can be expressed as follows

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_{in} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A_W & B_W \\ C_W & D_W \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_{out} \\ 0 & 1 \end{bmatrix} \tag{13}$$

where

$$A_W = \exp\left(-jM\frac{k\ell}{1-M^2}\right)\left[\cos\left(\frac{k\ell}{1-M^2}\right) + jM\sin\left(\frac{k\ell}{1-M^2}\right)\right] \tag{14}$$

$$B_W = jZ_W\exp\left(-jM\frac{k\ell}{1-M^2}\right)\left[(1+M^2)\sin\left(\frac{k\ell}{1-M^2}\right) - 2jM\cos\left(\frac{k\ell}{1-M^2}\right)\right] \tag{15}$$

$$C_W = j\frac{1}{Z_W}\exp\left(-jM\frac{k\ell}{1-M^2}\right)\sin\left(\frac{k\ell}{1-M^2}\right) \tag{16}$$

$$D_W = A_W \tag{17}$$

$$Z_{in} = Z_W \sum{}' \frac{k Q_{0,0}}{\gamma_{2n,l} + \left(\frac{kM}{1-M^2}\right)^2 \frac{1}{\gamma_{2n,l}}}\left(\frac{kM}{(1-M^2)\gamma_{2n,l}} + j\frac{1}{\tanh \gamma_{2n,l}\ell}\right) \tag{18}$$

$$Z_{out} = Z_W \sum{}' \frac{k Q_{\ell,\ell}}{\gamma_{2n,l} + \left(\frac{kM}{1-M^2}\right)^2 \frac{1}{\gamma_{2n,l}}}\left(\frac{-kM}{(1-M^2)\gamma_{2n,l}} + j\frac{1}{\tanh \gamma_{2n,l}\ell}\right) \tag{19}$$

When the cross section of chamber and both of the input and output pistons are in circular form, applying $e_l=1/\cosh\xi_l$ with $\xi_l \to \infty$, $Q_{0,0}$ and $Q_{\ell,\ell}$ in Eqs. (18) and (19) become

$$Q_{0,0} = \left(2\,\frac{r_w}{r_0}\,\frac{J_1(\lambda_{0,i} r_0/r_w)}{\lambda_{0,i}\, J_0(\lambda_{0,i})}\right)^2 \quad ; \quad Q_{\ell,\ell} = \left(2\,\frac{r_w}{r_\ell}\,\frac{J_1(\lambda_{0,i}\, r_\ell / r_w)}{\lambda_{0,i}\, J_0(\lambda_{0,i})}\right)^2 \qquad (20)$$

where r_0, r_ℓ and r_W are the radius of input, output piston and chamber, respectively, $J_n(x)$ is the Bessel function of nth-order.

An example of Z_{in} is shown in Fig. 3 where the elliptical chamber has e_W=0.5 and S_0/S_W=0. At $ka_W<\lambda_{2,0}$, Z_{in} increases corresponding to the increases of ka_W and it suddenly becomes great in frequency range near $\lambda_{2,0}$, especially for large value of M. Characteristic of Z_{in} have an effect on the fundamental resonance frequency of the four-pole parameters[2].

3. CONCLUSION

The four-pole parameters is given by Eq. (13) which is composed of the product of the input, chamber and the output section matrices. The matrices of input and output section include the term of higher-order modes which are defined by Eq. (18) for elliptical cylinder and Eq. (20) for circular cylinder, respectively.

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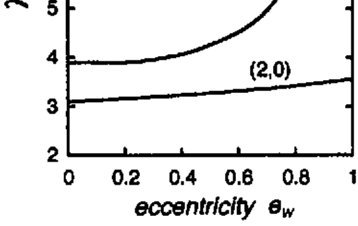

Fig. 2 resonance of even waves

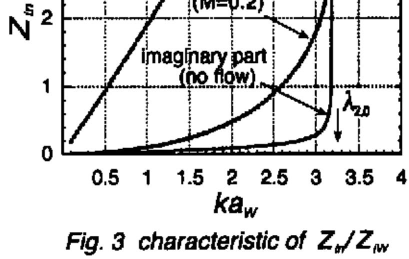

Fig. 3 characteristic of Z_{in}/Z_{iw}
(e_w=0.5, ℓ=0.4cm, S_0/S_w=0.21)