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CALCULATION OF WAVE PROPAGATION AND ENERGY TRANSMISSION IN RIB-STIFFENED PLATE STRUCTURES

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1. INTRODUCTION

The present work aims at analysing wave propagation and energy flux in built-up plate structures typical of ship-building. A semi-analytical finite element (FE) approach has been utilized to calculate dispersion curves for geometries typical of rib-stiffened hull plates. The approach applies finite elements to model the cross-section of the waveguide whereas the displacement in the direction of propagation is described by a harmonically oscillating function resembling a propagating wave. In an existing prediction model [1], the energy is assumed to propagate solely in the plate elements. However, at low frequencies the stiffeners are vibrating with the plates [2] and a different description of the wave field is preferable. The present calculations are used to validate simple analytical models representing the built-up rib-stiffened plating. The method also provides detailed information on the wave types that can propagate in rib-stiffened structures.

2. COMPUTATION OF DISPERSION PROPERTIES

For the virtual work formulation we assume the waveguide system to be built up of a finite number of flat, thin-walled strip elements. The geometry of such an element is defined by its width, denoted by $2a$, and its thickness $h(y)$. The waveguide is oriented in the x-direction and the elements are thus taken to be infinitely extended in the x-direction. Plate thickness and mass density may only vary with the local y-coordinate. To represent a wave propagating in the direction of the waveguide, displacements of the form: $u_k(x,y,z,t) = \mathrm{Re}\left(\hat{u}_k(y,z)e^{i(\omega t-\kappa x)}\right)$ are sought; κ being the unknown propagation constant and ω the angular frequency.

The thin-walled shell elements are able to withstand both membrane and bending strain and the virtual work for free waves is formulated for in-plane and bending displacement separately. We thus have [3]

$$\delta(\mathrm{L}_{ip}) = \int dx \int_{-a}^{a} h\left(\delta\{\hat{\varepsilon}^*\}^T \mathbf{D}\{\hat{\varepsilon}\} - \rho\omega^2(\delta\hat{u}^*\hat{u} + \delta\hat{v}^*\hat{v})\right) dy = 0, \tag{1}$$

$$\delta(\mathrm{L}_b) = \int dx \int_{-a}^{a} h\left(\frac{h^2}{12}\delta\{\hat{\chi}^*\}^T \mathbf{D}\{\hat{\chi}\} - \rho\omega^2\delta\hat{w}^*\hat{w}\right) dy = 0, \tag{2}$$

where and $\{\hat{\varepsilon}\}$ and $\{\hat{\chi}\}$ are defined as

$$\{\hat{\varepsilon}\} = \left[\frac{\partial\hat{u}}{\partial x}, \frac{\partial\hat{v}}{\partial y}, \frac{\partial\hat{u}}{\partial y} + \frac{\partial\hat{v}}{\partial x}\right]^T, \{\hat{\chi}\} = \left[\frac{\partial^2\hat{w}}{\partial x^2}, \frac{\partial^2\hat{w}}{\partial y^2}, 2\frac{\partial^2\hat{w}}{\partial x \partial y}\right]^T, \tag{3a-b}$$

D represents the stiffness matrix, $\rho(y)$ is the mass density of the plate element and $\hat{u}, \hat{v}$ and $\hat{w}$ are temporal Fourier transforms of the displacements in the x-, y- and z- directions. Virtual quantities are denoted δ while the asterisk denotes the complex conjugate. The displacement field of the cross-section of a strip-element is represented by simple polynomials [4].

Assembling the local mass and stiffness matrices according to standard procedures for finite elements, a system of equations representing the motion for he built-up waveguide system is arrived at. This linear system may be written as

$$[\mathbf{K}(\kappa) - \omega^2\mathbf{M}]\{\hat{\mathbf{u}}_g\} = \{0\}, \tag{4}$$

where $\{\hat{\mathbf{u}}_g\}$ represents the mode shapes of the cross-modes and $\mathbf{K}(\kappa)$ is a fourth order matrix polynomial in κ.

To rewrite equation (4) as a standard eigenproblem in κ, a series of linear transformations are applied. The procedure is detailed in [3]. The resulting eigenvalue system is roughly three times larger than the original system. It is unfortunately also non-symmetric and its numerical solution time consuming.

At each frequency we can expect to find eigenvalues corresponding to four times the number of degrees of freedom in the FE-model. Some of these are purely real, representing propagating waves. Others are purely imaginary, representing evanescent modes, i.e. modes that decay exponentially. In addition, complex eigenvalues appear. These are associated with exponentially decaying standing waves, generally found in systems with several different characteristic stiffnesses; e.g. orthotropic plates. Here, the focus is on propagating waves and for the calculational example only those eigenvalues are considered.

3. CALCULATION EXAMPLE

A waveguide system with three stiffeners, as illustrated in Figure 2, is analysed. The cross-section is chosen to correspond to typical geometries of a ship's hull. The material is steel. A mesh with 37 nodes, evenly distributed over the cross-section is established and the system is analysed with the FE-technique outlined above.

In Figure 3, the FE-results for propagating waves are plotted together with the six first flexural modes of an equivalent orthotropic plate for which the inertia and stiffness of the ribs is smeared out on the plating [3]. The correspondence between the FE-results and those calculated analytically is good at low frequencies.

At higher frequencies, in the plate region of the modes, the numerically obtained dispersion curves diverge from those obtained analytically. Here, the wavenumbers may be calculated from the properties of the base plate or the ribs, treated as separate structural members.

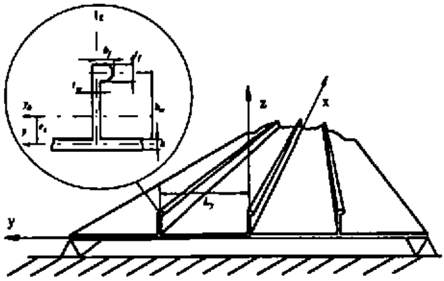

Figure 2. Plate waveguide system with three rib-stiffeners.

In the plate region, the dispersion curves are clustered in groups. For all modes in such a group, the number of nodal lines between two adjacent stiffeners is the same, as shown in Figure 4. At high frequencies, some of the dispersion curves in the second group tend to a line below, but parallel to, the dispersion curve of the base plate as seen in the upper right corner of Figure 3. These represent waves that predominantly propagate in the rib-stiffeners.

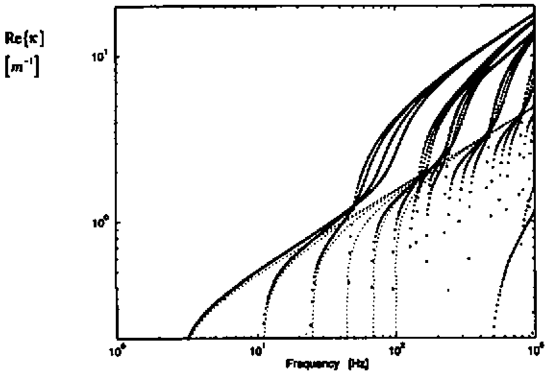

Figure 3. Calculated wavenumbers for propagating waves. Together with the FE-results, wavenumbers of an equivalent orthotropic plate are shown : ·····, FE-results; ‾ ‾, equivalent plate.

Also above the cut-on frequency of the first plate mode, most of the FE-modes have wavenumbers in the range of those determined by equivalent plate theory, i.e. show "stiff" behavior, in a frequency region just above cut-on. Observe that stiff modes appear in the entire frequency region analysed.

The modal group velocity corresponds to the rate the mode is propagating energy. In Figure 5 normalized group velocities, c_0, are shown for the first four modes obtained from numerical differentiation of the FE-dispersion curves. In the low frequency region, where the modes are stiff, c_0 is much larger than in the high frequency region. For e.g. the first mode, the rate of energy transfer is more than three times greater in the stiff region than in the plate region when comparing waves with the same amplitude.

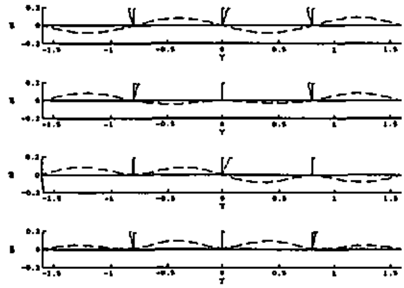

Figure 4. Displacement functions of the four lowest modes at 102 Hz illustrating typical plate modes.

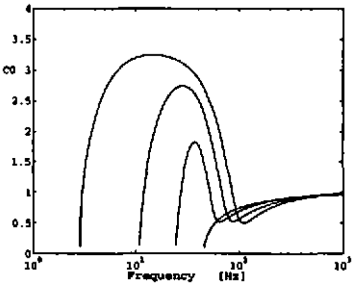

Figure 5. Non-dimensional modal group velocity for the first four modes of stiffened plate system.

4.CONCLUSIONS

Wave propagation in stiffened plate systems typical of a ship's hull has been analysed. It is shown that the flexural waves in such structures can be divided into three groups with respect to which parts of the structure that determine the wavenumbers. The first two groups of waves propagate either predominantly in the plate elements or in the ribs. The propagation constants of the modes in these groups can thus be determined from the properties of the base plate or the stiffening ribs respectively, treated as separate structural members.

For a third group of modes, the rib-stiffeners deflect with the base plate of the structure. These modes are referred to as *structurally stiff* in a frequency region just above cut-on. Below the cut-on frequency of the first plate mode, only stiff modes propagate energy. In this frequency region the calculated wave numbers compare well with those determined by so-called *equivalent plate* theory for which the plating is represented with an equivalent orthotropic plate. In the investigated frequency region, there are modes with stiff characteristics at practically all frequencies. Equivalent plate theory gives reasonable estimates to the wavenumbers in the stiff region of the modes also at higher frequencies.

Also the modal group velocities was formed. It is concluded that the rate of energy transfer in the stiff modes can be more than three times greater than in the plate modes when comparing waves with the same amplitude. For this reason it is important to correctly model the distribution of plate waves and waves propagating in stiff modes, when energy transmission predictions are attempted. This is significant, not only for applications of the waveguide model [1] but also when using e.g. Statistical Energy Analysis. It is suggested that equivalent plate theory is used below the cut-on frequencies of the first plate mode. At higher frequencies, both plate modes and stiff modes are likely to propagate energy. The relative importance of these wave types must be determined from the excitation.

5. REFERENCES

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