AN ITERATIVE SOLUTION TECHNIQUE FOR COUPLED VIBRO-ACOUSTIC ANALYSIS

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1 INTRODUCTION

The Finite Element Method (FEM) and Boundary Element Method (BEM) are commonly used as numerical prediction techniques for vibro-acoustic analysis and design. Due to the large model sizes and subsequent computational loads, the practical use of these techniques is restricted to low-frequency applications. However, there is a strong industrial need to extend vibro-acoustic analyses towards higher frequencies and to allow efficient design optimisation through fast recalculations. This tendency, along with the ever increasing performance of modern computer resources, turns parallel computation methods into an appealing approach for efficient vibro-acoustic calculations over a broad frequency range. Such an approach puts some particular requirements to the numerical solution schemes to become amenable to parallel computation.

In the last decades, parallel computation has been a major topic of research and a large amount of techniques have been developed to perform (vibro-)acoustic analysis. One of the most successful techniques is the Domain Decomposition Technique[1] which divides a global acoustic domain into several (non-overlapping) subdomains and enables the total computation cost to be distributed among several processors, each of them handling one particular acoustic subdomain problem. The steady-state acoustic behaviour of each subdomain is governed by the Helmholtz equation with suitable boundary conditions[2]. Besides the problem boundary conditions, additional boundary conditions must be specified at the interfaces between the various subdomains to restore the subdomain connectivity. Coupling the subdomains is achieved by solving a so-called ‘interface problem’. The solution on the global domain is therefore governed by the solution at the interfaces. A particular non-overlapping domain decomposition method called Finite Element Tearing and Interconnecting (FETI)[3] uses a direct solver for the local problems and an iterative method to deal with the interface problem. In each iteration step, one independent local problem per subdomain is solved directly. A global problem, which is usually preconditioned and evaluates the residuals of the interface continuity constraints, is solved iteratively and drives the iteration process until convergence has been reached.

This paper explores the feasibility of a domain decomposition approach for coupled (interior) vibro-acoustic problems. The basic idea is to separate the coupled vibro-acoustic problem into two distinctive domains, i.e. a structural and an acoustic problem, which can be solved in separate solution schemes that could be distributed over different processors. The explored approach is based on an iterative algorithm that involves only updates of the Right Hand Sides (RHS) of the two subdomain problems that are solved directly. The paper discusses the feasibility and convergence performance of such an approach through a simple one-dimensional validation example. Parallel implementation issues and subsequent performance analysis have not been addressed yet.

2 DIRECT SOLUTION SCHEME FOR COUPLED VIBRO-ACOUSTIC PROBLEMS

The most commonly used Eulerian formulation of a coupled vibro-acoustic problem leads to the following finite element formulation in physical co-ordinates[4] :

$$\begin{bmatrix} [K]+i\omega[C]-\omega^2[M] & [L] \\ -\omega^2\rho_o[L]^T & [H]+i\omega\rho_o[A]-\omega^2[Q] \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} F_s \\ F_a \end{Bmatrix} \tag{1}$$

where K, C and M are, respectively, the structural stiffness, damping and mass matrices; H, A and Q are, respectively, the fluid compressibility, acoustic damping and fluid inertia matrices. L is the coupling matrix between the structure and the cavity, ρ_o is the fluid density and ω the angular frequency. u is the vector of unknown nodal structural displacements and p the vector of unknown nodal acoustic pressures.
This matrix equation can be solved directly for the unknown structural displacements and acoustic pressures. Note however that, in contrast with uncoupled structural and uncoupled acoustic models, the direct solution of a coupled model of type (1) is much more expensive. This is not only due to the fact that the total model size is much larger since an acoustic and a structural problem must be solved simultaneously, but also due to the fact that the symmetry of the matrix equations is no longer preserved.

3 SOME ITERATIVE SOLUTION SCHEMES FOR COUPLED VIBRO-ACOUSTIC PROBLEMS

3.1 Scheme 1

From the global coupled matrix equation (1), a structural and an acoustic subproblem may be formulated as:

$$\left([K]+i\omega[C]-\omega^2[M]\right)\{u\}=\{F_s\}-[L]\{p\} \tag{2}$$

$$\left([H]+i\omega\rho_o[A]-\omega^2[Q]\right)\{p\}=\{F_a\}+\omega^2\rho_o[L]^T\{u\} \tag{3}$$

The left hand sides of these two subdomain equations involve sparse, symmetric matrices with possible complex coefficients. To facilitate the system solution, the left hand sides of both sub-domain equations can be preconditioned by using a complete Cholesky factorisation. In an iterative solution scheme, this factorisation needs to be executed only once, thus providing a gain in time and cost of computation. At a given iteration step *i*, the RHS of each subdomain problem can be updated according to the following formulas:

$$\left([K]+i\omega[C]-\omega^2[M]\right)\{u\}_i=\{F_s\}-[L]\{p\}_{i-1} \tag{4}$$

$$\left([H]+i\omega\rho_o[A]-\omega^2[Q]\right)\{p\}_i=\{F_a\}+\omega^2\rho_o[L]^T\{u\}_{i-1} \tag{5}$$

Updating terms results from back-substitutions at a previous iteration step, with initial conditions:

$$\{u\}_0=\left([K]+i\omega[C]-\omega^2[M]\right)^{-1}.\{F_s\} \tag{6}$$

$$\{p\}_0=\left([H]+i\omega\rho_o[A]-\omega^2[Q]\right)^{-1}.\{F_a\} \tag{7}$$

However, the above developed iterative scheme may be subjected to instabilities in the neighbourhood of any uncoupled acoustic and uncoupled structural resonance, where the system matrices of equations (2) and (3) become singular or nearly singular, such that the convergence of the iterative scheme is prohibited.

3.2 Scheme 2

Introducing the structural impedance matrix $[Z_s(\omega)]=([K]+i\omega[C]-\omega^2[M])$ and the acoustic impedance matrix $[Z_a(\omega)]=([H]+i\omega\rho_0[A]-\omega^2[Q])$, the coupled model (1) becomes:

$$\begin{bmatrix} [Z_s(\omega)] & [L] \\ -\omega^2\rho_o[L]^T & [Z_a(\omega)] \end{bmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix}=\begin{Bmatrix} F_s \\ F_a \end{Bmatrix} \tag{8}$$

leading to the following two subproblem formulations:

$$[Z_s(\omega)].\{u\}=\{F_s\}-[L]\{p\} \tag{9}$$

$$[Z_a(\omega)].\{p\}=\{F_a\}+\omega^2\rho_o[L]^T\{u\} \tag{10}$$

When $Z_a(\omega)$ and $Z_s(\omega)$ are non-singular, equations (9) and (10) can be rewritten as:

$$\{u\}=[Z_s(\omega)]^{-1}.\{F_s\}-[Z_s(\omega)]^{-1}.[L]\{p\} \tag{11}$$

$$\{p\}=[Z_a(\omega)]^{-1}.\{F_a\}+\omega^2\rho_o[Z_a(\omega)]^{-1}.[L]^T\{u\} \tag{12}$$

Let's assume that one can define non-singular approximations $[\tilde{Z}_s(\omega)]^{-1}$ and $[\tilde{Z}_a(\omega)]^{-1}$ of $Z_s(\omega)^{-1}$ and $Z_a(\omega)^{-1}$, respectively. By defining some new vectors of unknowns $\{\tilde{u}\}$ and $\{\tilde{p}\}$ as:

$$\{u\}=-[\tilde{Z}_s(\omega)]^{-1}.[L]\{p\}+\{\tilde{u}\} \tag{13}$$

$$\{p\}=\omega^2\rho_o[\tilde{Z}_a(\omega)]^{-1}.[L]^T\{u\}+\{\tilde{p}\} \tag{14}$$

equations (9) and (10) can be modified to:

$$\left([Z_s(\omega)]+\omega^2\rho_0[L][\tilde{Z}_a(\omega)]^{-1}[L]^T\right).\{u\}=\{F_s\}-[L]\{\tilde{p}\} \tag{15}$$

$$\left([Z_a(\omega)]+\omega^2\rho_0[L]^T[\tilde{Z}_s(\omega)]^{-1}[L]\right).\{p\}=\{F_a\}+\omega^2\rho_0[L]^T\{\tilde{u}\} \tag{16}$$

Therefore, one can develop the following iterative scheme:

$$\left([Z_s(\omega)]+\omega^2\rho_0[L][\tilde{Z}_a(\omega)]^{-1}[L]^T\right).\{u\}_i=\{F_s\}-[L]\{\tilde{p}\}_{i-1} \tag{17}$$

$$\left([Z_a(\omega)]+\omega^2\rho_0[L]^T[\tilde{Z}_s(\omega)]^{-1}[L]\right).\{p\}_i=\{F_a\}+\omega^2\rho_0[L]^T\{\tilde{u}\}_{i-1} \tag{18}$$

with

$$\{\tilde{u}\}_{i-1}=\{u\}_{i-1}+[\tilde{Z}_s(\omega)]^{-1}.[L]\{p\}_{i-1} \tag{19}$$

$$\{\tilde{p}\}_{i-1}=\{p\}_{i-1}-\omega^2\rho_o[\tilde{Z}_a(\omega)]^{-1}.[L]^T\{u\}_{i-1} \tag{20}$$

and with the following initial conditions:

$$\left(\left[Z_s(\omega)\right]+\omega^2\rho_0\left[L\right]\left[\widetilde{Z}_a(\omega)\right]^{-1}\left[L\right]^T\right).\{u\}_0=\{F_s\} \tag{21}$$

$$\left(\left[Z_a(\omega)\right]+\omega^2\rho_0\left[L\right]^T\left[\widetilde{Z}_s(\omega)\right]^{-1}\left[L\right]\right).\{p\}_0=\{F_a\} \tag{22}$$

With an appropriate definition of the approximations $[\tilde{Z}_s(\omega)]^{-1}$ and $[\tilde{Z}_a(\omega)]^{-1}$ possible convergence problems near uncoupled acoustic and uncoupled structural resonance frequencies, as encountered in iterative scheme 1, could be circumvented. However, due to the appearance of the $[L][..][L]^T$ matrix products in the left hand sides of the subdomain matrix equations (17) and (18), the sparsity of the original subdomain impedance matrices $Z_a(\omega)$ and $Z_s(\omega)$ can be altered. Indeed, with an appropriate numbering of the nodes in the FE discretisation, the non-zero entries of matrices can appear in a narrow band around the matrix diagonal yielding sparsely populated, banded matrices. The introduction of the matrix product terms can however lead to a non-sparse or even asymmetric complement that is added to the original subdomain system matrix. This may limit factorisation schemes and the use of efficient solvers. It is therefore essential to choose a judicious complementary term so as not to significantly increase the computation cost as well as to avoid singularities of the complementary term and its transformed system.

3.3 Scheme 3

A third iterative solution scheme starts from defining at the fluid-structure coupling interface two novel variables, i.e. a displacement vector $\vec{\tilde{u}}$ and an acoustic pressure $\widetilde{p}$, that are related to the original problem variables through:

$$\vec{u}=\left(\widetilde{Z}_s\right)^{-1}p.\vec{n}+\vec{\tilde{u}} \tag{23}$$

$$p=\omega^2\rho_o\left(\widetilde{Z}_a\right)^{-1}\vec{u}.\vec{n}+\widetilde{p} \tag{24}$$

with $(\tilde{Z}_s)^{-1}$ and $(\tilde{Z}_a)^{-1}$ scalar values and with $\vec{n}$ the normal vector to the coupling interface Ω_c.

The weak form of the weighted residual formulation of the structural part of a coupled vibro-acoustic problem can be expressed as:

$$\{\hat{u}\}^T\left[Z_s\right]\{u\}=\{\hat{u}\}^T\{F_s\}+\int_{\Omega_c}\vec{\hat{u}}.\vec{n}.p.d\Omega \tag{25}$$

in which the finite element shape function approximations of the structural displacement $\vec{u}\approx\left[N_s\right]\{u\}$ and of the weighting function $\vec{\hat{u}}\approx\left[N_s\right]\{\hat{u}\}$ have been used.

Based on (24), the integral in the RHS of the weak form (25) can be expressed as:

$$\int_{\Omega_c}\vec{\hat{u}}.\vec{n}.p.d\Omega=\int_{\Omega_c}\vec{\hat{u}}.\vec{n}.\left(\omega^2\rho_o\left(\widetilde{Z}_a\right)^{-1}\vec{u}.\vec{n}+\widetilde{p}\right).d\Omega=\omega^2\rho_o\int_{\Omega_c}\left(\widetilde{Z}_a\right)^{-1}\vec{\hat{u}}.\vec{n}.\vec{u}.\vec{n}.d\Omega+\int_{\Omega_c}\vec{\hat{u}}.\vec{n}.\widetilde{p}.d\Omega \tag{26}$$

By using a similar finite element shape function approximation for the novel variable $\widetilde{p}$ as for the acoustic pressure, i.e. $\widetilde{p}=\left[N_a\right]\{\widetilde{p}\}$, expression (26) takes the form:

$$\omega^2\rho_0\{\hat{u}\}^T\int_{\Omega_c}\left(\widetilde{Z}_a\right)^{-1}\left[N_s\right]^T\{n_e\}\{n_e\}^T\left[N_s\right]\{u\}.d\Omega+\{\hat{u}\}^T\int_{\Omega_c}\left[N_s\right]^T\{n_e\}\left[N_a\right]\{\widetilde{p}\}.d\Omega \tag{27}$$

Substituting (27) into (25) yields to the following weak form expression (28):

$$\{\hat{u}\}^T[Z_s]\{u\}=\{\hat{u}\}^T\{F_s\}+\omega^2\rho_0\{\hat{u}\}^T\left(\int_{\Omega_c}\left(\widetilde{Z}_a\right)^{-1}[N_s]^T\{n_e\}\{n_e\}^T[N_s].d\Omega\right)\{u\}+\{\hat{u}\}^T\left(\int_{\Omega_c}[N_s]^T\{n_e\}[N_a].d\Omega\right)\{\widetilde{p}\}$$

Since the above weighted formulation should hold for any expansion of the weighting function, a set of n_s equations in the n_s nodal structural unknowns and n_a nodal acoustic unknowns is obtained:

$$[Z_s]\{u\}=\{F_s\}+\omega^2\rho_0\left(\int_{\Omega_c}\left(\widetilde{Z}_a\right)^{-1}[N_s]^T\{n_e\}\{n_e\}^T[N_s].d\Omega\right)\{u\}+\left(\int_{\Omega_c}[N_s]^T\{n_e\}[N_a].d\Omega\right)\{\widetilde{p}\} \tag{29}$$

which can be reformulated as:

$$\left([Z_s]-\omega^2\rho_0[Z_1]\right)\{u\}=\{F_s\}+[L]\{\widetilde{p}\} \tag{30}$$

where

$$[Z_1]=\left(\int_{\Omega_c}\left(\widetilde{Z}_a\right)^{-1}[N_s]^T\{n_e\}\{n_e\}^T[N_s].d\Omega\right) \quad \text{and} \quad [L]=\int_{\Omega_c}[N_s]^T\{n_e\}[N_a].d\Omega$$

The weak form of the weighted residual formulation of the acoustic part of a coupled vibro-acoustic problem can be expressed as:

$$\{\hat{p}\}^T[Z_a]\{p\}=\{\hat{p}\}^T\{F_a\}+\omega^2\rho_0\int_{\Omega_c}\hat{p}.\vec{u}.\vec{n}.d\Omega \tag{31}$$

in which the finite element shape function approximation of the weighting function $\hat{p}\approx[N_a]\{\hat{p}\}$ has been used.

Based on (23), the integral in the RHS of the weak form (31) can be expressed as:

$$\omega^2\rho_0\int_{\Omega_c}\hat{p}.\vec{u}.\vec{n}.d\Omega=\omega^2\rho_0\int_{\Omega_c}\hat{p}.(\left(\widetilde{Z}_s\right)^{-1}p.\vec{n}+\vec{\widetilde{u}}).\vec{n}.d\Omega=\omega^2\rho_0\int_{\Omega_c}\left(\widetilde{Z}_s\right)^{-1}.\hat{p}.p.d\Omega+\omega^2\rho_0\int_{\Omega_c}\hat{p}.\vec{\widetilde{u}}.\vec{n}.d\Omega \tag{32}$$

By using a similar finite element shape function approximation for the novel variable $\vec{\widetilde{u}}$ as for the structural displacement, i.e. $\vec{\widetilde{u}}=[N_s]\{\widetilde{u}\}$, expression (32) takes the form:

$$\omega^2\rho_0\{\hat{p}\}^T\int_{\Omega_c}\left(\widetilde{Z}_s\right)^{-1}[N_a]^T[N_a]\{p\}.d\Omega+\omega^2\rho_0\{\hat{p}\}^T\int_{\Omega_c}[N_a]^T\{n_e\}^T[N_s]\{\widetilde{u}\}.d\Omega \tag{33}$$

Substituting (33) into (31) yields the following weak form expression (34):

$$\{\hat{p}\}^T[Z_a]\{p\}=\{\hat{p}\}^T\{F_a\}+\omega^2\rho_0\{\hat{p}\}^T\left(\int_{\Omega_c}\left(\widetilde{Z}_s\right)^{-1}[N_a]^T[N_a].d\Omega\right)\{p\}+\omega^2\rho_0\{\hat{p}\}^T\left(\int_{\Omega_c}[N_a]^T\{n_e\}^T[N_s].d\Omega\right)\{\widetilde{u}\}$$

Since the above weighted formulation should hold for any expansion of the weighting function, a set of n_a equations in the n_s nodal structural unknowns and n_a nodal acoustic unknowns is obtained:

$$[Z_a]\{p\} = \{F_a\} + \omega^2\rho_0\left(\int_{\Omega_c}\left(\tilde{Z}_s\right)^{-1}[N_a]^T[N_a]d\Omega\right)\{p\} + \omega^2\rho_0\left(\int_{\Omega_c}[N_a]^T\{n_e\}^T[N_s]d\Omega\right)\{\tilde{u}\} \quad (35)$$

which can be reformulated as:

$$([Z_a] - \omega^2\rho_0[Z_2])\{p\} = \{F_a\} + \omega^2\rho_0[L]^T\{\tilde{u}\} \quad (36)$$

where

$$[Z_2] = \int_{\Omega_c}\left(\tilde{Z}_s\right)^{-1}[N_a]^T[N_a]d\Omega \quad \text{and} \quad [L] = \int_{\Omega_c}[N_s]^T\{n_e\}[N_a]d\Omega$$

Based on the above formulations (30) and (36) of the subdomain problems, one can develop the following iteration scheme:

$$([Z_s] - \omega^2\rho_0[Z_1])\{u\}_i = \{F_s\} + [L]\{\tilde{p}\}_{i-1} \quad (37)$$

$$([Z_a] - \omega^2\rho_0[Z_2])\{p\}_i = \{F_a\} + \omega^2\rho_0[L]^T\{\tilde{u}\}_{i-1} \quad (38)$$

with

$$\{\tilde{u}\}_{i-1} = \{u\}_{i-1} - \left(\tilde{Z}_s\right)^{-1}[L]\{p\}_{i-1} \quad (39)$$

$$\{\tilde{p}\}_{i-1} = \{p\}_{i-1} - \omega^2\rho_o\left(\tilde{Z}_a\right)^{-1}[L]^T\{u\}_{i-1} \quad (40)$$

and with the following initial conditions:

$$([Z_s] - \omega^2\rho_0[Z_1])\{u\}_0 = \{F_s\} \quad (41)$$

$$([Z_a] - \omega^2\rho_0[Z_2])\{p\}_0 = \{F_a\} \quad (42)$$

With an appropriate definition of the matrices [Z_1] and [Z_2] and hence of the scalar values $\tilde{Z}_s$ and $\tilde{Z}_a$, possible convergence problems near uncoupled acoustic and uncoupled structural resonance frequencies, as encountered in iterative scheme 1, could be circumvented. In addition, the sparsity of the original subsystem impedance matrices $Z_a(\omega)$ and $Z_s(\omega)$ is preserved in the modified subsystem matrices in (37) and (38), as opposed to the previously described iterative scheme 2.

4 ONE-DIMENSIONAL VALIDATION EXAMPLE

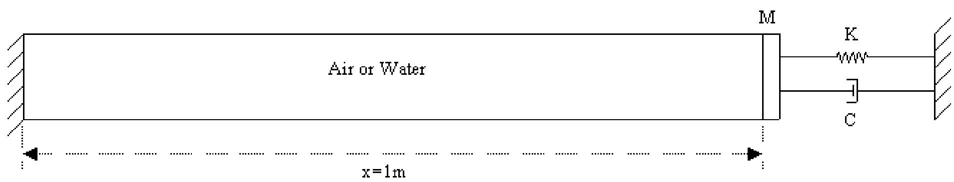

Figure 4.1: one-dimensional validation example

The above described iterative scheme 3 has been implemented in MATLAB for the one-dimensional validation case, shown in figure 4.1. One side of a 1 meter long fluid-filled tube ends with an elastically suspended piston-like structure, which is dynamically excited with a mechanical force. A force of 1 [N] was applied on the piston-like structure which features, per unit area, a spring stiffness

K of 400000 [N][m]$^{-3}$, a mechanical damping C of 50 [N][s][m]$^{-3}$ and a piston mass M of 1 [kg][m]$^{-2}$. The other end of the tube is rigid. The tube cavity was filled with air of density of 1.2250 [kg][m]$^{-3}$ or with water of density 1000 [kg][m]$^{-3}$. The speed of sound was considered to be in air 340 [m][s]$^{-1}$ and in water 1480 [m][s]$^{-1}$. The tube cavity was discretised by 20 linear elements.
The validation case allows to investigate the performance of the iterative scheme 3 for various values of the scalars $\tilde{Z}_a$ and $\tilde{Z}_s$ through comparison with the analytical expression for the exact solution of this one-dimensional coupled vibro-acoustic problem.

Three different types of (frequency-dependent) scalar values $\tilde{Z}_a$ and $\tilde{Z}_s$ have been considered:

- The determinant of the uncoupled impedance matrices, i.e.

$$\tilde{Z}_s = \det([Z_s(\omega)]) \quad \text{and} \quad \tilde{Z}_a = \det([Z_a(\omega)]) \tag{43}$$

  Since these determinant values may become zero at the uncoupled structural and acoustic resonance frequencies, a small offset is added to prevent the modified subsystem matrices in (37) and (38) to become singular. Note that the calculation of these frequency-dependent determinants may become a computationally expensive part of the iterative scheme.

- The traces of the uncoupled impedance matrices, i.e.

$$\tilde{Z}_s = Tr([Z_s(\omega)]) \quad \text{and} \quad \tilde{Z}_a = Tr([Z_a(\omega)]) \tag{44}$$

  which are less computationally demanding than the calculation of the previously defined values in (43).

- A third procedure is based on the modal stiffnesses and masses of a certain number of uncoupled structural and uncoupled acoustic modes, i.e.

$$\tilde{Z}_s = \prod_{i=1}^{N_s} \left(k_{si} - \omega^2 m_{si}\right) \quad \text{and} \quad \tilde{Z}_a = \prod_{i=1}^{N_a} \left(k_{ai} - \omega^2 m_{ai}\right) \tag{45}$$

  where k_{si} and m_{si} represent, respectively, the modal stiffness and modal mass values of the uncoupled structural mode *i* and where k_{ai} and m_{ai} represent, respectively, the modal stiffness and modal mass values of the uncoupled acoustic mode *i*. The calculation of the modal information is a computationally expensive procedure. However, for most practical vibro-acoustic engineering applications, the size of the coupled model (1) is that large that model size reduction techniques are mandatory. The most commonly used reduction technique is based on component mode synthesis in which the structural displacements and acoustic pressures are projected onto a base of uncoupled structural and uncoupled acoustic modes, respectively. Hence, the modal information is available anyway, even when considering a direct solution scheme, such that the additional cost of calculating the proposed modal based scalars in (45) is minor.

Figure 4.2 compares the results for an air-filled tube, obtained with a direct solution scheme as well as with the iterative scheme 3, using the determinant based and the trace based scalar values defined in (43) and (44). Both iterative methods seem to converge towards the results, obtained with a direct solution scheme. Figure 4.3 plots the frequency-dependent scalar values $\tilde{Z}_a$ and $\tilde{Z}_s$ that have been used in both iterative methods. The number of iteration steps to reach convergence is shown in figure 4.4. The stop criterion for convergence was set to a relative error of 0.1% between two successive iteration. It can be seen that the convergence behaviour of the determinant based approach is superior to the trace based approach, especially in the vicinity of resonances. This may be understood from the fact that the scalars in the determinant based approach reflect more the frequency-dependent resonant behaviour of the subdomains, as shown in figure 4.3.

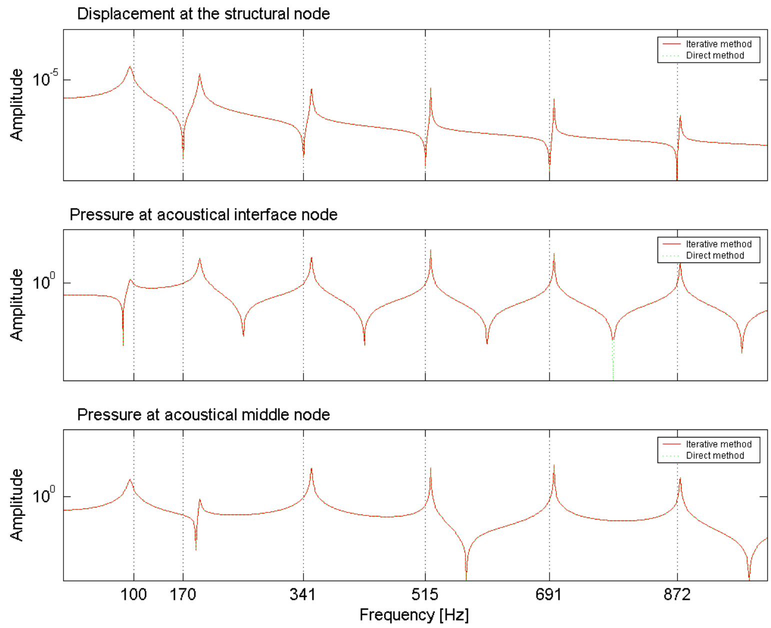

Figure 4.2: direct and iterative results for an air-filled tube

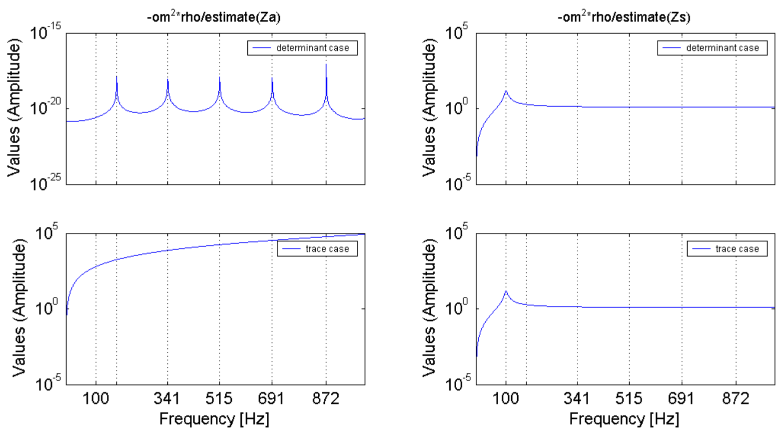

Figure 4.3: scalar values $\tilde{Z}_a$ and $\tilde{Z}_s$ for an air-filled tube (according to (43) and (44))

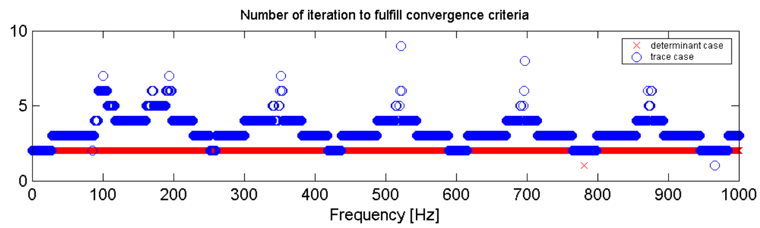

Figure 4.4: number of iteration steps, needed for convergence

Figure 4.5 plots the frequency response functions of the piston displacement, obtained with a direct solution scheme, against the results obtained with iterative solution scheme 3, using the modal based scalar values defined in (45). Since the structural part in the validation example has only one degree of freedom, only one structural mode can be considered (N_s=1), while a variable number of acoustic modes N_a has been considered. Figure 4.6 plots the frequency-dependent scalar values $\tilde{Z}_a$ and $\tilde{Z}_s$ that have been used in the modal based scheme. The corresponding number of iteration steps to reach convergence is shown in figure 4.7. It can be observed that a sufficient number of modes must be taken into account in the scalar value calculations according to equation (45) in order to obtain a reasonable convergence speed. Note that the acoustic rigid body mode had to be discarded to preserve computational stability.

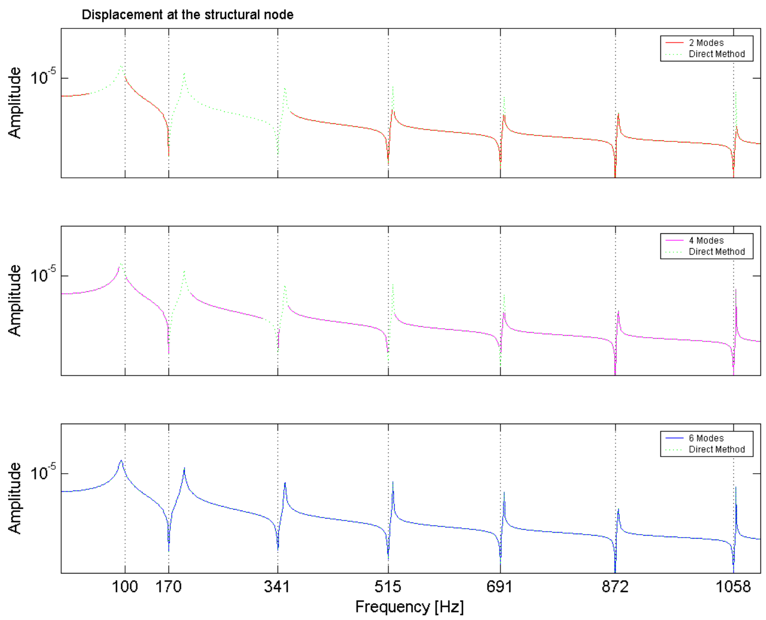

Figure 4.5: direct and iterative results for an air-filled tube (modal based scheme)

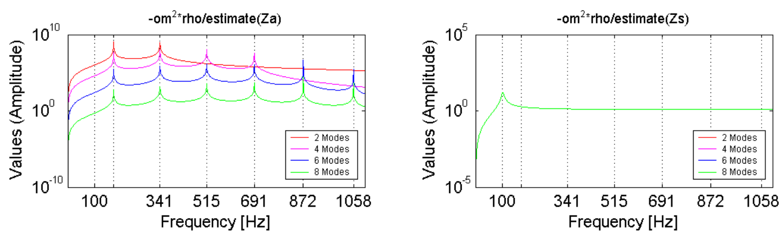

Figure 4.6: scalar values $\tilde{Z}_a$ and $\tilde{Z}_s$ for an air-filled tube (according to (45))

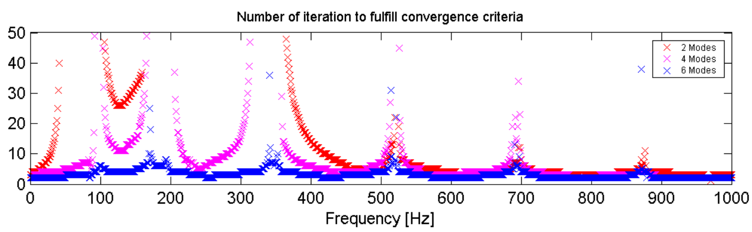

Figure 4.7: number of iteration steps, needed for convergence (modal based scheme)

Some additional investigations have been made for the case of a water-filled tube, in which stronger fluid-structure coupling effects occur. It was observed that in this case the trace based procedure does not ensure convergence, as opposed to the modal based procedure.
Figures 4.8, 4.9 and 4.10 confirm the findings that a modal based iterative scheme with a sufficient number of modes taken into account in the calculation of the scalar values according to equation (45) ensures convergence.

Note that the above conclusions must be considered with some precaution as the scalar term $\tilde{Z}_a$ only controls a single resonance of the structural subdomain. Indeed, the structural system is modelled by a single degree of freedom in this one-dimensional validation case. The method is nevertheless thought to adjust itself to general three-dimensional cases.

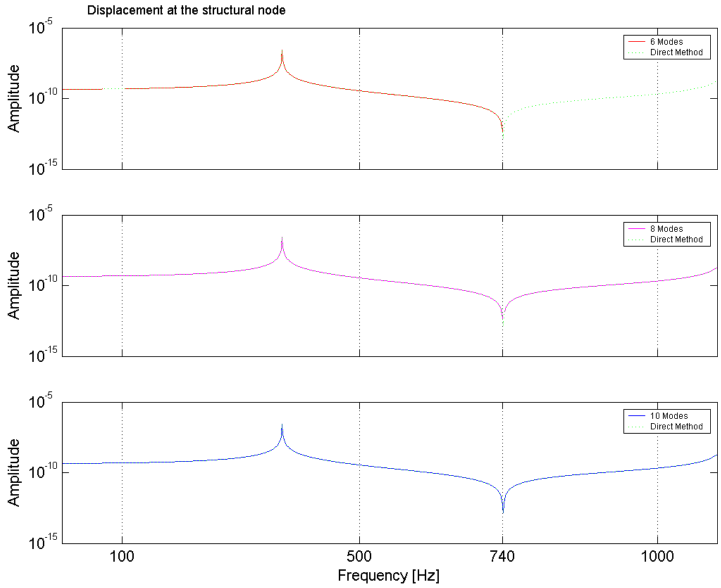

Figure 4.8: direct and iterative results for a water-filled tube (modal based scheme)

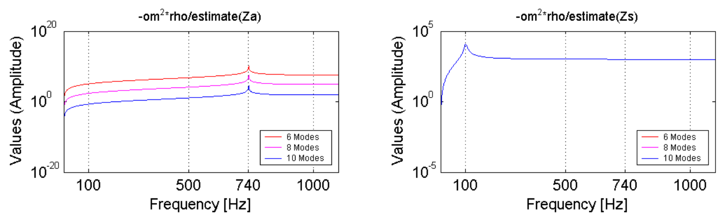

Figure 4.9: scalar values $\tilde{Z}_a$ and $\tilde{Z}_s$ for a water-filled tube (according to (45))

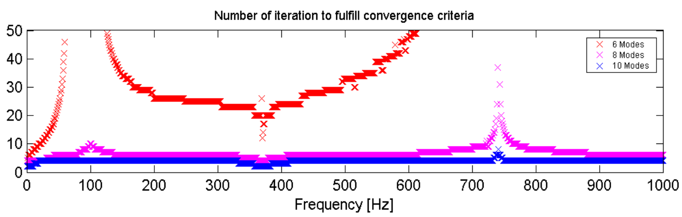

Figure 4.10: number of iteration steps, needed for convergence (modal based scheme)

5 CONCLUSION AND NEXT STEPS

This paper evaluates some (finite element based) iterative solution schemes for coupled vibro-acoustic analysis that are amenable to parallel implementation, such that an efficient prediction tool can be obtained for use over a broad frequency range.

A coupled system is decomposed into two subdomains, i.e. the structural and the acoustic part. In each iteration step, an acoustic and a structural subdomain problem is solved, using a right-hand side that is updated with the results from the previous iteration step. In order to avoid convergence problems due to singularities of the subdomain system matrices around resonances, an iterative scheme has been defined to circumvent these singularity problems through the introduction of some additional subdomain matrices that preserve the sparsity of the original structural and acoustic subdomain system matrices. In this way, an efficient solution scheme can be used in which a Cholesky factorisation of the system matrices is performed once and where conventional back-substitutions are used to solve the acoustic and the structural subdomain problems in each iteration step.

A one-dimensional validation example has been studied and revealed that a computationally efficient, converging iteration scheme can be obtained by using additional subsystem matrices based on modal stiffness and modal mass information of the uncoupled acoustic and structural problems.

Next steps will focus on the evaluation of the convergence performance of the proposed iterative schemes for three-dimensional problems and on the assessment of the computational gain of parallel implementations as compared to conventional direct solution strategies.

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