PREDICTION OF NEAR-FIELD BEHAVIOUR OF TRANSDUCERS: A COMPARISON OF THE FINITE ELEMENT, BOUNDARY ELEMENT AND ANGULAR PLANE WAVE SPECTRUM METHODS AND THE CONSEQUENCES FOR MEASUREMENTS IN THE NEAR FIELD

T. J. Esward & M. A. Bangash

Centre for Mechanical and Acoustical Metrology,
National Physical Laboratory, Teddington, Middlesex, TW11 0LW, U.K.

1. INTRODUCTION

Numerical methods of predicting the field radiated by underwater transducers are becoming increasingly popular. The Centre for Mechanical and Acoustical Metrology at the National Physical Laboratory is establishing a range of software tools for predicting both linear and nonlinear acoustic fields. In the nonlinear case, we have established methods based on propagating beams with a Gaussian beam profile [1] and have recently implemented a version of the Bergen code [2], which uses the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation to predict fields close to the beam axis of a propagating acoustic field. In the linear case, we have established a finite element and boundary element capability based on the PAFEC vibroacoustics software package. However, we also need ways of rapidly making field predictions based on experimental measurements and we are therefore investigating the angular plane wave spectrum method of forward and back-propagation of measured data.

As part of this investigation, we decided to test the field prediction capabilities of the angular plane wave spectrum method against finite and boundary element methods. This paper sets out some of the key results of that work. Predictions of the near-field behaviour of an ideal plane piston source are presented and compared for three different numerical techniques: finite elements, boundary elements, and the angular plane wave spectrum method. It will be seen that the angular plane wave spectrum method produces results that have some significant differences from those obtained by finite element and boundary element methods. The question of whether such differences, which may also arise in the modelling of realistic rather than ideal transducers, can be investigated experimentally is discussed. The results of the predictions are used to provide guidance on the experimental techniques to be adopted for near-field measurements and consequent comparisons with theory, with particular reference to the selection of the measurement position, the extent of the measurement field and spatial sampling interval, and the choice of hydrophone.

2. IDEAL PLANE PISTON SOURCE

2.1 Finite elements and boundary elements compared.

A 10 mm diameter ideal circular plane piston transducer radiating in water at 250 kHz was modelled by means of a stiff circular plate over whose face all points vibrate with the same amplitude and phase. The first test to be applied to the three numerical methods was a comparison between the finite element (FE) and boundary element (BE) methods for the field of this ideal plane piston source in an infinite baffle. In each case the PAFEC vibroacoustics package was used to generate data. Axial symmetry was assumed in both models. For the finite element case, the fluid was modelled by acoustic finite elements out to a distance of 10 mm (one transducer radius) from the transducer's front face, with nodes being spaced at 0.25 mm intervals, and with a boundary element external to the acoustic finite element region. The spatial sampling interval is equivalent to $\lambda/24$. Pressure amplitude and phase were then extracted for the plane perpendicular to the acoustic axis at 10 mm from the transducer's front face. For the boundary element solution, the region of acoustic finite elements was restricted to a distance of 5 mm from the front face, the remainder of the fluid being modelled by a boundary element. Pressure data were then requested at points in the boundary element region corresponding to those points at which finite element data had been obtained, that is, the plane at 10 mm from the transducer's front face. The normalised pressure amplitude results for the two cases are presented graphically in figure 1, where the abscissa of the plot represents distance perpendicular to the acoustic axis of the beam. On the scale of the graph, the two plots are overlaid and it is not possible to distinguish the finite element from the boundary element results. The normalised pressure amplitude results for the two methods differ only in the third decimal place.

2.2 Angular plane wave spectrum and finite elements compared

The next investigation was a comparison of the angular plane wave spectrum method and acoustic finite elements. The angular plane wave spectrum method is now well-established as a means of predicting acoustic field propagation and has been described by a number of authors including Reibold [3] and Stepanishen and Benjamin [4]. A computer program to implement the method described in [3] was produced using the Matlab language and validated by comparison with results previously obtained by one of the present authors [5]. For the work reported here, a 0.25 mm spatial sampling interval was employed to correspond with the node spacing used for the finite element work. The transducer diameter (20 mm) was modelled by a "top-hat" function using 81 points at 0.25 mm intervals and these data were then rotated to produce a two-dimensional distribution. A 162 by 162 two-dimensional Discrete Fourier Transform (DFT) was employed for all projections and the input data at the plane of the transducer face were taken to represent acoustic particle velocity, the prediction of pressure in the field from particle velocity at the source being achieved by the use of equations 5 and 6 from [1]. The sound speed and density of water were chosen to be the same in the FE/BE and angular plane wave spectrum models (1500 m s^{-1} and 1000 kg m^{-3}, respectively). For the finite element model, as explained above, the fluid was represented by pressure-based acoustic finite elements out to 10 mm from the front face, the remainder of the fluid space being modelled by a boundary element.

A comparison of the angular plane wave spectrum method and acoustic finite elements was made by examining how well each predicted the field of the 10 mm radius, 250 kHz piston source in the region of axial minima and maxima. The on-axis field of a plane piston source has a well-known closed form analytical solution [4] and for the transducer in question there are on-axis maxima at 1.1 mm and 15.2 mm from the transducer's front face and a minimum at 5.3 mm from the face. Figure 2 presents a comparison of the PAFEC finite element results with the angular plane wave spectrum results in the plane of the axial minimum at 5.3 mm. Once again, symmetry has been assumed, and normalised pressure amplitudes are plotted as a function of distance from the beam axis in the plane perpendicular to that axis. Note that the finite element results more closely reflect the theoretical zero pressure at the minimum and that, although there appears to be reasonable agreement between the two methods at points close to the axis, this agreement breaks down further from the beam axis. It may be that the difference between the performance of the finite element method and the DFT-based method is due to truncation and windowing effects in the DFT results, and the imposition of a Cartesian co-ordinate system on essentially circularly-symmetrical data. The wavelength of sound in water at 250 kHz is 6 mm, and the position of the axial minimum at 5.3 mm is less than one wavelength from the transducer face. An even more severe test of the angular plane wave spectrum method is provided by the axial maximum at 1.1 mm, less than one fifth of a wavelength from the front face, where the sound field might be expected to possess substantial content at high spatial frequencies. Figure 3 presents the results of the comparison of the angular plane wave spectrum method with acoustic finite elements at 1.1 mm from the transducer. Although the two plots show qualitative similarities, there are differences in the position and amplitude of the off-axis maxima and minima. The angular plane wave spectrum result appears to show minima beyond the edge of the transducer at 11 mm and 15 mm from the axis, whereas these are not present in the finite element results, which show a monotonically decaying pressure amplitude in the same region of the field.

Two further calculations based on the ideal plane piston were carried out. Figure 4 shows a comparison of finite elements and the angular plane wave spectrum method at a distance from the transducer of 10 mm (the radius of the transducer). Although there are differences between the two data sets, there is better qualitative and quantitative agreement than was achieved at 1.1 mm. Once again, note the side lobes in the angular plane wave spectrum result. Figure 5 shows a comparison between the boundary element method and the angular plane wave spectrum method at the position of the last axial maximum at 15.2 mm from the transducer face (a distance just exceeding 2.5 wavelengths). There is good agreement between the two methods up to 5 mm off-axis. Both methods give virtually identical values for the -6 dB beam width of the main lobe. It is in the side-lobe structure of the angular plane wave spectrum results further from the axis where the main differences between the two data sets can be found. There are thus clear differences between the finite and boundary element results and the angular plane wave spectrum predictions and it is of interest to consider whether these might be investigated experimentally. It should be noted that a calculation of the far-field directivity pattern of the transducer in question using boundary element methods shows two side lobes on either side of the beam axis in addition to the main lobe of the beam.

In the view of the authors, the differences between the angular plane wave spectrum results and those of the finite element and boundary element calculations are likely to be due to:

1. the imposition, in the angular plane wave spectrum method, of a square sampling grid on an essentially circular problem. The presence of this "squareness" can easily be detected in the data if two-dimensional plots are displayed for any plane at which pressure values have been derived. A larger grid may reduce this effect, at the expense of longer computation times;
2. the relatively small ka value (10.5) of the transducer compared with those for which the angular plane wave spectrum method is most commonly used. If the field has components which spread rapidly in directions at large angles to the acoustic axis, combined with a relatively short near-field region, then the limitations of a finite sampling plane and a finite sampling interval may cause high spatial frequency data to be misrepresented, leading to reconstruction errors.

A possible cause of differences between the angular plane wave spectrum method and the finite element and boundary element results may arise from the effects of evanescent waves over the relatively short propagation distances employed here [3]. However, comparisons have been performed between angular plane wave spectrum predictions which include and exclude evanescent waves. Although there are small differences between the pressure predictions for the two cases at the planes of interest, these differences are insufficient to account for the discrepancies with the FE/BE results.

3. CAN THESE RESULTS BE TESTED EXPERIMENTALLY?

In the case of an ideal radiator one can resolve differences between the predictions using other modelling techniques. However, suppose the model represented a non-ideal radiator (which is likely to be the case for any attempt to model a real transducer) and that one wished to resolve the question of the differences between the various predictions experimentally. Ignoring the question of whether an ideal plane piston radiator can be manufactured, is it practicable to make measurements on a device of the kind modelled here and at the field positions of interest?

Consider the position of the first axial maximum at 1.1 mm from the transducer front face. Is it feasible, for example, to attempt a narrowband, toneburst measurement at this position? There are a number of experimental difficulties which for the purposes of this discussion will be ignored. Assume that our measurement hydrophone is free of electrical pick-up from the drive signal to the transducer, that there are no acoustic reflections back from the hydrophone to the transducer, and that the hydrophone can be mounted so that no reflections from the mount interfere with our measurement. Suppose, then, that it is possible to detect an uncorrupted signal less than $\lambda/5$ from the transducer front face. For planar scanning and beam plotting measurements, there remains the question of the directivity of the receiver to be overcome. When the hydrophone is located on the beam axis, the edge of the 10 mm radius transducer disk is at an angle of nearly 84° from the measurement position. As the hydrophone is scanned away from the axis, the angle subtended by the further edge of the piston becomes even greater. One therefore requires a hydrophone which can approximate a point receiver at 250 kHz. Note that figure 3 shows side lobe structure in the angular plane wave spectrum prediction outside the region directly in front of the transducer, but this is not present in the finite element results. To test this experimentally one needs to be able to make measurements down to pressures lower than 5% of the on-axis pressure.

An alternative approach is to use a larger, more directional receiver and to attempt to deconvolve the hydrophone's directional response from the measurements. This method has been applied by one of the authors [7] to measurements of fields scattered by tissue-mimicking materials, but requires that accurate and precise measurements of the directivity response of hydrophones be made over a wide angular range.

At the position of the axial minimum, 5.3 mm from the transducer, the directivity requirements are not so extreme, the angle between the acoustic axis and the position of the transducer edge being 62°. However, one still requires a relatively small but sensitive hydrophone in order to detect the on-axis minimum and to ensure that one is not misled as to the value of the on-axis pressure by spatial-averaging effects. Similar considerations apply to measurements to investigate the side lobe structure of the beam.

Given the apparent difficulties presented by measurements very close to the transducer, one may decide to measure further away. In the case of the angular plane wave spectrum method it is relatively easy to use amplitude and phase data at one plane to predict the amplitude and phase distribution at another plane, provided that one is able to measure phase in the plane of interest, or to reconstruct it from two sets of amplitude measurements by use of techniques such as the Gerchberg-Saxton algorithm [8] [9]. However, to obtain data which can be of use in reconstructing sharply defined features of the field such as maxima and minima one requires small spatial sampling intervals and measurements over large areas, to ensure that the highest spatial frequencies required for the reconstruction are detected. In the case of a non-ideal radiator, in which circular symmetry in the measurement plane cannot be assumed, the number of required data points can become very large. Consider the plane at the last axial maximum in the case of our model. A two-dimensional data set which encompassed a plane surface of 40 mm by 40 mm, sampled at 0.25 mm intervals, requires 25,921 data points, and even then figure 5 shows that acoustic information still appears to be present at the edge of the measurement area, and that truncation at this point may introduce DFT artefacts into the angular plane wave reconstruction. A further problem arises if one were to attempt to reconstruct the field at 1.1 mm or 5.3 mm from measurements at the last axial maximum. When one is so close to the radiating transducer, less than 3 wavelengths away in this case, evanescent waves contribute to the field and accurate back-projection in the presence of such waves may be problematic [3].

To make hydrophone measurements of sufficient precision and accuracy to allow validation of a model of the kind considered here is clearly a challenging experimental task. In the light of the considerations outlined above, it appears likely that confirmation of the size and shape of the main lobe of the acoustic beam may be possible, but that obtaining sufficient information to confirm detailed field structure such as the relative size and position of side lobes may be more problematic for transducers of the size and frequency in question.

4. CONCLUSIONS

Results of comparisons between FE and BE methods and the angular plane wave spectrum method for predicting the acoustic field radiated from a 250 kHz, 10 mm radius, ideal plane piston transducer show that whereas the finite element and boundary element predictions demonstrate close agreement with each other, the angular plane wave spectrum approach produces clear differences from FE and BE, especially in the detail of side lobe structure. Possible sources of these differences have been suggested. Analysis of experimental methods available has shown that the experimental uncertainties associated with near-field measurements limit the ability to resolve the differences between the theoretical methods.

It may be argued that the example presented here represents an extreme case. Nevertheless, it demonstrates in a very clear manner some of the important questions which have to be considered when one is attempting to compare theoretical modelling results with experimental measurements. The limitations of available experimental techniques should be borne in mind during the initial stages at which the theoretical approach to the problem is defined, otherwise one may find that the experimental measurements are not able to contribute to a resolution of the matters in question.

5. REFERENCES

1. S. I. Aanonsen et al, Distortion and harmonic generation in the nearfield of a finite amplitude sound beam, J. Acoust. Soc. Am, 75, pp 749-768, 1984
2. D.R. Bacon and A.C. Baker, Comparison of two theoretical models for predicting non-linear propagation in medical ultrasound fields, Phys. Med. Biol. Vol 34(11) pp 1633-1643, 1989
3. R. Reibold, Sound source reconstruction using Fourier optics, Acustica, 63 , pp 60-64, 1987
4. P. R. Stepanishen and K. C. Benjamin, Forward and backprojection of acoustic fields using FFT methods, J. Acoust. Soc. Am. 71, pp 803-812, 1982
5. M. A. Bangash, An investigation of the detailed characterisation of ultrasonic transducer behaviour, Ph.D thesis, University of Surrey, Guildford, UK, 1995
6. L.E. Kinsler, A.R. Frey et al, Fundamentals of acoustics, 3rd edition, John Wiley, pp 176-178, 1982
7. T.J. Esward, Problems in the ultrasonic characterisation of inhomogeneous materials due to scattering, Ph.D thesis, University of Surrey, Guildford, UK, 1995
8. A. Hurrell, The reconstruction of transducer vibration patterns from the amplitude distribution of the radiated pressure field alone, Ultrasonics, 34, pp 91-97, 1996
9. A. Hurrell and J.R.Blakey, The application of phase recovery techniques to the determination of transducer source distributions, Ultrasonics, 34, pp 99-102, 1996

ACKNOWLEDGEMENTS

The authors wish to acknowledge the support and funding of the National Measurement System Policy Unit of the UK Department of Trade and Industry.

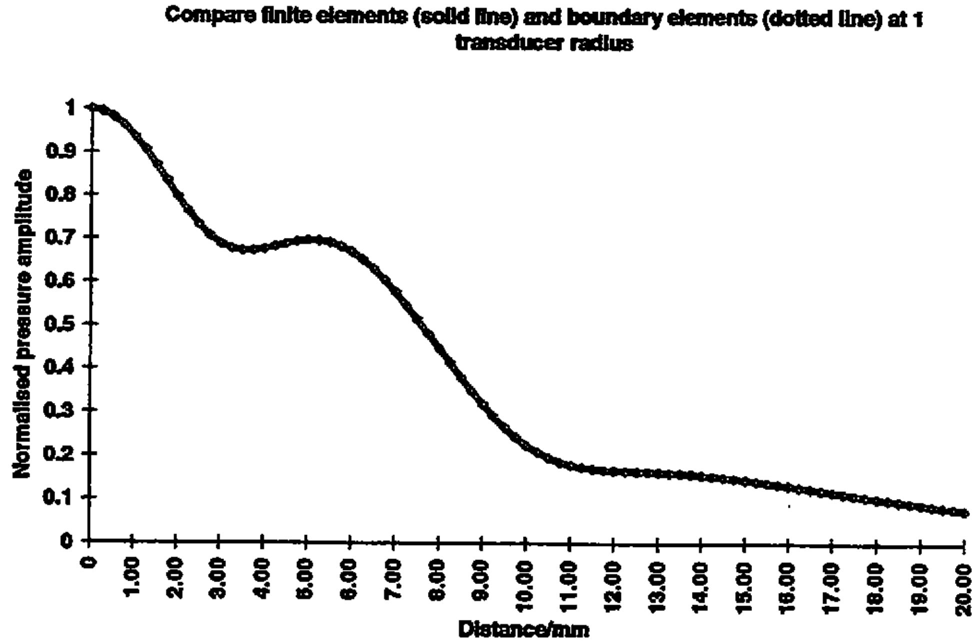

Figure 1: Field of an ideal 10 mm radius, 250 kHz plane piston: finite elements and boundary element compared at 10 mm from transducer front face.

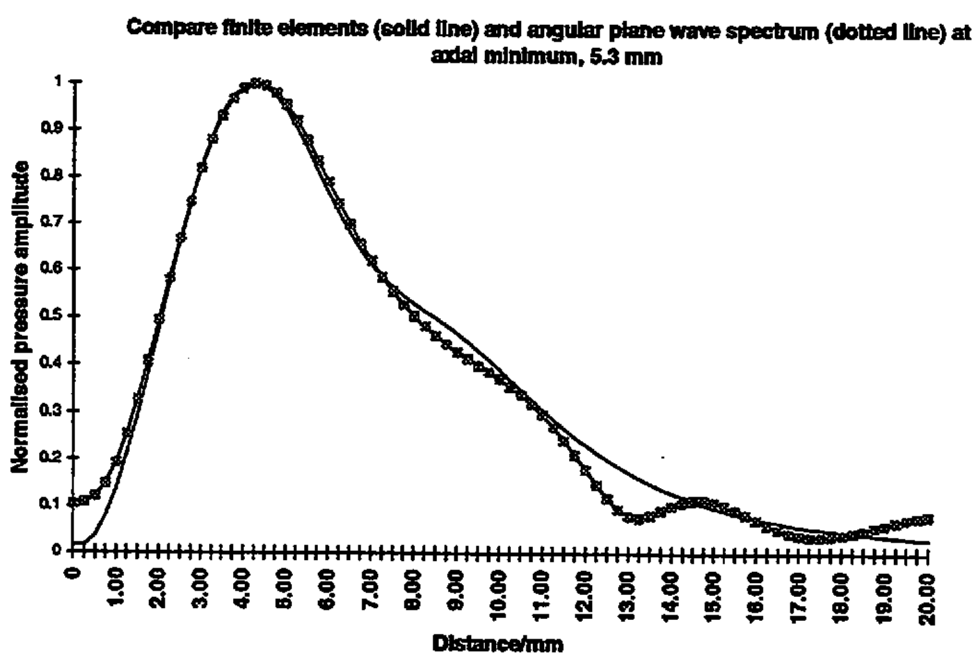

Figure 2: Field of an ideal 10 mm radius, 250 kHz plane piston: finite elements and angular plane wave spectrum compared at 5.3 mm from transducer front face.

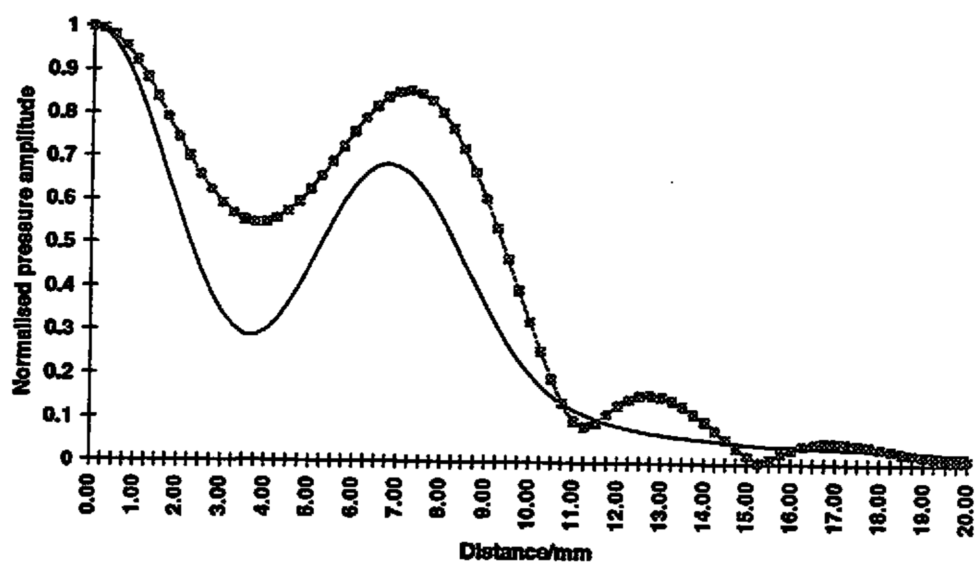

Figure 3: Field of an ideal 10 mm radius, 250 kHz plane piston: finite elements and angular plane wave spectrum compared at 1.1 mm from transducer front face.

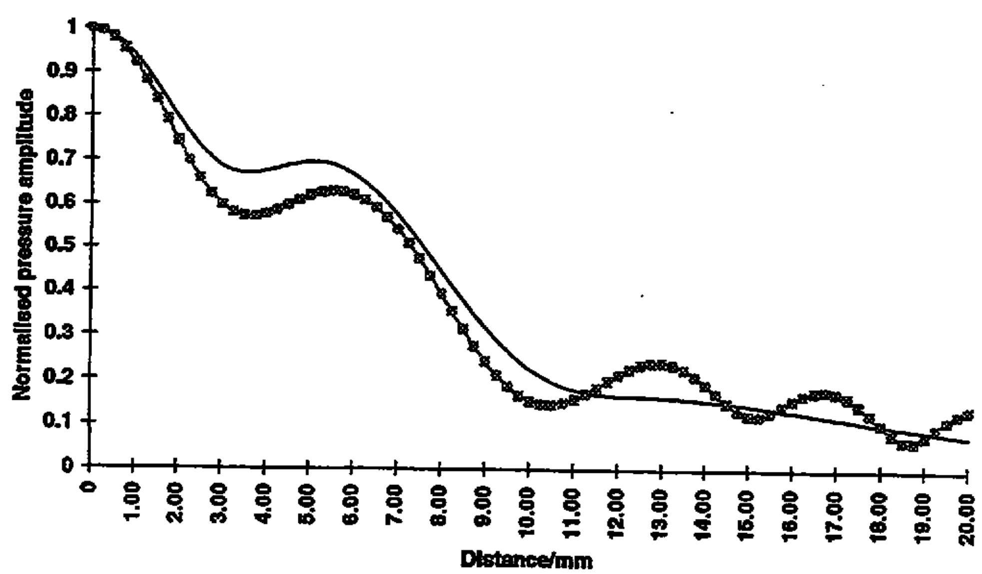

Figure 4: Field of an ideal 10 mm radius, 250 kHz plane piston: finite elements and angular plane wave spectrum compared at 10mm from transducer front face.

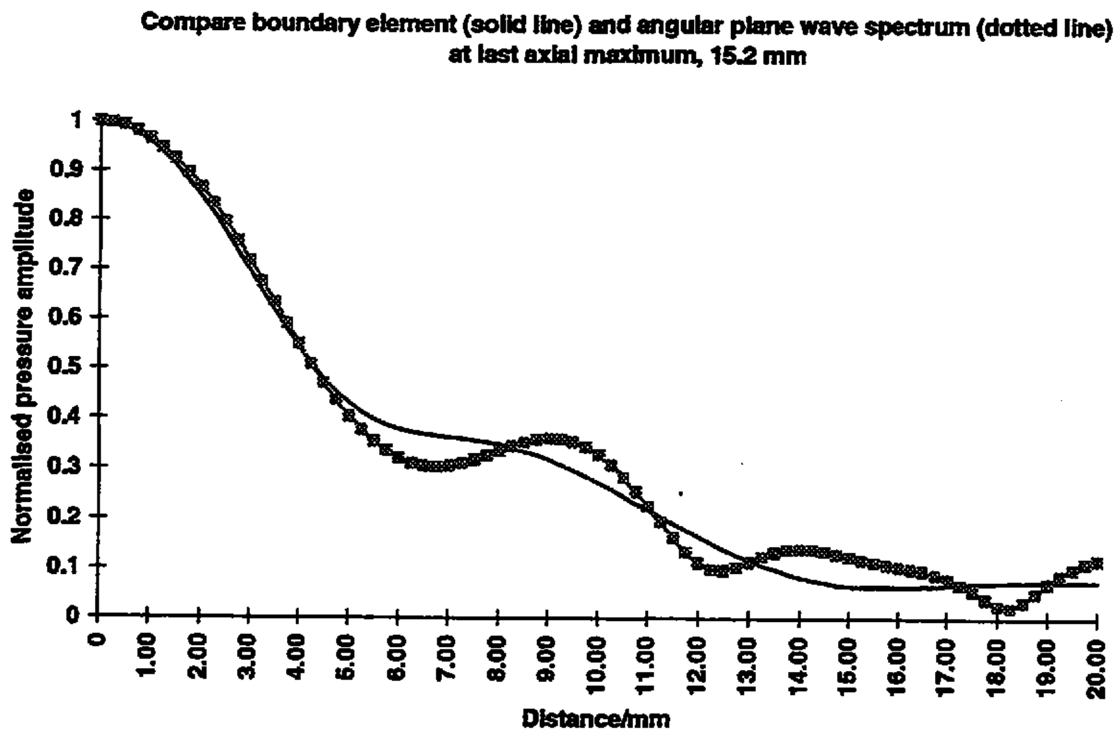

Figure 5: Field of an ideal 10 mm radius, 250 kHz plane piston: boundary element and angular plane wave spectrum compared at 15.2mm from transducer front face.