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DIFFUSION PARAMETERS FOR AUDITORIUM SURFACES

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1. INTRODUCTION

When a propagating wave strikes a surface or an object, a proportion of the incident energy will be reflected. Energy which is not reflected is either absorbed by the surface or transmitted through it. If the wavelength is small in comparison to the dimensions of the surface, Snell's law will dominate and the angle of reflection will equal the angle of incidence. Furthermore, the direction of the reflected wave will be co-planar with the incident wave and the surface normal. This is known as a specular reflection and is the basis of ray tracing geometric models. If however the wavelength is similar to a dimension of the obstruction then some of the reflected energy will be scattered and the intensity in the specular direction will consequently be reduced. If a significant fraction of the reflected energy is scattered then the reflection is termed diffuse.

Diffuse reflection of sound by surfaces in auditoria contributes to the acoustic, although the exact importance of its contribution is still unclear. Further investigation is required but it is accepted that diffusion should be considered during the design process and must be implemented in prediction models if accuracy is not to be compromised [1].

Conceptually, the most straightforward incorporation of diffusion into designs and models would be to define a metric which could quantify the diffusivity of any surface or object. This *diffusion coefficient* would be complementary to Sabine's absorption coefficient in describing the acoustic properties of a surface. From the value of the diffusion coefficient, architects, consultants and modellers would know how the diffusivity of the surface rates between the two extremes of *specular reflection* and *ideally diffuse*. This would ultimately be of benefit to the users of auditoria.

In recent years a number of metrics have been proposed [2-12] and D'Antonio chairs an Audio Engineering Society Subcommittee on Acoustics for the Characterization of Acoustical Materials, SC-04-02, which has the aim of establishing standards in this area. However, no accepted diffusion coefficient for characterising surfaces presently exists, although some prediction programs have included a form of diffuse reflection modelling. This paper presents some preliminary results from a study titled *"The Development of a Diffusion Coefficient for Room Acoustics"*.

2. QUANTIFYING DIFFUSION

2.1 Methods not using the Directional Distribution of Scattered Energy.

One measure which quantifies the diffusivity of a surface or object is the ratio of diffuse and total energy reflected. (The diffuse energy being that which is not reflected in a specular direction). Two methods for evaluating this diffusion coefficient have been developed by Mommertz and Vorländer [11], both of which involve measuring the impulse response for various orientations of the sample using maximum length sequences. One method requires anechoic conditions and the other a reverberant space. Whichever method is used, the total reflected energy can be readily extracted from the measurements and phase-locked superposition of a sufficient number of impulse responses yields the specular energy. Evaluation of the required ratio from these two quantities is then straightforward.

Lam [12] has suggested a method for rating the diffusion created by a sample based upon room diffusivity. A non-diffuse space is created - for example a reverberant room with a large amount of absorption on one surface. The degree of non-diffuseness is quantified by, say, measuring pressure distributions around the room. The surface or object is then introduced into the room and the degree of non-diffuseness re-measured. From the improvement in diffusivity, a diffusion coefficient is derived.

All of these methods are simple in concept and the measurements are relatively straightforward. However, the influence of edge effects remains to be investigated and there are no simple prediction methods to replicate the measured results. It is important that a diffusion coefficient is predictable given the cost and time required for measurements. Furthermore, it is not known whether either method correctly ranks the diffusivity of different surfaces and objects.

Although the methods of Mommertz and Lam will be examined as part of the study, the remainder of this paper is limited to consideration of parameters which quantify the diffusivity of samples by examining how closely the directional distribution of scattered energy resembles the ideal.

2.2 Defining Ideal Diffusion.

When considering the propagation of sound in auditoria, analogies are sometimes drawn with optics, where an ideal diffuser is defined as scattering the energy in an incident light ray in accordance with Lambert's cosine law [13]. Indeed, this is the definition commonly used in geometric models. However, Lambert's law should not be applied to room acoustics except at high frequencies because it is valid only in the case of incoherent small wavelength scattering from a point. The large wavelengths and finite sized surfaces encountered in auditoria lead to coherent diffuse reflection of sound, the pressure in the reflected sound field being determined by interference effects. If this were not the case then profiled diffusers based on wells, such as those described by Schroeder [14], would not work.

The simplest definition of an ideal diffuser is that it scatters the energy contained in an incident sound field uniformly into 2π space, i.e. the reflected intensity in all directions - including the specular - is the same. In some applications it may be more desirable for the diffuser to direct the reflected energy away from the specular directions, i.e. to create a notch in the directional distribution of scattered energy. Cox [4] has used this definition to evaluate the quality of small to medium sized diffusers mounted on large baffles. An example of this type of application is the use of diffusers attached to the rear wall of a concert hall to prevent echoes from directional musical instruments such as trumpets.

2.3 Practical Considerations.

A problem with any diffusion coefficient which is calculated from the directional distribution of scattered intensity is that it is only in the far field that the shape of this distribution is independent of the distance from the sample. This is illustrated in Figure 1, which shows how the directional distribution of scattered Sound Pressure Level (SPL) from two plane profiles of different widths varies with measurement distance at different frequencies. These two-dimensional distributions were predicted using a BEM technique [15] and the results are in octave bands. The thick solid line is the far field distribution, with the thin solid and dotted lines corresponding to the distribution obtained using measurement distances of 10m and 3m respectively. In all cases the distributions are symmetrical about 0° and the source is positioned 0.1m behind the 0° measurement position.

In order for the value of the diffusion coefficient not to depend upon the measurement distance, the radius of the semicircle or hemisphere on which the measurement positions lie must be large enough to ensure that in every direction the measurement is made in the far field. It is evident from Figure 1 that this is difficult to achieve because the near field increases in extent as the product of profile width and frequency increases, which means that the required radius can become impractically large. Furthermore, as the angle of incidence measured from the surface normal increases, the near field becomes even more extensive and reaching the far field becomes an impossibility for the sizes of surfaces and frequencies of interest in auditoria as grazing incidence is approached. Consequently, the perceived ideal of characterising diffusivity from the far field distribution of scattered intensity may not be practically realisable.

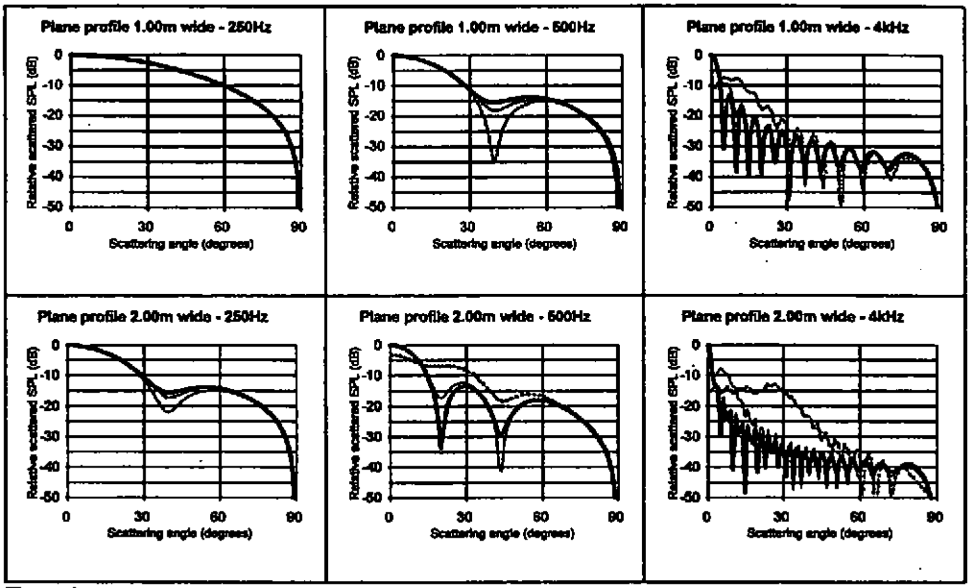


Figure 1

In any case, audiences in auditoria are usually situated within the near field of large structural elements such as walls, floors and ceilings and this presents a serious problem. For example, consider the case shown in Figure 2 where the focal point of a large concave surface is within an audience area. If measurements of scattered intensity are made at typical audience positions, focussing will be observed. If, however, measurements are made in the far field, the reflection from the concave surface will appear diffuse.

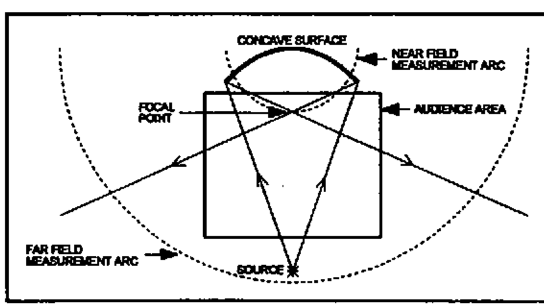


Figure 2

With a plane surface the situation is reversed: if the directional distribution of scattered energy is measured in the far field then a distinct specular reflection is observed and the surface will be classified as a poor diffuser. However, if all the measurement positions are in the near field then to each position a specular reflection path will exist and as a result the plane surface will be rated as an ideal diffuser!

Within auditoria, surfaces and objects which have the potential to produce diffuse reflections can be loosely divided into three categories:

- Objects situated in free space, such as suspended reflectors and some structural pillars.
- Large rough surfaces which may effectively extend to infinity, such as coffered ceilings and the audience.
- Objects mounted on or within large plane surfaces, such as decorative figurines and half pillars on otherwise flat walls.

Defining these categories is useful because it provides three different situations in which diffusion coefficients can be tested. It may be that a different parameter is required for quantifying diffusion in each situation; indeed none of the metrics reported in the literature can be successfully applied in all three cases, generally they are applicable only to the first. However, it is the aim of this study to develop a unified parameter.

3. BRIEF REVIEW OF DIFFUSION COEFFICIENTS IN THE LITERATURE

3.1 Requirements of the Ideal Diffusion Coefficient.

The ideal diffusion coefficient would:

- have a firm physical basis.
- be applicable to all three situations outlined in Section 2.3.
- be consistent in evaluating and ranking the diffusivity of samples.
- be bounded between zero and unity.
- yield values for practical surfaces and objects which are evenly spread over the range bounded by specular reflection - the worst case - and ideal diffusion.
- be straightforward to measure and predict.

3.2 Standard Deviation and Directivity Types.

Different approaches to judging the closeness of measured or predicted directional distributions of scattered intensity or pressure to the ideal case have resulted in a number of metrics for quantifying diffusion being proposed [2-10]. Although these metrics all differ from one another in some aspect, each of them can be classified as being an example of one of the two broad types of diffusion parameter shown in Equations 1 and 2 below: Standard Deviation (modified), $\epsilon_{s.d.}$ and Directivity, ϵ_{dir}.

$$\epsilon_{s.d.} = \sqrt{\frac{\sum_{i=1}^{n}\left[L_{p,i}-\overline{L_p}\right]^2}{(n-1)}} \quad , \quad \overline{L_p} = 10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}10^{\frac{L_{p,i}}{10}}\right] \tag{1}$$

$$\epsilon_{dir} = \sqrt{\sum_{i=1}^{n}\left[\frac{I_i}{I_{total}}-\frac{1}{n}\right]^2} \quad , \quad I_{total} = \sum_{i=1}^{n} I_i \tag{2}$$

where n is the number of measurement directions, $L_{p,i}$ is the scattered SPL in direction i and I_i is the scattered intensity in direction i.

3.2.1 Physical Basis. The basis of standard deviation type diffusion coefficients is that for a perfect diffuser, the difference between the scattered SPL in direction i and the mean scattered SPL is zero. Directivity type coefficients, on the other hand, utilise the feature of the ideal case that the fraction of the total intensity scattered in direction i is a constant equal to the reciprocal of the number of measurement directions. Note that $\epsilon_{s.d.}$ is termed a modified standard deviation of the scattered SPL because $\bar{L}_p$ is the level corresponding to the mean scattered intensity and not the arithmetic mean SPL. It has been shown that calculating $\bar{L}_p$ in this manner results in a more intuitive rating of diffusivity because it penalises distributions which contain large sections of low SPL [5].

Although both $\epsilon_{s.d.}$ and ϵ_{dir} are based on ideal diffusion being defined as equal energy scattered in all directions, their application is not limited to situations where this is the ideal case. If the ideal is not uniform scattering, all that is required is to multiply the directional energy distribution being rated by the inverse of the ideal distribution prior to evaluation. The resulting product will be independent of direction for an ideal diffuser, regardless of the shape of the ideal distribution.

3.2.2 Practical Values. One of the problems with both types of coefficient is that although they are zero in the case of ideal diffusion, their values in the theoretical worst case - where there is non-zero scattered intensity in only a single direction - is not unity. A value between zero and unity can be obtained by dividing the coefficients by their worst case values. However, because practical scattered intensity distributions remain somewhat removed from the worst case, even when the reflection would be regarded as specular, the majority of practical values of these coefficients are bunched together in the lower end of the range as shown in Figure 3. All of these distributions are for normal incidence and, as indicated, some are predictions whereas others are the results of measurements made at 1:5 scale [8].

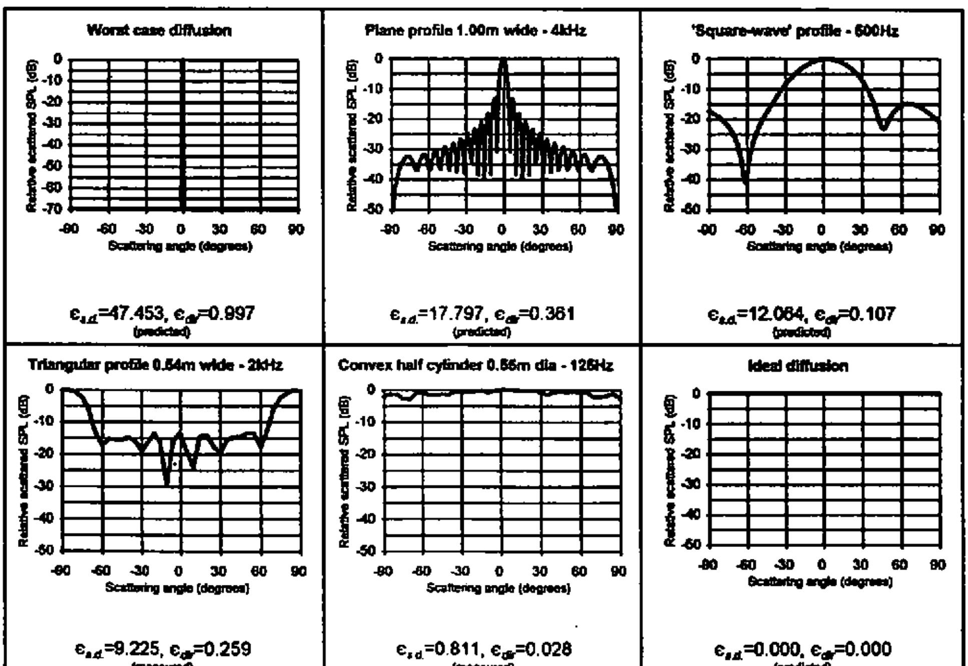


Figure 3

To overcome the problem of bunching described above, the normalisation could involve division by the worst practical value instead of by the theoretical worst case limit. However, what should this worst case value be? If a diffusion coefficient in excess of the chosen 'worst case' were subsequently measured then the resulting normalised value would be greater than unity, thus negating the purpose of the normalisation.

3.2.3 Relationship between $\epsilon_{s.d.}$ and ϵ_{dir}. The similarity between the two coefficients is greater than might initially be apparent. If the SPLs in the expression for $\epsilon_{s.d.}$ in Equation 1 are written in terms of the corresponding intensities and the resulting logarithm is represented by the first term of its series expansion, the expression for $\epsilon_{s.d.}$ becomes Equation 3. Also, since the total scattered intensity must be equal to the product of the mean scattered intensity and the number of measurement directions, ϵ_{dir} can be written as shown in Equation 4.

$$\epsilon_{s.d.} = \frac{10}{\bar{I}\ln(10)}\sqrt{\frac{\sum_{i=1}^{n}\left[I_i-\bar{I}\right]^2}{(n-1)}} \qquad (3)$$

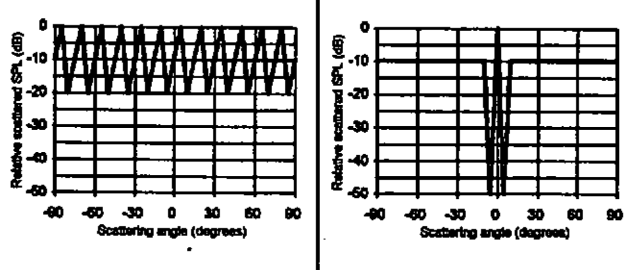
$$\epsilon_{dir} = \frac{1}{\bar{I}\sqrt{n}}\sqrt{\frac{\sum_{i=1}^{n}\left[I_i-\bar{I}\right]^2}{n}} \qquad (4)$$

where $\bar{I}$ is the mean scattered intensity.

However, since the expansion of the natural logarithm is valid only when the scattered intensity in every direction is similar to the mean, Equation 3 is only applicable to good diffusers. In fact in order for the higher terms in the series to be neglected, the distribution must approach uniformity. If this is the case then comparison of Equations 3 and 4 shows that the two coefficients differ only by a constant factor if n is large.

3.2.4 Physical Interpretation. In addition to these two types of diffusion coefficients not being bounded between zero and unity and practical values not being distributed over the whole range, it is difficult to interpret the physical significance of values other than the extremes. What, for example, can be learnt about the diffusion properties of a surface or object from the knowledge that its $\epsilon_{s.d.}$ or ϵ_{dir} value is, say, 0.5?

Figure 4 shows that neither coefficient discriminates between the case of the scattered intensity exhibiting perhaps significant variation about the mean but never deviating from it by a large amount and that where in the majority of directions, the scattered intensity is very similar to the mean but in a few directions there are considerable aberrations.



$\epsilon_{s.d.}$=9.942, ϵ_{dir}=0.201 $\epsilon_{s.d.}$=9.756, ϵ_{dir}=0.205

Figure 4

Furthermore, $\epsilon_{s.d.}$ and ϵ_{dir} are dependent only on the set of values which comprises the scattered intensity distribution, not the order in which these values are arranged. Consequently, two samples which produce apparently quite different distributions will have identical values of $\epsilon_{s.d.}$ and ϵ_{dir} if both distributions are simply different arrangements of the same set of values. In most applications this would be inconsequential but the parameters are always vulnerable to a special case where the shape of the distribution is important.

3.3 Specular Zone Type.

One metric for quantifying diffusion which is conceptually simple, similar to some which are presently used in geometric models and where the physical significance of all values can be easily understood is the ratio of diffuse and total reflected energy. When evaluating this parameter, the range of directions in which specular reflections are possible - the *specular zone* - is usually defined as that of a plane profile identical in size to the normal incidence projection of the diffuser, as shown in Figure 5.

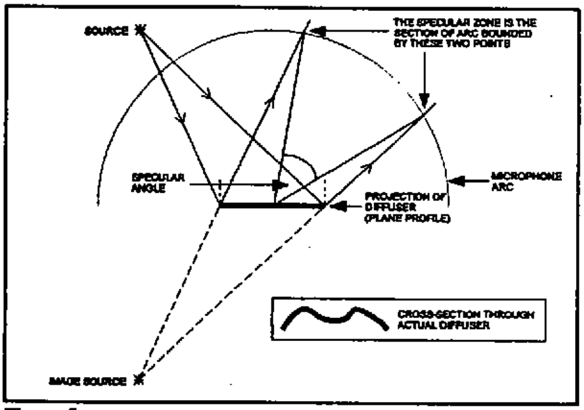


Figure 5

The simplicity of interpreting this metric, however, leads to it being fooled by some very simple cases. One example is a plane profile orientated so as to direct the reflected sound out of the specular zone. This would be rated as an excellent diffuser when in it is in fact simply re-directing a specular reflection. Furthermore, although the parameter is theoretically bounded between zero and unity, it is unlikely that values close to unity would be obtained in practice because no known diffuser can produce a perfect notch with zero energy in the specular zone. Difficulties in formulating an alternative definition of the specular zone have limited the development of this initially promising type of diffusion coefficient, although Mommertz method is a fresh approach.

3.4 Averaging over Frequency and Angles of Incidence.

The value of any diffusion coefficient calculated from the directional distribution of scattered intensity will be dependent on both the frequency content of the incident sound and its angle of incidence. In order to reduce the size of a potentially very large set of diffusivity values, it is necessary to perform some kind of averaging.

The diffusivity of a surface or object is a function of frequency because it is dependent on the size of a wavelength in comparison to its dimensions. If all the dimensions of the sample are much smaller than half a wavelength then its presence will have little effect; the sound will simply diffract around it. However, if the frequency of the incident sound is such that half a wavelength is approximately equal to a dimension of the sample, for example its width or the extent of its surface roughness, then a diffuse reflection is likely to result. At higher frequencies, where half a wavelength is small in comparison to the size of the sample, a more specular reflection would be expected.

The situation as regards averaging is very similar to that which presently exists with absorption coefficients. The standard procedure for measurement of absorption [16] involves diffuse sound which is incident from all angles in 2π space, the results are thus averages over these angles of incidence. If however the absorption coefficient of a material for a particular angle of incidence is required, it can be calculated from the results of an impedance tube measurement. If this approach were applied to the characterisation of diffusivity then normal practice would be to quote ⅓-octave diffusion coefficient values which are averaged over angles of incidence. If the diffusion coefficient for a particular incident angle were required, it could be measured or predicted separately

4. A NEW DIFFUSION COEFFICIENT

4.1 Definition.

The property of ideal diffusion that the scattered intensity in each direction is the same as that in all other directions has led to the development of a new metric for quantifying diffusion which satisfies more of the requirements of an ideal diffusion coefficient than standard deviation or directivity types. This parameter, ϵ_{auto}, is based upon the mean of the circular autocorrelation coefficient of the directional distribution of scattered intensity and can be expressed as shown in Equation 5. In the ideal case the value of ϵ_{auto} is unity and in the worst case of non-zero intensity in only a single direction, its value is $^1/_n$. A coefficient ϵ'_{auto} which is bounded between unity and zero can thus be obtained by the normalisation procedure shown in Equation 6

$$\epsilon_{auto} = \frac{\left[\sum_{i=1}^{n} I_i\right]^2}{n\sum_{i=1}^{n} I_i^2} \qquad (5) \qquad\qquad \epsilon'_{auto} = \frac{\epsilon_{auto}-\frac{1}{n}}{1-\frac{1}{n}} = \frac{n\epsilon_{auto}-1}{n-1} \qquad (6)$$

4.2 Properties.

Practical values of ϵ'_{auto} are distributed between zero and unity in a different manner to the corresponding values of $\epsilon_{s.d.}$ and ϵ_{dir}, as shown in Figure 6.

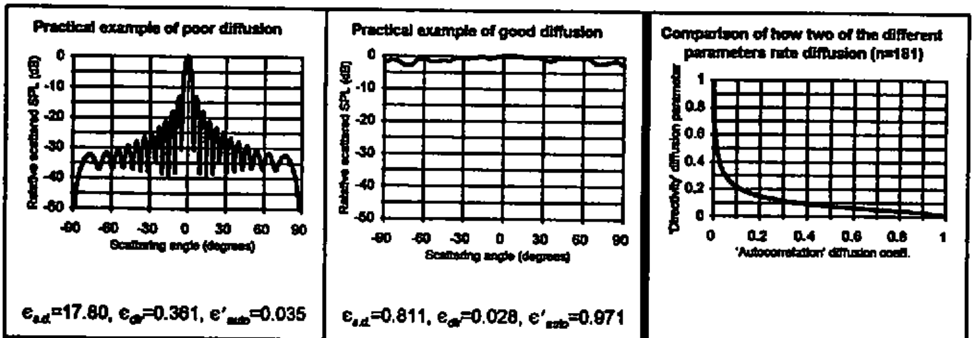


Figure 6

Although ϵ'_{auto} ranks the diffusivity of samples in the same order as ϵ_{dir} - in fact there is quite a simple mathematical relationship between the two parameters - the upper half of its range is compressed into less than the bottom tenth of the range of ϵ_{dir}. The effect of this is that two samples which are rated as moderate and good by ϵ'_{auto} have very similar ϵ_{dir} values but two which are both rated as poor by ϵ'_{auto} have quite different values of ϵ_{dir}. This is also true to a lesser extent for the corresponding $\epsilon_{s.d.}$ values. For quantifying the diffusivity of surfaces and objects in auditoria, ϵ'_{auto} is the better measure because values obtained in practice are distributed over the whole range of possible values rather than being bunched together.

ϵ'_{auto} shares two important features with standard deviation and directivity type coefficients. Firstly, its value is determined by the set of scattered intensity values of which the directional distribution is comprised and not the order or position in which individual values occur. Secondly, it is independent of the absolute values of the scattered intensities; if in every direction the scattered intensity changes by xdB, the values of these diffusion coefficients remain the same. This is a desirable feature because it is diffusivity that the coefficients are intended to characterise, not absorption. In order to ascertain whether a surface is primarily diffusing or primarily absorptive at any particular frequency, the values of both the diffusion and the absorption coefficients are required.

The chief shortcoming of ϵ'_{auto} is that it is no easier to physically interpret non-extreme values than it is with standard deviation and directivity type coefficients. Knowledge that a surface has an ϵ'_{auto} value of, say, 0.5 reveals only that its diffusivity is rated as moderate. It does not provide any information about the shape of the scattered intensity distribution.

6. CONCLUSIONS

A review of methods for characterising diffusion from surfaces has been carried out. The criteria for an ideal diffusion coefficient has been presented. Several different diffusion coefficient and parameter definitions are published in the literature, although none is ideal. As well as summarising those existing parameters, this paper has introduced a new parameter based on the autocorrelation function. Although the autocorrelation parameter still fails to meet all the criteria for an ideal diffusion coefficient, it appears to be superior to other diffusion coefficients based upon the directional distribution of scattered intensity.

7. ACKNOWLEDGEMENTS

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