VOLUMETRIC DIFFUSERS INSPIRED BY PERCOLATION FRACTALS

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1 INTRODUCTION

Diffusers can be used in auditoria to treat various acoustic problems such as echoes. Diffusers need to contain roughness at different scales to effectively scatter sound across a wide bandwidth. Fractals are structures which inherently have scaling invariance. They have roughness at different magnifications, and consequently should be ideal structures for diffusing surfaces across a broad bandwidth. A fractal Schroeder diffuser has been commercially available for many years[1], an example of which is shown in Figure 1. Small Schroeder diffusers[2] are imbedded in a larger device, each causing diffusion in different bandwidths. The system is analogous to a two-way loudspeaker, with its two drivers producing low and high frequency sound.

Figure 1. A Diffractal® (photo courtesy of RPG Diffusor Systems Inc.)

There are many other ways of generating fractal structures. Of interest in this paper are those based on percolation. These are very different constructions, with very different properties to previous fractal diffusers[3].

2 PERCOLATION

Consider the structure shown in Figure 2 left. This is a complex grid construction with black lines based on a square lattice. It has a multitude of complex and tortuous paths running between the lines. For scientists and mathematicians working with percolation, the issue is whether there is a path running through the whole structure; in other words, if a liquid was poured from the top, would it seep all the way through to the bottom?

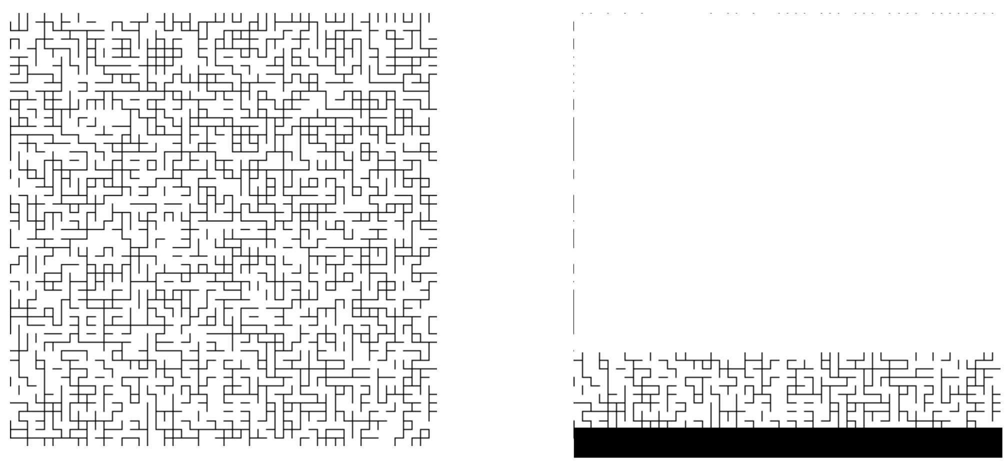

Figure 2. Left, an example of a 2D bond percolation[4]; right, a similar structure with a rigid backing.

Consider a similar structure where the spacing between the lattice lines is a few centimeters, and the material forming the lines are hard and acoustically rigid so for the time being absorption can be ignored. What happens if an acoustic wave is incident from the top onto this structure? Like the liquid, sound can propagate through the structure provided there is a pathway. In addition, some of the sound will reflect from the structure back upwards, while other sound will seep out of the sides. This is a surface which can be placed in the volume of a space and create dispersion.

A way of understanding the performance of volumetric diffusers is still being developed, and consequently to simplify analysis, a surface diffuser will be formed using a similar principle. Such a structure is shown in Figure 2 right which now has a rigid backing. If the percolation structure has many more lines, one which has a more completely filled grid, then the chances of there being a path through the structure to the backing wall and back is small. Conversely with a more open structure, one where the grid is less completely filled in, there is a greater probability of sound passing through to the bottom and back out again.

Why might this make an interesting structure for a diffuser? The pathways through the array are complex and tortuous, in other words sound has to travel a considerable distance to enter, and then finally exit the structure. Consequently, waves interacting with the structure undergo a large range of long delays resulting in considerable phase changes on reflection. The phase changes should be greater than that achieved by a Schroeder diffuser. For a Schroeder diffuser, the maximum phase change achievable is $-2kd_{max}$ where k is the wavenumber and d_{max} the maximum well depth. By forming channels that generate lateral sound propagation, the maximum phase change achievable increases considerably. This should mean such a structure should produce scattering at a lower frequency.

In the past, well folding within Schroeder diffusers has been investigated. By folding wells, it is possible to gain a greater bandwidth from a given overall depth, see Jrvinen *et al.*[5], Mechel[6] and Hargreaves *et al.*[7]. A standard Schroeder diffuser has much wasted space at the rear which can be better utilized. Figure 3 shows the use of well folding to reduce the depth of an N=7 quadratic residue diffuser (QRD). For normal incidence, the central divider shown in red is not actually needed, because the device is symmetrical.

The diffuser with the folded well is in many ways similar to the percolation structure shown in Figure 2, except that the pathways through the percolation fractal are much more complex.

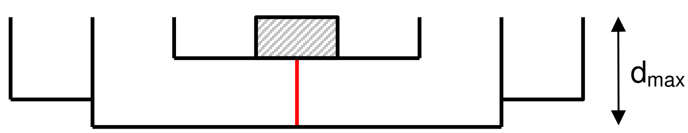

Figure 3. An N = 7 quadratic residue diffuser with folded wells to reduce the depth to 63% of its original size. The central divider shown in red is optional for normal incidence. (After Cox and D'Antonio[8])

3 EXAMINATION

A study has been undertaken to explore the performance of percolation structures as diffusers. The study is very much in its early stages, so the results presented here are rather tentative and incomplete. Figure 4 (left) shows a cross section through the type of structure that has been examined. These are single plane devices because they allow faster 2D prediction models to examine behaviour. The test diffuser is 0.6m wide and 0.12m deep. The vertical and horizontal elements that make up the structure are spaced on a 3cm grid. There are 2^{164} possible diffusers that can be made by randomly populating the grid; consequently, a method needs to be devised for designing the diffusers.

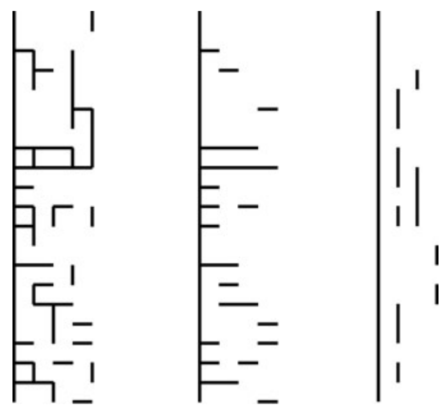

Figure 4. Three types of surface being examined. The left figure is the percolation structure of most interest. The middle and right structures lack vertical and horizontal walls respectively.

3.1 Monte Carlo

First, a random sample of diffusers was made to find out what this revealed about the performance. Three thousand diffusers were formed, one thousand of each of the diffusers shown in Figure 4. The locations of the lines were chosen using a random number generator. The scattering from the surfaces was then predicted using a 2D boundary element method based on the Helmholtz Kirchhoff equation using a 'thin panel' method. Normal incidence only has been considered. The predictions produced polar responses for 500 – 3000Hz, which were then converted into diffusion coefficients for evaluation. Because the diffuser is rather narrow, the diffusion coefficients calculated are rather larger than normal.

Figure 5 shows a frequency distribution for the three types of diffusers. When there are no vertical lines, a diffuser like the middle of Figure 4, then the diffusion coefficients are all clustered around 0.5. The quality of the scattering is poor, as might be expected. Consequently this design will not be discussed further. For a diffuser with just vertical lines, an example of which is shown in Figure 4 right, a much wider range of diffusion coefficients is produced. However, the best diffusers are created using both horizontal and vertical lines, such as that shown in Figure 4 left. It is assumed this happens because having both enables sound to be channelled in complex tortuous paths more easily. Consequently, the rest of the diffusers discussed in this paper have both horizontal and vertical lines.

The random set of diffusers was examined further to see what other features might be important for good scattering. Figure 6 shows how the diffusion coefficient varies with the number of lines in the diffuser, in other words the fullness of the lattice. If there are few lines in the diffuser, to the left of the figure, then the scattering is dominated by the reflection from the back wall, and the quality of the device is poor. If there are too many lines in the diffuser, to the far right of the figure, then sound can not easily penetrate the device, and it looks rather like a plane surface, and again the diffusion is poor.

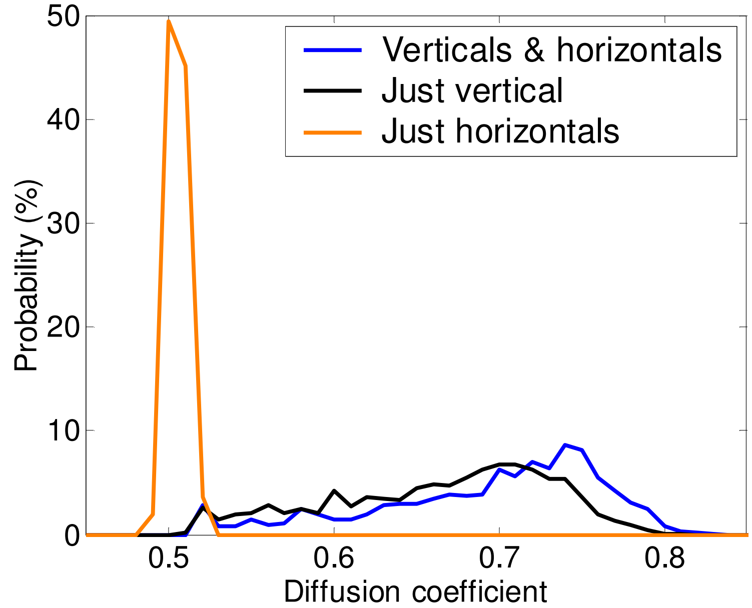

Figure 5. Frequency distribution for randomly created diffusers as a function of the diffusion coefficient.

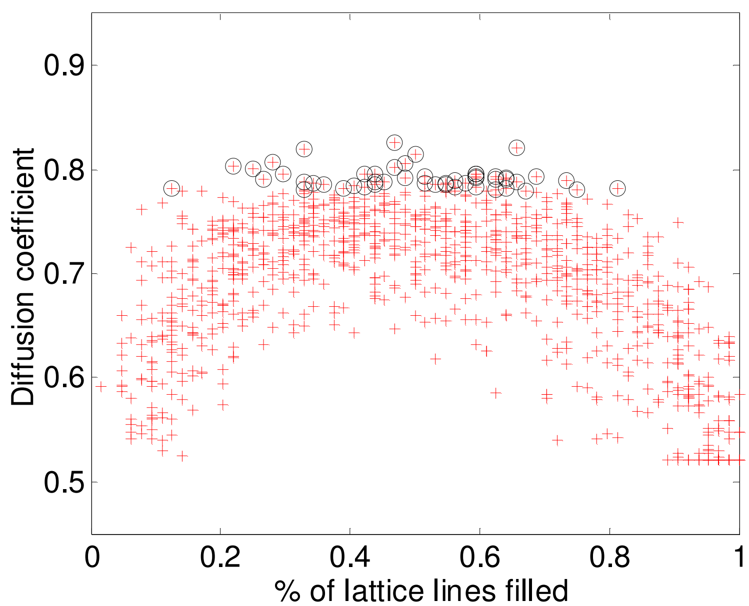

Figure 6. The diffusion coefficient for 1000 randomly generated diffusers verses how full the lattice is. The top 50 diffusers are highlighted by black circles.

The best diffusers are marked by black circles. It shows that a good diffuser can be produced with anywhere from 20% to 80% of the lattice lines filled.

Figure 7 examines how important line of sight is to the design. If the diffuser designs shown in Figure 4 are viewed from the right, how much of the back wall can be seen? It is suggested that if too much of the back wall is visible, then the diffuser will be poor. In Figure 7, when the abscissa is zero, the back wall is completely visible and when the abscissa is one, the back wall is completely hidden by the diffuser. Good diffusers need to cover at least 65% of the back wall. At first this seems surprisingly low, but provided reflections from the rest of the diffuser are out of phase with the back wall reflection, then good diffusion can be produced using interference.

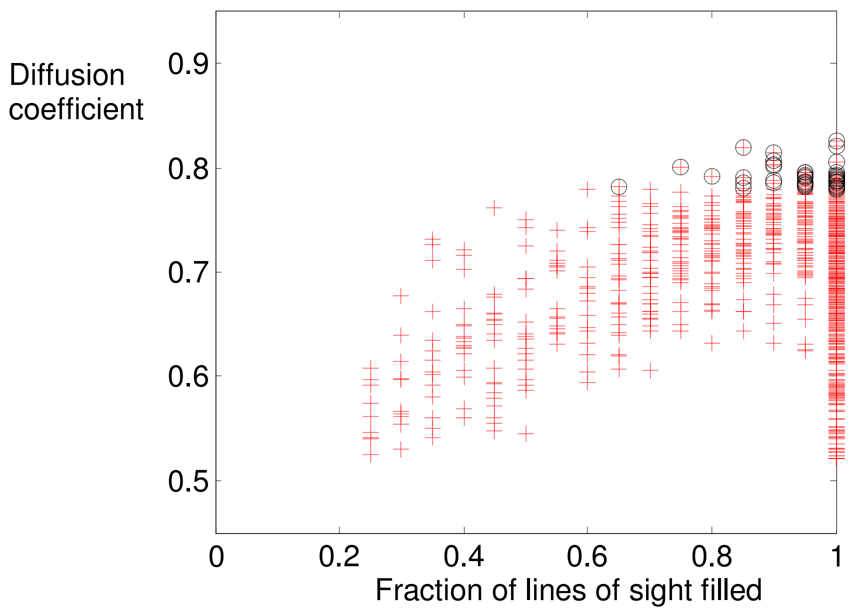

Figure 7. Diffusion coefficient for 1000 randomly generated diffusers; the top 50 are highlighted by black circles. Plot verses line of sight.

Future work will examine how important the channels and tortuous paths within the percolation structure are. This will be done by examining how the diffusion coefficient varies with a measure of the tortuosity of the path length distribution through the structure.

3.2 Numerical optimisation

A computer can be tasked to find the best percolation structure in a numerical optimisation algorithm. Such an approach has been used before for other diffusers[8]. As this is a discrete optimisation problem, a genetic algorithm is most appropriate. However, given the vast number of combinations to be searched, the algorithm is not going to be able to find a global minimum.

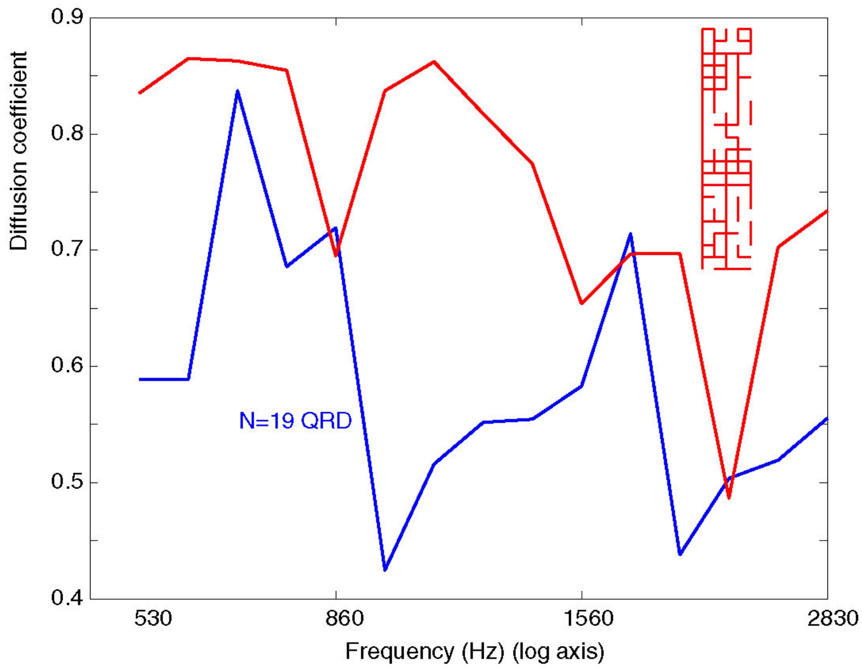

Figure 8. The diffusion coefficient for an optimised percolation fractal (red) and one period of an N=19 QRD (blue). The optimised shape is also shown.

Figure 8 shows the result from a first attempt at optimisation. The diffusion coefficient for an optimised structure is compared to the coefficient for a standard QRD. An improvement in performance is seen indicating a better diffuser. However, the structure shown as an insert is not as expected. Although it seems to be a complex percolation structure, closer inspection reveals that many of the lines are not important to sound, because they are completely enclosed. Further optimisations are required to see if this is an atypical result, or whether the assumption of the need for tortuous paths is incorrect.

4 CONCLUSIONS

The concept of designing a diffuser based on a percolation fractal has been outlined. A Monte Carlo simulation was used to try and understand the underlying requirements for good diffusion. It was found that between 20% and 80% of lattice lines need to be filled, and that 65% of the lines of sight through the whole surface need to be obscured by lattice lines. A first attempt at optimisation was presented.

This showed that better performance than a QRD could be achieved. However, the structure produced was rather simpler than expected. Further investigation is needed to understand the important features of the structure for producing good scattering.

5 REFERENCES

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