DESIGNING DIFFUSERS IN THE TIME DOMAIN

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1 INTRODUCTION

Manfred Schroeder's pioneering paper from 1975 on number theoretic diffusers begins, "What wall shape has the highest possible sound diffusion in the sense that an incident wave from any direction is scattered evenly in all directions?"[1] This was then the catalyst for many studies that examined how to get surfaces to scatter sound efficiently in all directions.

From the selfish standpoint of a listener hearing image shift, echoes or coloration, what is important is not a diffuser's ability to spatially distribute the sound, but how the diffuser alters the impulse response between the sound source and the listener. If a diffuser achieves spatial scattering through a complex surface profile, then inevitably the impulse response to listeners will be significantly altered as a by-product of meeting Schroeder's requirement for high sound diffusion. But what happens if a diffuser is designed first and foremost for the selfish listener, rather than for some desired reflected directivity? This first exploration of this idea focuses on Schroeder diffusers.

2 PREDICTION MODELS

In recent years there has been great interest in time domain solutions of the wave equation, especially Finite Difference Time Domain (FDTD) models. This is one motivation for examining diffuser design in the time domain. While FDTD is a popular wave-based approach, it uses a volumetric mesh and an iterative solution, which means calculation times can become excessively long. To allow more efficient exploration of impulse responses, new time-domain formulations have been derived that exploit the Kirchhoff boundary conditions[2].

The sound field at receiver point $\boldsymbol{r}$ and time t in the vicinity of a diffuser S is represented by the pressure $p_t(\boldsymbol{r},t) = p_i + p_s$, where p_i represents the sound travelling directly from the source to the receiver along vector $\boldsymbol{r_d}$, and p_s the sound scattered off the surface. As is common in many time-domain approaches, the source emits a Gaussian pulse:

$$p_i(\boldsymbol{r},t) = \frac{1}{4\sqrt{2}\pi^{\frac{3}{2}}\sigma r_d} e^{-\frac{(t-r_d/c)^2}{2\sigma^2}} \qquad (1)$$

where c is the speed of sound and σ controls the width of the source pulse. σ is chosen so the source creates sufficient energy only over the bandwidth of interest.

The scattered pressure is calculated by discretising the front face of the Schroeder diffuser into N elements whose dimensions are small compared to wavelength. Surface discretization like this is done for many frequency domain models. If the n^{th} element is at the mouth of a well of depth d_n then it can be shown that the scattered pressure is given by:

$$p_s(\boldsymbol{r},t) \approx -\sum_{n=1}^{N} \frac{cos(\theta)\Delta s}{8\sqrt{2}\pi^{\frac{5}{2}} r_1 r_2 \sigma^3 c} \left(t - \frac{2d_n + r_1 + r_2}{c}\right) e^{-\frac{\left(t-\frac{2d_n+r_1+r_2}{c}\right)^2}{2\sigma^2}} \qquad (2)$$

where θ is the angle of reflection, r_1 and r_2 are the respective distances from the source and receiver to the centre of the element, and Δs is the surface area of the element.

This is the time domain equivalent of a Schroeder diffuser model that is commonly used in the frequency domain[3]. For this model to be accurate: (i) the frequency content of the Gaussian pulse must be low enough so that plane wave propagation in the wells dominate; (ii) the radiation coupling between the wells has to be small, and (iii) the radiation impedance of each well must be small.

3 SPECULAR REFLECTION

First the scattering is examined in the specular reflection direction. Figure 1 shows the impulse response for three surfaces with the same overall dimensions of 2.8 x 2 m. The source was ten metres from the diffuser and the receiver five metres. For the Schroeder diffusers the well width was 2 cm and the maximum well depth 15 cm. These were one dimensional devices, with twenty periods for the N=7 Quadratic Residue Diffuser (QRD) and twenty-three periods for the *N=7* Primitive Root Diffuser (PRD). *t=0* is the time when the sound travelling direct from the source to receiver arrived. This is not shown in the figure to allow more detail of reflected impulse response to be seen.

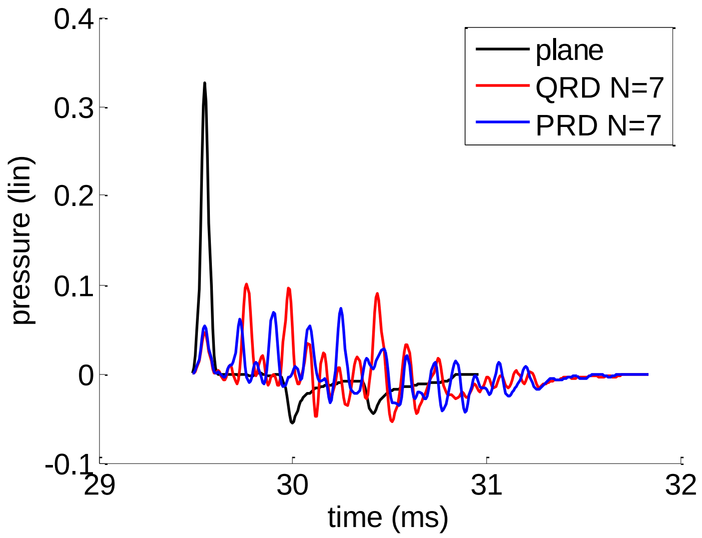

Figure 1 Scattered impulse response for three surfaces, specular reflection direction.

One feature of the impulse responses for the Schroeder diffusers is the regularity of the peaks. The delay of each of these corresponds to the time it takes the sound to travel down and back up each well. For example, the QRD has a prominent peak at 30.44 ms, which represents an additional delay of 0.89 ms compared to the plane surface. 0.89 ms corresponds to a propagation distance of 30 cm, or $2 \times$ the maximum well depth. The impulse response peaks have underlying periodicity because the QRD and PRD diffusers are based on integer number sequences. For example, the sequence for the QRD is 0,1,4,2,2,4,1 leading to a series of depths and delays that have periodicity.

Periodicity in time sequences is often clearer when an autocorrelation is calculated. Figure 2 shows the autocorrelation function of the scattered pressure for the three devices. The first peak for the QRD is at 0.219 ms and for the PRD at 0.176 ms. If a pure tone with a period of 0.219 ms (i.e. a frequency of 4.57 kHz) is incident on the QRD, the reflection will be strong because all the wells return sound that is in phase. Figure 3 shows the scattered pressure for each of the surfaces as a function of frequency. At 4.57 kHz the QRD produces the same scattered pressure level as the plane surface. This is a flat plate frequency, something that has been recognised in integer based Schroeder diffusers before[4].

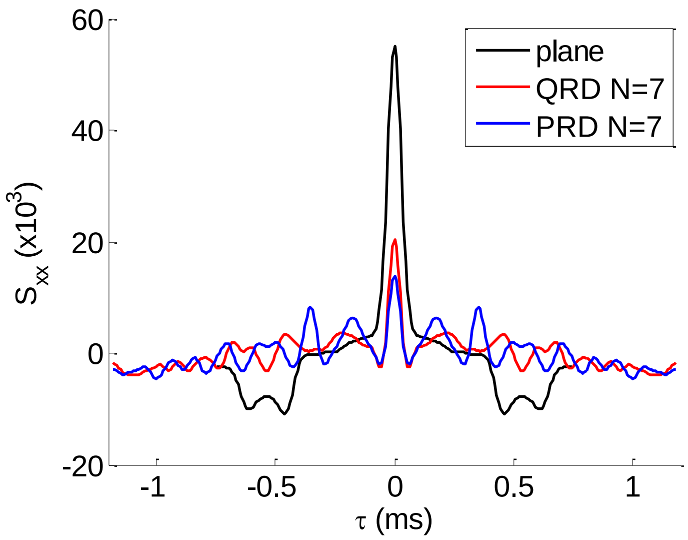

Figure 2 Autocorrelation of scattered impulse responses for three surfaces, specular reflection direction.

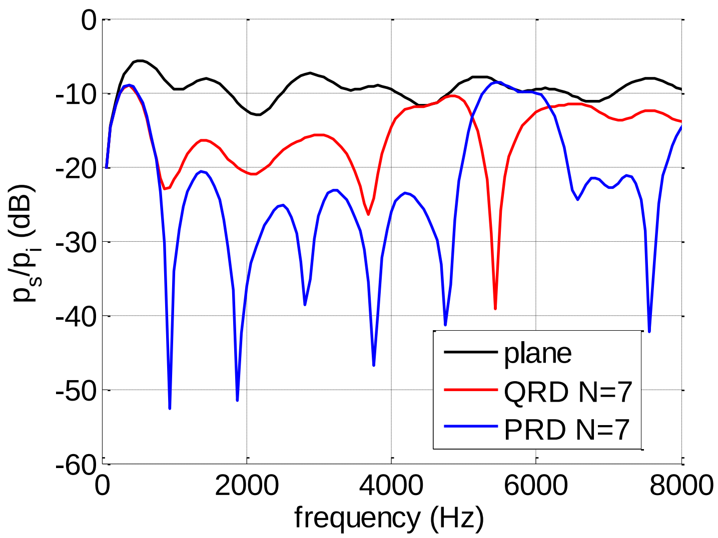

Figure 3 Scattered pressure spectrum for three surfaces, specular reflection direction.

As Figure 1 shows, the delay between the direct sound and the first reflection is about 30 ms. For this size of delay, the dominant audible effect is temporal fluctuations, changes in the envelope due to interference between the direct sound and the reflections. To illustrate this, the impulse responses were convolved with some anechoic music and the low frequency envelopes calculated. In Figure 4, the black lines show the low frequency envelope for a snippet of saxophone music. This consisted of three notes, with heavy vibrato on the last note. The low frequency envelope was calculated by first using the Hilbert transform and then applying a 4th order low frequency Butterworth filter with a -3dB point of 100 Hz.

When an additional reflection from the plane surface is added (red line, top plot), the corruption of the envelope by the temporal fluctuations are apparent. When the impulse response involving either diffuser are considered instead (red lines, middle and bottom plots), something much closer to the envelope of the original music is recovered. The primitive root diffuser does better than a quadratic residue diffuser in this case because it suppresses the reflection more effectively in the specular reflection direction (see Figure 3).

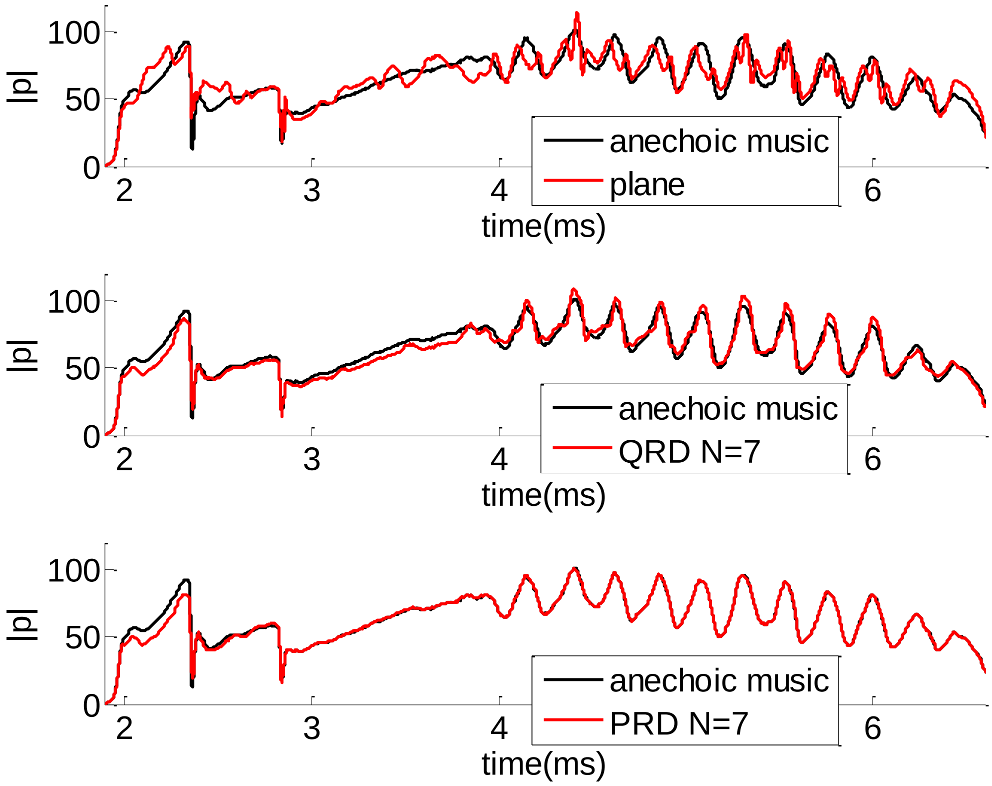

Figure 4 Low frequency envelope for a short snippet of saxophone playing. Anechoic music is just the direct sound, and the other three plots in red are the full impulse responses from the surfaces previously described. Specular reflection direction

It has been shown that a periodic arrangement of diffusers leads to sound being concentrated in certain directions where grating lobes form. If there are only a small number grating lobes, this can lead to uneven scattering. Consequently, using an aperiodic arrangement of diffusers can improve the spatial distribution of sound. What does this do for the listener in the specular reflection direction? To examine this, a QRD was made based on N=139. For this diffuser, there are no flat plate frequencies within the bandwidth being examined. Comparing the music envelopes in Figure 5, the N=139 QRD has a similar amount of envelope modulation as the periodic arrangement of N=7 QRDs.

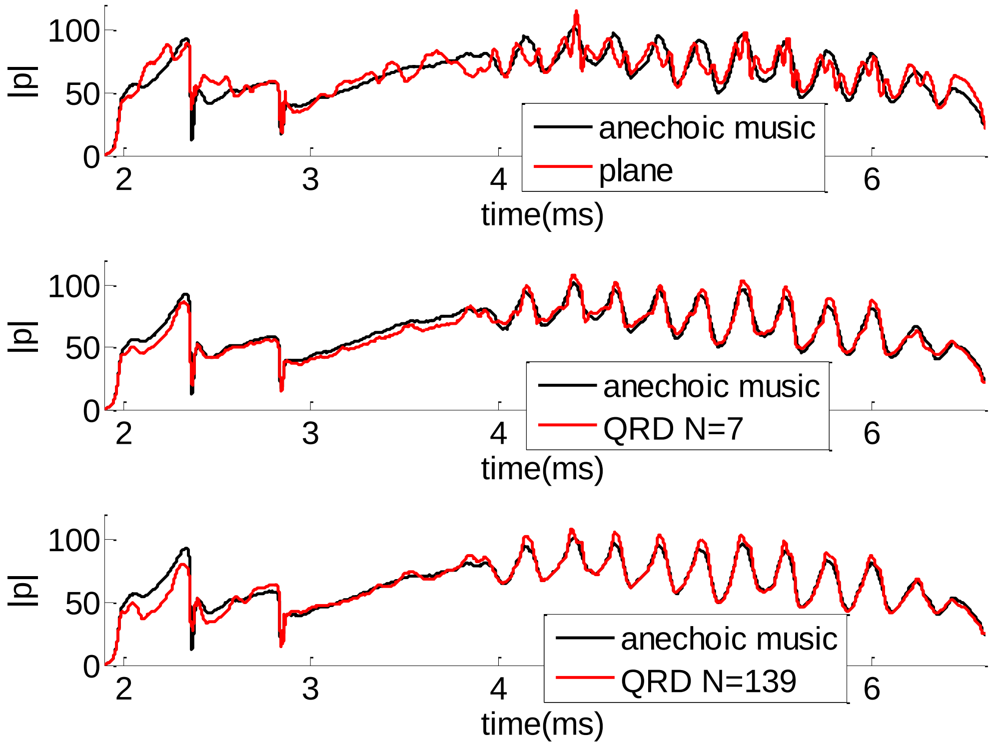

Figure 5 Low frequency envelope for a short snippet of saxophone playing, specular reflection direction.

4 OBLIQUE RECEIVER

What about listeners who do not receive a specular reflection from the panel? The receiver was moved so the angle of reflection was 30° to the panel normal. The receiver radius was increased to 6.09m so that the reflection from the plane panel arrived with the same delay as for the on-axis case for a fair comparison.

Figure 6 shows the impulse response for the plane surface and one Schroeder diffuser. The plane surface scattering is weaker as would be expected. The Schroeder diffuser produces more reflected energy as might be expected because it is designed to produce spatial scattering. Does this mean that the Schroeder diffusers produce more colouration because of the higher reflected energy? Figure 7 shows the envelopes for the three saxophone notes for a plane surface and two Schroeder diffusers. It appears the envelope fluctuations are fairly similar in all three cases shown.

Figure 8 shows the autocorrelation and Figure 9 the scattered pressure spectrum for a plane surface, an N=7 QRD and N=139 QRD. The autocorrelation shows no evidence of periodicity due to flat plate frequencies. This happens because for oblique receivers, there is a lack of energy being received at the flat plate frequency. The longer QRD (N=139) scatters more energy laterally at low frequency, and some periodicity shows up in autocorrelation and the temporal modulation appears worse. However, listening the sound files, there appears to be no obvious audible differences.

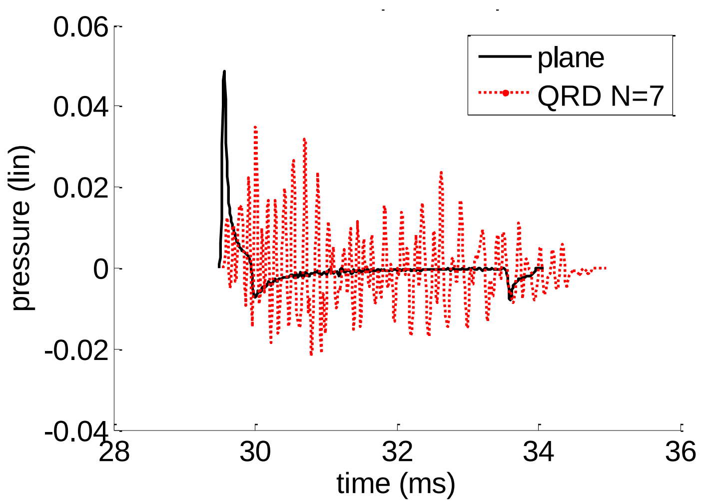

Figure 6 Scattered impulse response for three surfaces. Oblique receiver.

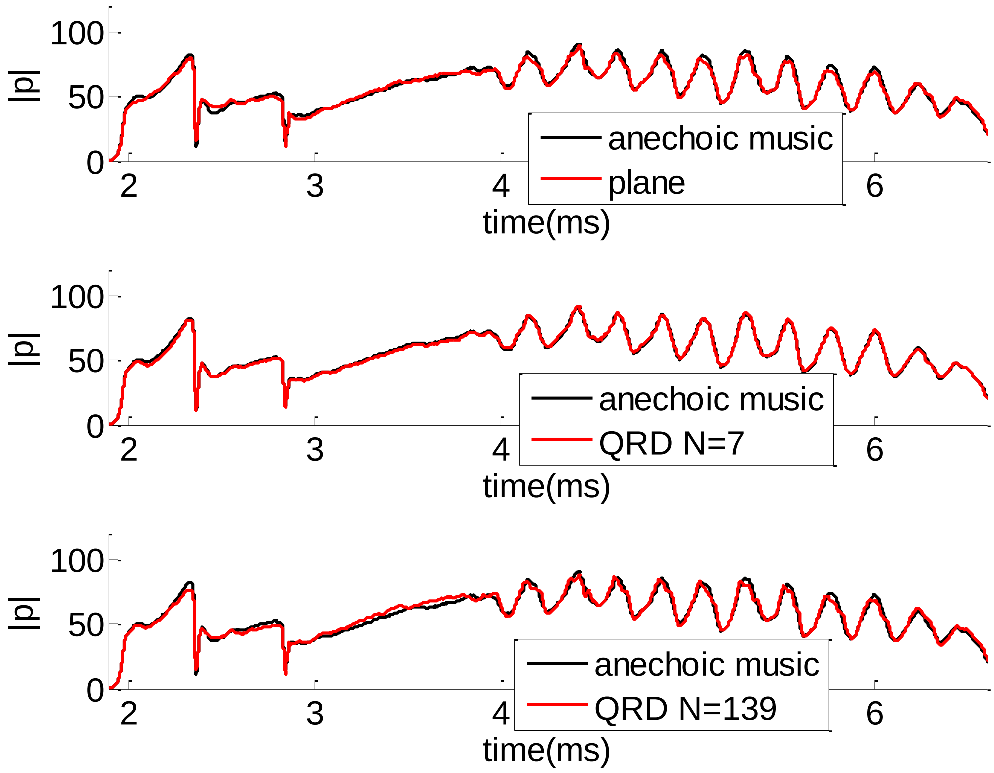

Figure 7 Low frequency envelope for a short snippet of saxophone playing. Oblique receiver.

5 CONCLUSIONS

The reflections from various Schroeder diffusers have been examined in the time domain. For the reflection delays considered in this paper, colouration results in temporal fluctuations in the envelope. To illustrate this, examples were given for a few saxophone notes. Unfortunately, there does not appear to be an established method for characterising and quantifying the audible effects of such fluctuations. There is a need for listening tests to better understand the colouration, and to produce a metric to quantify the perceptual effects.

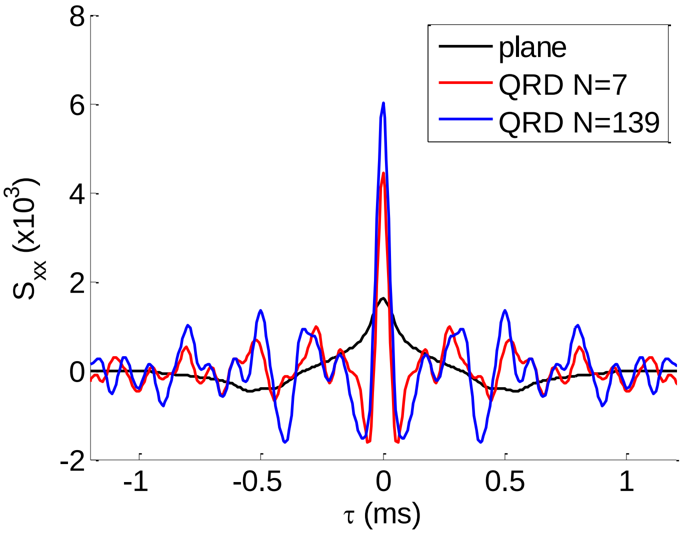

Figure 8 Autocorrelation of scattered impulse responses for three surfaces, oblique receiver.

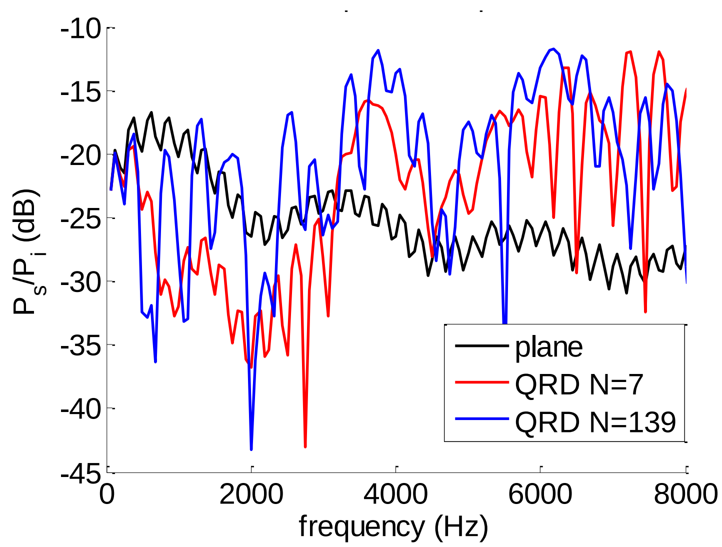

Figure 9 Scattered pressure spectrum for three surfaces, oblique receiver.

6 REFERENCES

1. Schroeder, M. R. (1975). Diffuse sound reflection by maximum− length sequences. *The Journal of the Acoustical Society of America*, *57*(1), 149-150.
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4. Angus, J. A., & McManmom, C. I. (1996, May). Orthogonal sequence modulated phase reflection gratings for wideband diffusion. In *Audio Engineering Society Convention 100*. Audio Engineering Society.