THE PERFORMANCE OF SCHROEDER DIFFUSERS: A WIDE BAND BEM INVESTIGATION

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1 INTRODUCTION

A Schroeder diffuser[1,2] is a reflection phase grating, an example of which is shown in Figure 1. It consists of a series of wells of the same width and different depths. The wells are separated by thin fins. The depths of the wells are determined by a mathematical sequence, such as the quadratic residue sequence. Single plane diffusers cause scattering in one plane, in the other direction, the extruded nature of the surface makes it behave like a plane surface. Because of this it is normal to just consider a cross-section through the diffuser, which contains the plane of maximum dispersion. This paper presents results only for the plane of maximum dispersion; the results can be qualitatively extended to multi-plane diffusers.

Figure 1 A standard one dimensional Quadratic Residue Diffuser

Since their conception almost thirty years ago, Schroeder diffusers have evolved from their simplest design, to counter inherent performance weaknesses. Fractal constructions (Figure 2), diffusers formed from optimised well depth sequences and modulation to remove periodicity, have all been shown to enhance the scattering abilities of these surfaces. In additional, alternative generation sequences, including non-integer based sequences to deal with critical frequencies, have been suggested. Although the performance of standard and enhanced Schroeder diffusers has been evaluated by several authors, the work has mainly been conducted using prediction models which have been limited by the computing power available. This has meant that the scattering was either predicted with an approximate model or limited in bandwidth. With modern computing power, it is possible to carry out a wide bandwidth evaluation on large surfaces, using accurate boundary element modelling and the results from such an investigation are presented in this paper. This allows a more complete investigation of the scattering properties of Schroeder diffusers.

Figure 2 A fractal Schroeder diffuser (order = 2)

The paper will show that standard designs, those applying the simplest form of the diffuser, suffer from a variety of problems. In theory a primitive root sequence should produce reduced energy in the specular reflection direction, but this is only achieved for a limited set of frequencies. Quadratic residue diffusers based on a small number of wells per period suffer from flat plate frequencies, and this is seen even above the plane wave cut off frequency of the wells and influences the scattering for a considerable bandwidth. For other frequencies, low N number quadratic residue diffusers appear to suffer from a lack of phase variation across the surface and consequently the scattering performance is poor. It will be shown that these problems can be solved by moving to an optimized or non-integer based sequence. Modulation can be used to improve the low to mid frequency performance of diffusers, especially for Schroeder diffusers with narrow period width. Results will show how the use of fractal construction enables improved bass performance, and this improved bass response is achieved without excess absorption.

2 TEST BED

The scattering from the surfaces will be predicted using a Boundary Element Model (BEM) utilising a thin panel solution which explicitly meshes the entire diffuser surface. This solution technique has been verified against measurement and has been shown to be robust and accurate[3].

The scattering is calculated in the form of a polar response on a receiver arc 50m from the surface of the diffuser. Source positions are on a 100m radius from surface. These large distances are used to ensure the tests are in the far field. From the polar responses, the diffusion coefficient is derived to test the quality of the dispersion generated as a function of frequency. The methodology follows that set out in AES-4id-2001 (AES Information document for room acoustics and sound reinforcement systems – characterisation and measurement of surface scattering uniformity)[4]. Nine source positions are used to investigate the variation in dispersion with angle of incidence. The results from the different source positions are combined to form a random incidence diffusion coefficient by a simple arithmetic average.

With faster computers now available, it is possible to carry out a broad bandwidth analysis of the diffusing ability for reasonably wide surfaces. The test surfaces used are all 3.6m wide, which includes 6 periods of most of the diffusers tested. The analysis is carried out from 100 to 5-8kHz, depending on the diffuser tested. The results presented are averaged over 1/3 octaves unless otherwise indicated. A variety of surfaces are tested as detailed in Table 1. The reasons for testing these surfaces will be discussed in the following sections. References are given in the table which describes the design methodology of the surfaces.

Table 1. The surfaces tested.

Test surface	Description	Notes
QRD®	6 periods of an N=7 QRD[5], maximum depth of 0.2m,each period 0.6m wide	Standard construction using quadratic residue sequence
PRD	6 periods of a modified N=7 PRD[6], maximum depth of 0.2m, each period 0.6m wide	Uses the primitive root number sequence
Flat	Reference flat non-absorbing surface 3.6m wide	
Diffractal™	Third order N=7 QRD fractal construction[7], maximum depth 0.5m (largest diffuser only uses 6 wells)	See Figure 2
Optimised 8	6 periods of an optimised phase grating diffuser[8] with 8 wells in a modulated array, 3.6m wide, max. depth of 0.17m,	
Optimised 12	6 periods of an optimised phase grating diffuser with 12 wells in a modulated array, max. depth of 0.2m,	

3 PERFORMANCE OF TRADITIONAL N=7 QRD

The standard N=7 diffuser based on the quadratic residue sequence is one of the most popular forms of the Schroeder diffuser. Using N=7 produces improved bass response and is relatively cheap to manufacture. Figure 3 shows the diffusion coefficient spectra for a number of diffusers including the N=7 QRD. Because of the evaluation technique used, the diffusion coefficients calculated using AES-4id-2001 tend to be numerically small, but for this study it is the relative values to other surfaces which is of most importance. A higher figure indicates better dispersion.

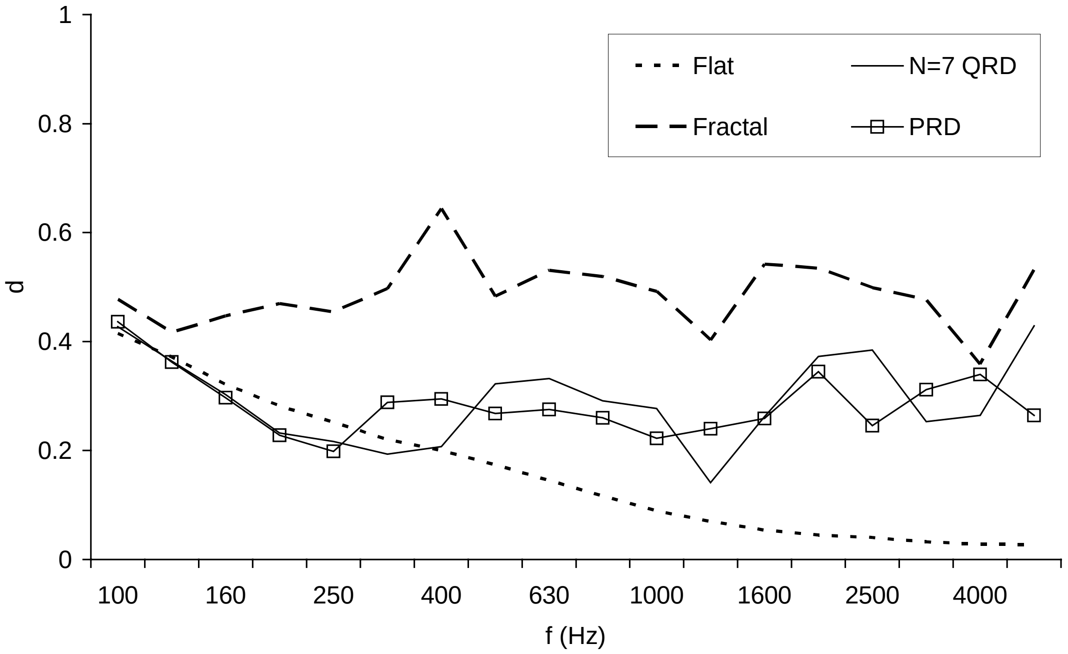

Figure 3 Random incidence diffusion coefficients for some Schroeder diffusers

Schroeder originally produced a diffuser based on a maximum length sequence, but moved to a design based on other number sequences, such as the quadratic residue sequence, because of problems of critical frequencies (sometimes called flat plate frequencies). An example of a maximum length sequence is shown in Figure 4. An octave above the design frequency, when the well depths equals 1/2 the wavelength, the surface behaves like a plane surface, because all waves reradiate with the same phase; this is a critical frequency and the dispersion will be poor. Consequently, the maximum length sequence diffuser is only useful over an octave.

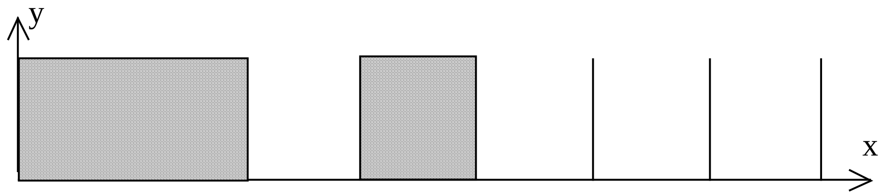

Figure 4 A cross-section through one period of an N=7 maximum length sequence diffuser. (After Cox and D'Antonio[3])

The quadratic residue sequence suffers from the same problem, but there will be fewer critical frequencies in the audio bandwidth. This fact was highlighted by the work of Angus[9], but his analysis only used an approximate prediction model. It is now possible to more correctly analyse whether critical frequencies are robust. For most integer based sequences generated using a prime number N, the critical frequency occurs at frequencies of nNf_0 , where n is an integer, and f_0 is the design frequency. The quadratic residue diffuser tested here has a design frequency of about 490 Hz, so the first critical frequency is at 3.4kHz. A dip in the diffusion coefficient response can be seen for the 3.15 and 4kHz 1/3 octave bands in Figure 3. The dip in the 1.25 kHz 1/3 octave band will be explained below. Consequently, this phenomenon is robust to a more accurate analysis. Furthermore, these results have emphasised the importance of critical frequencies to quadratic residue diffuser performance:

1. The effects of the critical frequencies extend over a broader bandwidth than might at first be expected. At the critical frequency, the scattering is identical to that from the flat surface, but the scattering is also poor for surrounding frequencies. This is why the dip in the diffusion coefficient extends over two 1/3 octave bands. For frequencies near to the critical frequency, the phase variation introduced to the reflected wavefront is limited, as the phases of the reflection factors of the wells are all similar.

2. The critical frequencies are apparent above the cut off frequency of the wells. When the wavelength of sound is smaller than twice the well width, it is assumed that cross modes in the wells exist, and the plane wave propagation assumption inherent in the number theory design of Schroeder diffusers breaks down. It had been previously assumed that if the first critical frequency was placed above the plane wave cut off frequency, that critical frequencies would not be a problem, because the cross modes in the wells would complicate the scattering from the surface. However, the critical frequencies of this diffuser are above the plane wave cut-off frequency of the wells, and they still cause a decrease in scattering performance. Indeed the second critical frequency (at about 6.8kHz, affecting the 6.3 and 8 kHz 1/3 octave band, not shown in Figure 3) also causes a decrease in scattering performance, even though this is nearly two octaves above the plane wave cut-off frequency.

3. It appears that low N number sequences suffer from other frequencies where the phase variation in the reflection factors is limited and so the dispersion is poor. An example of this is evident for the 1.25kHz 1/3 octave band in Figure 3 where a dip in performance is seen. The depth sequence of the N=7 QRD is based on a number sequence of s_m = 0,1,4,2,2,4,1. Table 2 shows the reflection factor phase for different frequencies for the 7 wells. For instance, at n=7 - the first critical frequency - all wells have a reflection factor phase of zero, and so the surface behaves as though it is a flat surface.

Table 2. Variation in reflection factor phase (in radians) with well depth sequence number and frequency. The frequency index should be multiplied by the design frequency to get the frequency in Hz.

Frequency index n	Well depth sequence number						
	0	1	4	2	2	4	1
1	0	0.90	3.59	1.80	1.80	3.59	0.90
2	0	1.80	0.90	3.59	3.59	0.90	1.80
7/3	0	2.09	2.09	4.19	4.19	2.09	2.09
3	0	2.69	4.49	5.39	5.39	4.49	2.69
4	0	3.59	1.80	0.90	0.90	1.80	3.59
5	0	4.49	5.39	2.69	2.69	5.39	4.49
6	0	5.39	2.69	4.49	4.49	2.69	5.39
7	0	0.00	0.00	0.00	0.00	0.00	0.00

The case of n=7/3, is an example where the phase variation for the different wells is reduced. This is the case where the depth of the s_m=1 well is a sixth of a wavelength. For this frequency, the reflection factor for the s_m=4 well is identical as shown in Table 2. There are only three unique reflection phases, and consequently the wavefront dispersion is limited. It is unclear, however, why this effect is only evident at n=7/3, because this also occurs at n=m7/3 where m is an integer.

A final feature of note is that the bass response of the Schroeder diffuser is being limited by the period width and not the depth – this will be returned to later.

4 PERFORMANCE OF TRADITIONAL N=7 PRD

Schroeder proposed the used of a primitive root sequence to make a diffuser with reduced specular reflection. Later Feldman[10], Cox[11] and Schroeder[12] returned to this design. Although in all cases only a simple prediction model was used to test the performance of the diffuser. The primitive root diffuser is intended to produce a notch response – the specular lobe is suppressed and the other diffraction lobes have even energy. The notch is only achieved at the design frequencies, and multiples of the design frequency. For this reason, a broadband notch is not achieved. The boundary element method polar responses showed that broad bandwidth notches are not achieved.

The primitive root diffuser has a depth sequence of s_m = 3,2,6,4,5,1 which means it has a greater number of different depth wells than the quadratic residue diffuser. This means it has a more even diffusion coefficient frequency response, see Figure 3, having fewer frequencies where the reflection phase variation across the front face is small. The PRD still has critical frequencies where it produces reflection identical to a flat surface, but because the PRD has a higher design frequency than the N=7 QRD, the first critical frequency for the PRD is at 5.1kHz on the far right of the graph and so maybe less obvious.

Although failing to provide a notch response, it appears that the PRD is a better disperser of sound than a QRD. However, much better designs can be achieved, as discussed in the following sections.

5 PERFORMANCE OF A DIFFRACTAL®

The bandwidth of a Schroeder diffuser is limited at high frequencies by the well width and at low frequencies by the maximum depth. Additionally, wide area coverage with periodic arrays focuses energy into certain diffraction directions. To provide full spectrum sound diffusion in a single integrated diffuser, D'Antonio and Konnert were inspired by the self similarity property of fractals to form a fractal diffuser. The surface consists of nested self similar scaled diffusers, each of which covers a specific frequency range and offers wide area coverage, see Figure 2 for an order 2 fractal based on an N=7 QRD. Each diffuser size is designed to provide uniform scattering over a specific range of frequencies, in the same way that multi-way loudspeakers use different sized drivers to cover different frequency ranges. Using this technique, the effective bandwidth is extended. Again the performance of the fractal diffusers has only previously been tested using a simple Fourier model, and a more accurate and wide band analysis can now be given here. Furthermore, the commercial implementations of a fractal Schroeder diffuser do not have all the diffuser fins extending to front face of the diffuser, because that would lead to excess absorption in the narrow deep wells, and this might affect the performance. Consequently, a three-way fractal diffuser, using three different scales of a N=7 quadratic residue diffuser was tested.

The first feature of note in Figure 3 is that the diffuser has a significantly improved bass response in comparison to a traditional Schroeder diffuser. This occurs because the diffuser is now very much deeper than the previous diffusers being tested. Furthermore, because the surface is modulated, the period width is now 3.6m and the performance is limited by the depth and not the period width.

The principle of the fractal diffuser was that the different scales of diffuser would disperse different frequency regions, and the results show this to be true. From 500Hz to 5kHz, the Diffractal diffusion coefficient very much follows the diffusion coefficient of the mid-frequency N=7 QRD it is based on. The minima and maxima are in similar positions, and evidence for critical frequencies are still seen. Below 500Hz, the largest QRD becomes significant and additional dispersion is generated. The smallest diffuser is only 2.5cm deep, and becomes significant for the 3.15kHz 1/3 octave band and above as expected (this was clearer in results for a second order Diffractal, which did not include the bass unit, graph not shown).

6 MODULATION

Angus[13,14,15,16,17,18] presented a series of papers outlining methods for using two phase grating base shapes in a modulation scheme to deal with the problems of periodicity. In Schroeder's original design concept, an "optimum" diffuser is one that produced grating lobes with the same energy. Angus highlighted the fact that these grating lobes are undesirable, and better dispersion could be generated by modulating a diffuser array. Figure 5 shows such a modulation arrangement for two quadratic residue diffusers, one based on N=7, the other on N=5. The idea is to use two or more base shapes, and arrange them according to a pseudorandom arrangement so there is no repetition. The diffusers are arranged using sequences with optimal autocorrelation properties such as the Barker sequence. By using two different quadratic residue sequences, it is possible to reduce the effect of critical frequencies. Another technique is to use one Schroeder diffuser and its inverse, which is a surface that scatters sound 180° out of phase. Analysis using a simple Fourier theory has shown that the modulation is effective, with the modulation using a diffuser and its inverse being most effective provided critical frequencies are considered in the design. However, this requires the manufacture of two base shape diffusers, and it is possible to carry out modulation using a single asymmetric diffuser, which is useful as it reduces manufacturing costs. The second base shape is obtained by flipping the order of the wells, which can be achieved simply by turning the diffuser upside down. An example of this is shown in Figure 6.

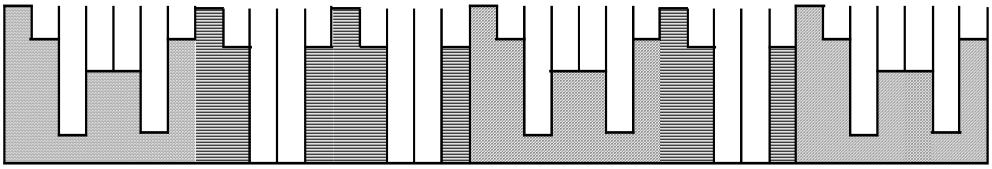

Figure 6 A cross-section through a modulation scheme using N=5 and N=7 quadratic residue diffusers and the modulation sequence {1,0,0,1,0,1}. (After Cox and D'Antonio[3])

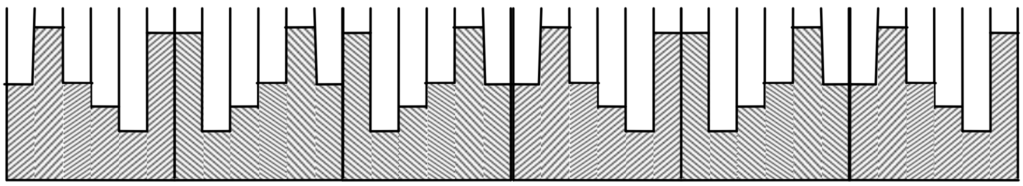

Figure 5 A cross-section through a modulation scheme using a single (asymmetric) diffuser and the modulation sequence {1,0,0,1,0,1}.

The modulation technique tested was the single base shape asymmetric modulation using a reflection phase grating which was designed using numerical optimisation. Two cases were tested: one of the optimised surfaces has 8 wells, the other has 12 wells in the 0.6m wide period. The results from this test are shown in Figure 7.

As has been shown previously, the optimised surfaces produce better scattering than traditional Schroeder designs. Whereas the original optimisation was done on single periods of devices for a limited bandwidth, increased computing power now allows broad bandwidth optimisation for arrays of diffusers. The 12 welled optimised diffuser has better diffusion over a larger bandwidth than the 8 welled diffuser. It is speculated that with 8 wells the optimiser did not have sufficient number of degrees of freedom to generate dispersion at all frequencies – the optimiser could produce good dispersion at high or low frequencies, but not both.

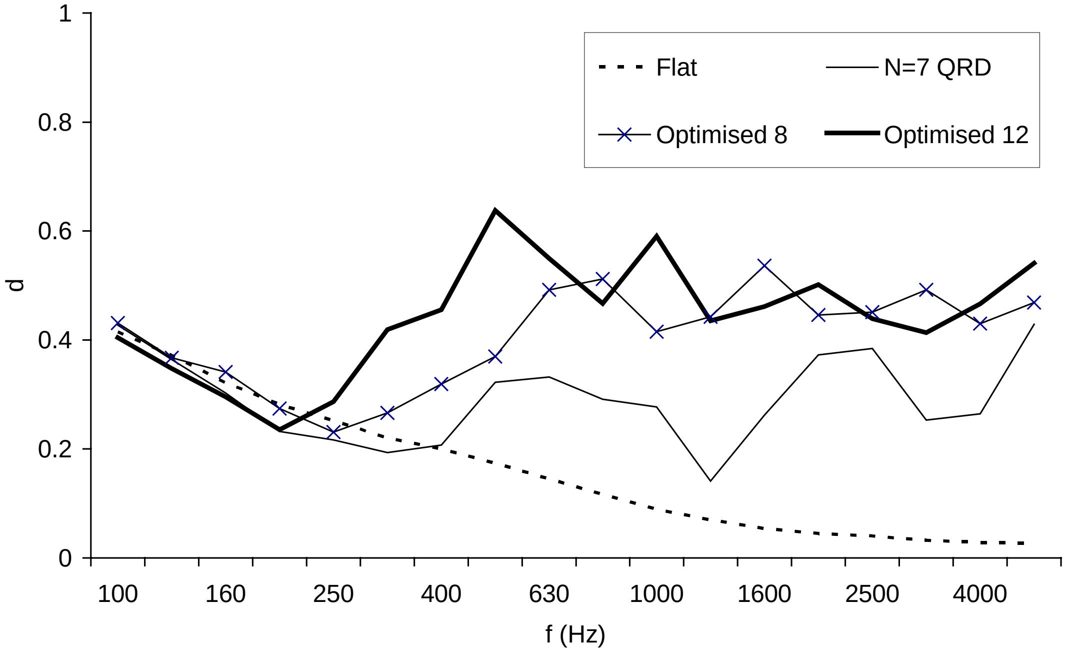

Figure 7 More random incidence diffusion coefficients.

The first feature of modulation being demonstrated, is the ability of modulation to produce better low frequency dispersion from a given diffuser depth. With a periodic diffuser arrangement, the dispersion generated is often limited by the period width. Until the frequency is high enough that the first grating lobe appears (when the wavelength is smaller than the period width) then little dispersion is generated, because only the specular lobe exists. Consequently, modulation is useful because it increases the period width and so generates additional grating lobes. This is the reason that the optimised diffuser with 12 wells produces significantly more diffusion at a lower frequency than the QRD.

Modulation produces better diffusion at low-mid frequencies, but at high frequencies, say greater than 5kHz for the typical geometries used in practical diffusers, the dispersion by a modulated array summed over a one third octave band is often worse than for a periodic arrangement. This happens because the number of grating lobes in the periodic case saturates, so when the polar responses are summed over a one third octave band the grating lobes average out. Having said this, the improvements generated by modulation in the more important low-mid frequency ranges far outweigh any slight decrease in performance at higher frequencies.

7 CONCLUSIONS

This paper has analysed the performance of Schroeder diffusers using an accurate prediction model over a wide bandwidth and for random incidence. It has highlighted the weaknesses of traditional designs, illustrating problems of critical frequencies and limited low frequency response from periodic arrangements.

It has shown that improvements to the dispersion of traditional diffusers can be made by using modulation to remove periodicity effects and numerical optimisation to select appropriate well depths. Fractal geometries enable a better bass response without the risk of excess absorption.

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