QUANTIZED PYRAMIDAL BEAM TRACING OR A SOUND-PARTICLE-RADIOSITY- ALGORITHM ?

New solutions for the simulation of diffraction without explosion of computation time

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1. Introduction - State of the Art of numerical methods in room acoustics

In geometrical room acoustics, noise immission as well as in illumination prognosis – fields with the common assumption of wavelengths being small compared with significant room dimensions - three basic algorithms are used. These are, with increasing reflection order: The mirror image source method (MISM), Ray Tracing Methods (RTM) and the radiosity method (RadM).
The MISM is (for the optical limiting case) an exact and deterministic method replacing any situation of multiple surface reflections by the direct immission from equivalent mirror image sources, naturally only handling geometric reflections. However, the MISM suffers under creating a huge number of MIS and most of them are invisible. Creating these MIS recursively, as with the classical MISM, this can principally not be avoided (s. fig. 1 an text below).

Ray tracing methods (RTM), "monte carlo", resp. statistical and numerical methods, simulate sound (or light) propagation directly, thereby allowing as well geometric as diffuse reflections being detected in spatially extended detectors; although statistic, ray tracing turns out to be (with higher reflection) order much more efficient than the MISM [1], however tends to under-sample the rooms structure with even higher order; therefore RTM are often combined with the MISM, i.e. RT serves in finding mirror sources which are then verified by back-tracing.

A method, well known in optical, seldom in acoustical ray tracing, is beam tracing (BTM). Beams are rays with increasing cross section, therefore the receivers may be points; beams, in turn, may be subdivided in cones (advantage: simple, disadvantage: overlapping and creating multiple but not existing sources), and pyramidal beams (originating from connecting a point source with a wall polygon). Neighbouring pyramidal beams, in the following simply called ***pyrs***, perfectly fit together, there are neither overlaps nor wholes. Pyramidal beam tracing (PBT), however, is geometrically so complicated, that (having anyway some scattering in mind) it has been seldom applied in room acoustics up to now. This is the more the case, if pyrs hitting by part different walls are recursively split-up. This version of PBT means nothing else than a very efficient mirror source method without waste. In room acoustics, PBT was still seldom applied, firstly in *Virtual Reality* resp. real time applications with its typical limitation to low order reflections or without scattering[3].

On the other side, mainly in order to roughly simulate diffuse reflections in late reverberation, well known from the simulation of illumination, the radiosity method has increasingly been employed,. For acousticians, the RadM is nothing else than the numerical formulation of Kuttruffs integral equation[4.] Basis idea is the pair-wise illumination of surfaces patches for which, once in advance, a constant matrix of form factors is set up, dependant only from geometry. The result, i.e. the illumination distribution, is obtained by simply solving a (large) linear equation system. If time dependant, the RadM works as an iteration yielding the sound decay at any point in the room[4]. Remarkable is: the RadM (in opposite to the RTM) gets along with a finite number of paths of energy interchange, which are not only split-up but also re-unified, hence do not grow exponentially. But the decisive restriction is: The pair-wise approach of the RadM works only with completely diffuse reflections, "forgetting" their pasts. Also some hybrid methods have been suggested, for ex. combining BTM and RadM[2], or using by part the "lumping-together-effect" of the RadM[5].

However, none of these geometric methods is able to account for diffraction or specific angle dependant scattering, mainly happening at by-passing of edges. (Diffuse reflections may be considered as only the most simple way of improvement.) Up to today, this is still their main deficit becoming important mainly if room acoustics is extended to sound propagation in open rooms (built-up-areas), but also in the quality of auralization.

Introduction of diffraction was first tried by the author in 1986 in the way to extend the sound particle model. Problem: An exact hitting of diffracting edges is, with such monte- carlo-methods, never the case. So, deterministic diffraction models [6,7,8] basing on Fermat´s principle are not applicable. The critical situation is therefore only the by-passing of inner edges (wedges). The idea was to utilize the “uncertainty relation”: the nearer the by-passing, the stronger the particle´s deflection (either energies of deflected particles or the deflection angle probability density (DAPD)). This heuristical model[9] yielded astonishingly good agreements with the given angle functions as well with single screens as with slits, however was quite heuristic. In 1993 the author succeeded with another approach for the diffraction of rays near edges analytically deriving a by-pass-distance-dependant DAPD function from considering the change of the energy flow around the edge with a shift of the screen[10]. However, this model was not extendable to simultaneous by-passing at multiple edges. Also, the crucial basic algorithmic problem was not solved:

With the introduction of any scattering or diffraction, rays would have to be split up recursively, the number of rays and hence computation time would grow exponentially –
the problem of the tree structure.

It is therefore the goal, to extend the geometrical methods by diffraction effects, keeping at the model of energetic superposition, assuming rooms to be large, but not very large compared with wavelengths, fulfilling at least the well known law of detour valid at simple screens[6], generalized to arbitrary combinations of reflections and diffractions - but avoiding explosion of computation time. Wanted is, rather than any patched-up-method, one single new universal algorithm. In the following, rather than the physical, mainly this algorithmic problem is addressed.

2. Summary of earlier ideas for the solution: „Quantized Pyramidal Beam Tracing” (QPBT)

The invention of QPBT[11] started thinking over: Why is the radiosity method so efficient although it employs diffuse reflections, i.e. scattering with, in principal, an exponential increase of rays ? Obviously, because there is happening also a re-unification - the key of the solution. An analogy is: Why do populations not grow exponentially although each person has children and a “tree” of descendants ? The reason is simple, as known: men must die. It can not make sense, also, to follow an exponentially increasing number of particles while the total energy remains constant, and details are even less audible with decay time.

But the RadM is principally not extendable by geometric reflections. So, it was first thought about, how to re-unify particles or rays. Further statistical analysis showed, however, that the chance of rays “travelling in about the same direction at almost the same place” (coincidence) is so extremely small that the effort to search for that would compensate the gain. So, the particle model had to be given up. Anyway, to allow some angle tolerance, beams instead of rays have to be used. In this context, tracing and recursively splitting Pyrs (PBT), with other words, the efficient version of the MISM was the first charming idea.

2.1. The Algorithm of Unquantized Pyr Tracing

shall only roughly sketched out here as in principle published elsewhere (s. for ex.[12]). Its core is the recursive mirroring of pyrs at the room´s surfaces involving, each time the module is called up (by it self) , the loop over all (plane) surfaces, the projection of the Pyr, "clipping", mirroring, diminishing of its energy by absorption, by the way delivering sound intensity to receiver points having laid within the pyr (s. fig.1,2).

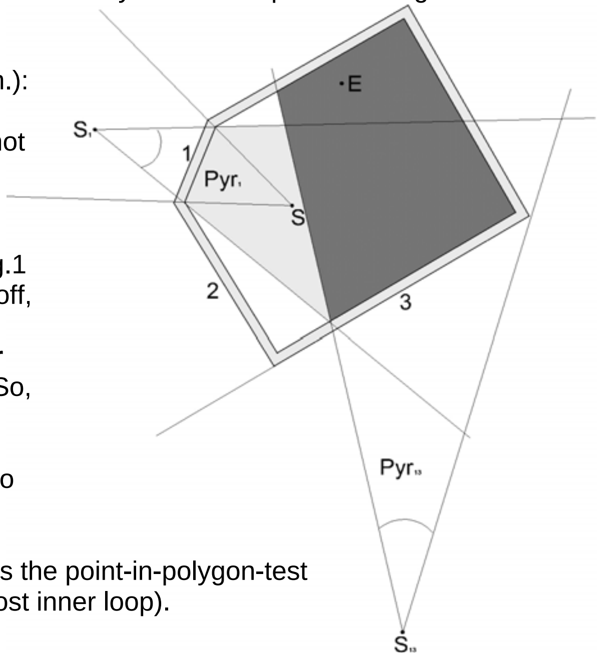


Fig. 1. Exampel for Pyramidal Beam Tracing (2-dim.): receiving point E lies in the "illumination" of pyr 13 (dark field), i.e. "sees" source S13, however does not "see" "mother-source" S1 (bright pyr from the left)

The vertices of the pyrs are equivalent to mirror sources and hence mirrored in the familiar way. Fig.1 also shows, that a "sub-tree" of MIS cannot be cut off, as "children" of MIS may be visible from a receiver point while their "mother" is it not. **Pyrs are „mirror image sources with built-in visibility borders".** So, during pyr tracing, the splitting-off-degree of pyrs is decreasing, thus the exponential increase of the numbers of MIS is avoided, pyr tracing converges to ray tracing.

Clipping is the most crucial point with pyr tracing (as the point-in-polygon-test with ray-tracing, billions of times called up in the most inner loop).

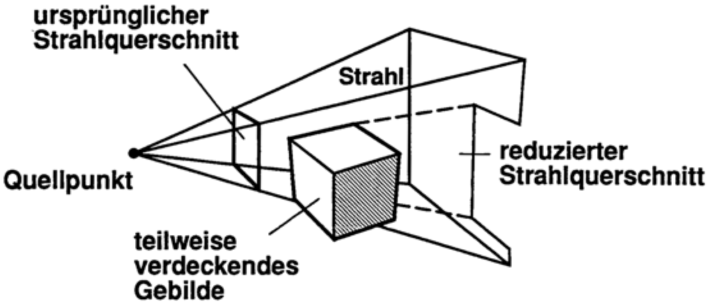


fig.2. Clipping of a pyr at an obstacle (here cube as an example for a partially hit surface): a part of the pyr, i.e. its original cross section is transmitted passing by the cube (and hitting other surfaces), the other part will be reflected (not shown here, but shown in fig. 1 as dark pyr coming from below).

With pyrs having many edges (usually 3-5) this problem is multiplied respectively. Projected in 2 dimensions, the problem reduces to a polygon-polygon-intersection. To avoid testing all edge-edge-intersections, many algorithms have been suggested in literature (for ex. [13]). With previous sub-division in convex sub-spaces and surfaces, this problem may be much simplified; furthermore, in order to avoid testing all surfaces for partial intersection, pre-checks are recommendable, utilizing bounding volumes around the surfaces. A sub-division of the room into convex sub-spaces was found to be practically necessary.

2.1.1. Subdivision of rooms into convex sub-spaces as a pre-condition

There are three reasons for doing this:

1. simplifying beam tracing and hence saving computation time.
The polygon-polygon intersection is handled much quicker, cross sections would even become non-connected otherwise. (It can be proved that with convex surfaces and, hence pyramidal beams, those become more and more simple-shaped, number of edges converges to 3.)

Convex sub-division is performed in (essentially) 4 steps:

1. sub-division of all surface polygons in convex sub- polygons
   (two for ex. for the "ceiling" surface in fig.3., 4 in fig.4) by introducing dividing edges;

2. sub-division of the room into sub-spaces setting up new “transparent walls” (in the middle of fig. 3) based on the dividing edges of step 1, beginning for ex. (s.fig.3.) with an edge no.1, looking for continuing edges lying in a plane (if not possible creating another dividing edge no.5);
3. defining sub-spaces and assigning them to the polygons;

4. assigning the sub-spaces also to all source and receiving points.

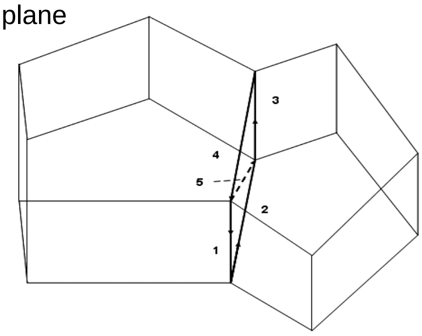


fig.3. sub-division of a room in two convex sub-rooms, bold lined the dividing “transparent wall”.

At transparent walls, rest pyrs, after clipping, travel further instead of being reflected carrying as an information the sub-space number. Thus, in searching for next hit surfaces in the space behind, computation can be considerably spared.

2. advantage, connected with that and the radiosity method:
the number of possible surface-surface-connecting paths and hence also the required computer storage is reduced (divided by the number of sub-spaces).

3. simplifying finding of “inner”, hence diffracting edges automatically with clipping and crossing transparent walls.
This is important to implement diffraction models (s. section 1 and 6.). Otherwise complicated (and in some cases contradicting) methods to detect near-by passing of edges can be dropped.

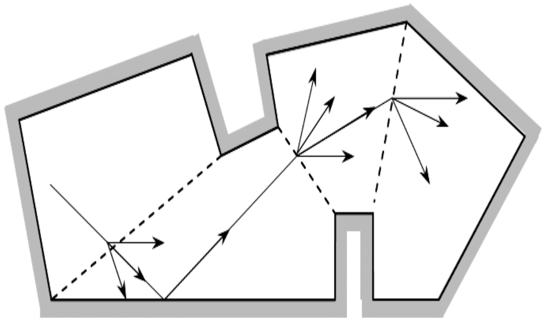


fig. 4. room divided in 4 convex sub-spaces by transparent walls (dotted lines) at which some rays are diffracted depending on their by-passing distance to the eges (comp. fig.17)

2.2. The Quantization of the mirror image source space

Although the exponential growth is truncated with PBT, it is, of course, again happening with introduction of scattering, hence the re-unification problem had still to be solved.

How to unify pyrs now ?
With radiosity, beam energies are collected (unified) at the moment they hit the surface patches as “sites of re-distribution” of energies – a method which is obviously much more efficient as the somewhat artificial searching for the coincidence of rays. Summarizing: Quantization is needed;
Not only the solide angle of beams should be quantized (as with pyrs at the beginning) but also the surface of the room (as with radiosity).

And from the parable of the population another idea arose:
as pyrs are re-distributed at the patches, loose their identity, they “die”, so to say, and are “new-born”, inheriting the energies of died pyrs - pyrs as a kind of new elementary particles, only transient carriers of sound energy.

But simply generalizing the radiosity method to geometrical reflections is impossible as “forgetting” of the past of the rays is its pre-condition. Hence, introduction simply of “other directivities” and staying at the radiosity as a frame algorithm is not possible. The preceding travelling distance of each ray or its source point has to be saved to preserve the correct distance law in future.

Collecting arguments (surfaces and distances), it almost suggests itself, to quantize the room. This means, in terms of the MISM, to quantize the "sky of mirror image sources" (SMIS) to a finite number of small volumes where sound energies of pyrs (i.e. the branches of the tree) are re-unified:

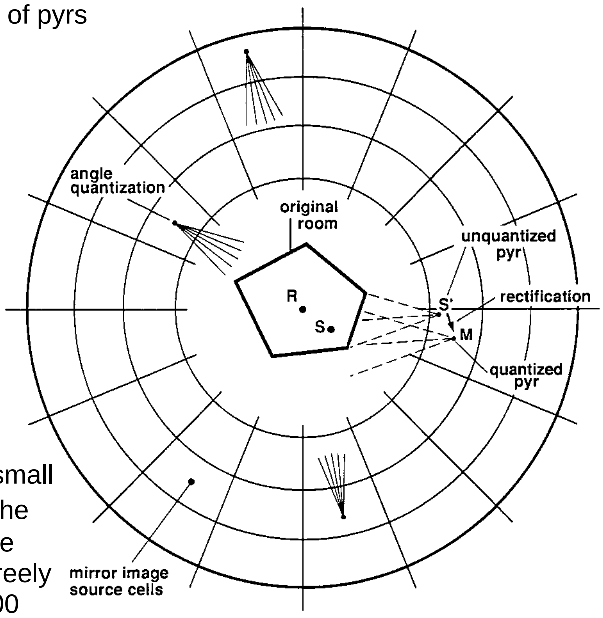


Fig. 5: The Quantization of the mirror image source space: In the middle the original source S and a receiving point R. Around R, beginning from a minimum radius r_0, concentric spheres subdivided in cells of equal solid angle; in one of those cells a rectifying process of a mirror source S´ to a cell centre M happens (a parallel-shifting without change of orientation) from which again apyr is re-emitted, directed towards the original room quantized according to its surface polygons.

The distance quantization may be chosen as a small part of a free path length $\Delta r << \Lambda = 4V / S$ of the room as a measure of one generation of MIS, the solid angle (azimuth and polar angles) may be freely chosen by the user (for ex. each about 10° or 500 segments); although statistically each MIS is in a volume of the original room, solid angles may be constant (and volumes increasing) as an increasing re-unification effect is even desired, they may even decrease, as it becomes less important with distance (reverberation time) from where the sound arrives.

From each cell of the MIS-space the pyr´s solid angles have again to be quantized (sketched with the dotted lines in fig.5.). So, it seemed[11], that even a fivefold quantization had to be performed: threefold according to the MIS space (quantum numbers n_r, n_δ, n_φ) and twofold to describe the solid angle of re-direction (n_α, n_β). This means a huge storage for pyr data – apparently even an infinite, as r is unlimited. So far, only the rectification process in the MIS cells is introduced to pyr tracing (taking into account, of course, certain visibility errors). To allow re-unification (to have partners to be re-unified with), a qualitative change in algorithm now is required:

pyrs have to be followed simultaneously; the recursion has to be replaced by an iteration.

Thereby lots of pyrs (physically propagating simultaneously) are handled pyr by pyr always looking for an "oldest" one, i.e. on an inner ring of the SMIS, to be mirrored and to deliver its energy to always existing "younger" and "neighbouring" pyrs on outer rings ("oldest" because "energy must first arrive in pyrs resp. cells before being distributed further"). Due to causality, the distance r of the vertex of a pyr to the receiver increases with mirroring ("energy from a mirrored source can not arrive earlier than from an un-mirrored".) So, with respect to radius (or time s. fig.14), a kind of continuous overtaking process happens. During the mirroring process, inner cells of the SMIS will become free for ever while outer become "occupied" by pyr energies. The maximum increase in distance r is just two room diagonals. Thus the radial range of occupied cells is limited. This is very usefull, even deciding for practicability: The required computer storage is not infinite but may be re-used.

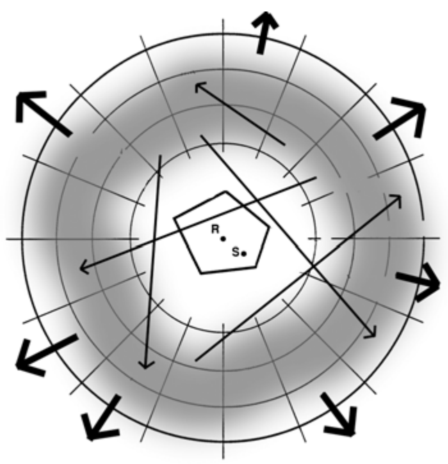


Fig.6. mirroring of pyrs in the SMIS of fig. 5:
A "cloud" of occupied cells drifting outwards.

2.3. The Re-unification process

Now, assumed such an “oldest pyr” and neighbouring already found, the energy of the oldest has to be distributed to the new ones due to the degree of overlap in solid angles (as, without directivity, all pyrs have same energy per solid angle. For later pyrs with small solid angles also the surface proportion may be taken as an approach.).

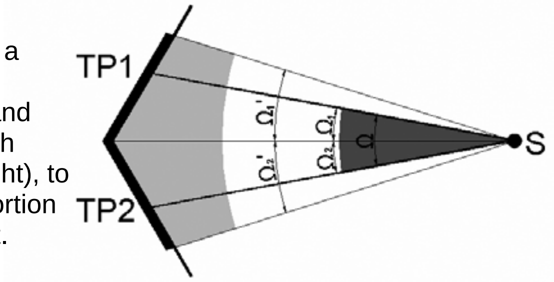


Fig.7 : From a rectified source point S (=M in fig. 5) a mirrored Pyr is emitted, originally unquantized, i.e. without change of boundaries (here 2-dim., dark), and projected unto two polygons (TP1, TP2), which each define larger solid angles resp. quantized pyrs (bright), to which the energy is transferred according the proportion of overlap; the energy per solid angle is thinned out.

Rather than unquantized pyrs, quantized ones do not become narrower. As in each solid angle and quantized pyr energies from different pyrs arrive (without loss in total without absorption), re-unification is achieved.

The space of MIS may be quantized only from a certain distance from which quantization pays (the re-unification factor is >1), s. figs. 5 and 8.

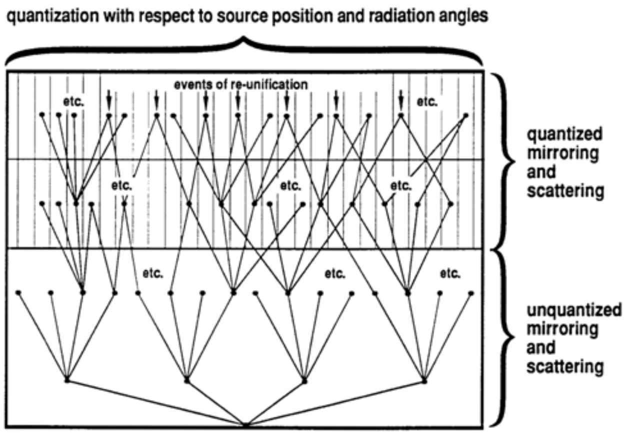


Fig.8**:** below: tree structure of pyr tracing with ***decreasing*** degree of splitting off without but about constant degree with scattering;
above a certain reflection order: redistribution; pyrs are sorted into quantized standard beams (here symbolically columns) thus avoiding the further exponential increase

2.4. Remaining problems of QPBT

Difficult performing problems of QPBT were left unsolved in 1996[11] :

1. How is the complicated but necessary polygon-polygon-intersection solved in detail ?
2. Which pre-check should be inserted to accelerate polygon finding or excluding ?
3. How works the algorithm of convex sub-division in detail ?

All these problems were solved, however shall not be lined out here. More interesting may be the following problems by which QPBT in 1996 still was looking to fail:

4. By which criterion quantize the solid angles of the new generated pyrs?
5. How to find quickly (under billions) “the oldest pyr” and also the pyrs being next neighbours for re-unification ? With other words: how to save and to organize memory ?

3. RECENT SOLUTIONS

3.1. Solution of the Quantization Problem:

Obviously, subdivision of the whole solid angle 4π into equal pieces from any point of the MIS space (s. dotted fans in fig. 5) does not make sense, is waste (most of the pyrs would never reach the room surfaces). Also solid angles should become smaller with increasing distance to optimally resolve the room structure. Therefore, the most consequent solution is to quantize the new pyr´s solid angles by the polygons of the room itself, even better, as more regularly and accurate, by

small patches of about even size into which each polygon of the room surface should be divided (s. fig.17) A measure should be mean wavelength. This means further, rather than a fixed quantization with quantized pyrs of pre-defined shape as it was planned before, a dynamical one:
After mirroring and rectifying of an old pyr, the original room is aimed-for, and by its polygons and patches (directed towards the mirror source) the new quantized pyrs are built ("born"). This happens renewed generation for generation.

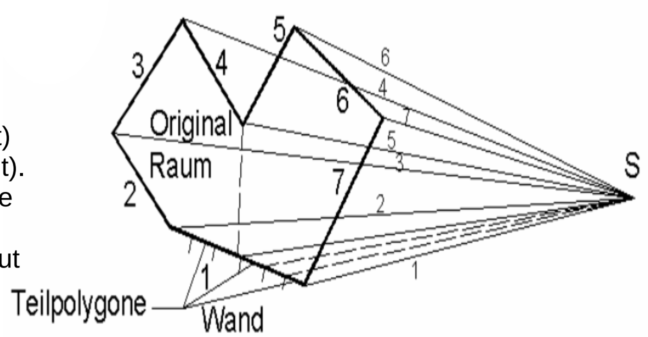


Fig.9 : **The new simple quantization procedure:** Aiming for the original room (left) by a source S (vertex of pyrs, here fans, right). S is the rectified point M in fig. 5; involved are only surfaces directed towards S, each subdivided into small patches as sketched out here only for surface 1

Each time a pyr has just been rectified in a cell, a loop over all polygons (resp. patches), the clipping and the distribution of the pyrs energy to the new ones starts, exactly as with the unquantized PBT as described in 2.3. So, neighboured pyrs are found automatically, without any additional "searching". The decisive difference to un-quantized PBT is:
Cross sections of new pyrs are not built by the intersection of pyr-polygons with surface-polygons, but are identical with the last ones, while the energy contribution per cross section is thinned-out.

So, as the actual geometry is defined by the patches indirectly, the

data set of a pyr consists only of the following few numbers: line and column number N_z and N_s in the memory (s. fig.14); quantum numbers of the SMIS: for radius: n_R, for solid angle: n_Ω (including n_δ, n_φ) number of the patch aimed for: n_p number of the last surface having mirrored (to clip the space behind for further mirroring)

with other words: **"Pyrs" exist only virtually.**

3.2. The re-unification process

Overlapping of pyrs and energy distribution in detail:

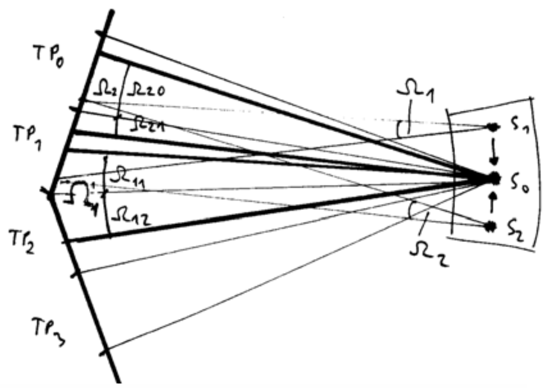


Fig. 10. Interpolation of solid angles (s. fig.7):
Left: 4 polygons TP_0..TP_3 (2-dim.)

Right: cell of the SMIS (compare fig. 5) where two sources (vertices of pyrs) are shifted (rectified) into the same centre S_0.

From S_1 and S_2 emitting each an original un-quantized pyr (thin lines, with solid angles Ω_1 and Ω_2), also shifted to the vertex S_0 (bold lines), each, in turn, divided in sub-pyrs with solid angles Ω_{11} and Ω_{12} resp. Ω_{20} and Ω_{21} by the clipping of the edge between polygons TP_1 /TP_2 resp. TP_0 /TP_1

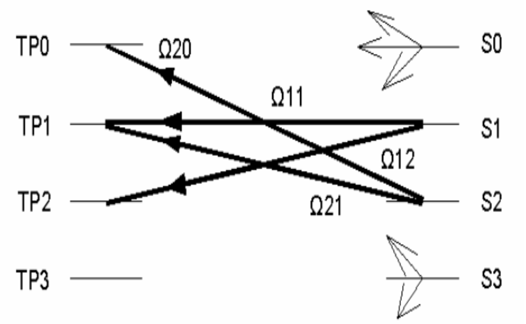


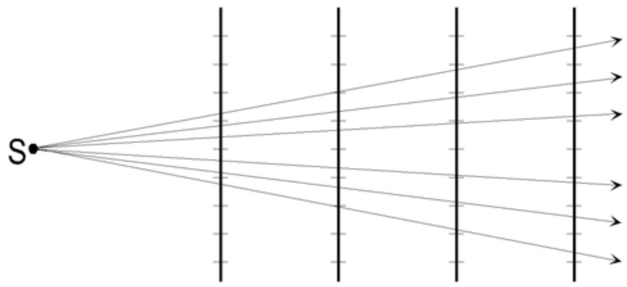
Fig. 11. energy flow of fig.10 from sources S_1 and S_2 right to the polygons TP_0…TP_2 via the solid angles defined by the geometry (detail of fig.8.)

4. AN ALTERNATIVE: COMBINATION OF THE SOUND PARTICLE WITH THE RADIOSITY-METHOD (SPR)

Due to the clipping and the quantization process, QPBT is a rather complicated method. With increasing reflection order, pyrs get more and more narrow, the effort of clipping, each time to perform a whole polygon-polygon intersection, seems to become inefficient. So, how easy would it be to return to the “good, old” sound particle method !

On the other side, it has become clear, that the basically desired re-unification effect is achieved only by a quasi-simultaneous tracing of all physically existing beams as it is performed by the time-iterative formulation of the radiosity method. So, why not use this as a frame algorithm. However, this method fails in extending to geometrical reflections because of the neglected correct divergence of rays (resp. the correct distance from the source, which led to the quantization of the SSMIS with QPBT). Sound rays being ideally thin lines show no divergence. In spite of that, the sound particle simulation works correctly. As known, the reason is: the correct distance laws are obtained simply in a statistic way in counting particles in detectors.

Fig. 12 bunch of rays homogenously emitted from a source S crossing some surfaces divided into small patches (here ticks); the surfaces are shown only symbolically as parallel (as if rays, actually reflected, are travelling into mirrored spaces behind): decreasing intensity is measured by the number of rays hitting a patch, here, on the first surface sometimes two times.



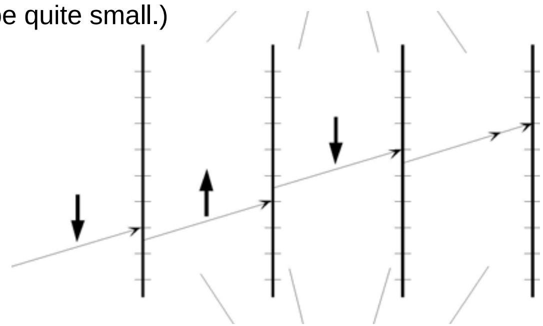
Isn´t it possible to solve that in the same way with radiosity? Yes, it is. The deciding idea:

> **To achieve the finite number of paths of energy interchange, particles hitting surfaces have to be rectified to the centres of the surface patches - as the „stations for the re-unification and re-distribution of sound energy”.**

Thus, the quantization of the surface is utilized (as finally also with QPBT) two times: first to quantize space (as with QPBT the MIS space) and second (as with radiosity) to quantize solid angles.
An astonishingly simple solution, which was found in 2000[14].

Of course, the disadvantage will be a certain un-accuracy in shifting the reflection points depending of the size of the patches (so that these should be quite small.)

Fig.13: rectifying of a ray (one of fig.12) to the centres of the patches: the ray direction is kept at each crossing (actually at a reflection), the effective direction might be changed due to shifting, however, on average over many reflections and random shifting, only slightly



This ray tracing now has to be synchronized with a time-iterative radiosity algorithm. Rays (paths between patches) are traced not each by each and reflection by reflection but quasi-simultaneously, more precisely, not (as with the stationary version of radiosity in a matrix) generation by generation, again according to the criterion: ”from the oldest first”, i.e. first from the patch containing energy least travelled from the original source. This frame procedure is similar to that of QPBT and explained in detail in section 5.

In principal, sound particles carrying geometrically reflected energy, are traced parallel to others distributing diffuse sound energy as with radiosity or carry even both energies mixed.

Presupposed are the “form factors” $H_{k,k´}$ (describing the diffusely transmitted energies between patches k and k´, also the respective time interval numbers $m_{k,k´}$) and the organization of the room data according to a sub-division into convex sub-spaces (also useful for radiosity as excluding impossible patch-patch-connections).

To be stored in computer memory are (similar to classical radiosity) the energies delivered to and stored on the surface patches, numbered with the patch number k and the time interval number n mod n_{max} ($n_{max} = R_{max} / \Delta r$ =maximum number, R_{max}=room diagonal, Δr = chosen radius interval, s.5.)
Further more also number of sound particles $J_{0,k,n}$ and for each (j) its energy $e_{J,k,n}$ and its directional vector $\mathbf{v}_{J,k,n}$. The whole pre-history of the particles need not to be stored.
But in this way, the crucial problem is solved how to extend radiosity by geometrical reflections !

Frame Algorithm (like time-dependant, future orientated radiosity):

1. initialization: emission of particles form the original source,

then, also after each generation, in the sequence “the oldest first”:

2. handling of the immissions from all patches to all receiver points,
3. distribution of the diffusely reflected energy (according the form factors),
4. emission of sound particles
   (first homogeneously, later according stored and reflected directions);
5. iteration: if all patches with energy of same “age” are “empty”:
   back to 2.
   - until some abortion criteria (time or energy dependant) is reached.

Sub- Algorithm Particle Tracing (for non-diffuse parts)

1. multiplication of particle energies to be emitted with the surface´s degree of reflection,
2. multiplication with degree of non-diffusivity (if given),
3. finding next reflection points on other surfaces,
   getting patch number and time interval of arrival,
4. rectifying the hit point to the patch´s centre
5. transfer and superposition of energy to the already received energy on the patch,
6. computation of the new direction of reflection
7. counting of received particles on the patch in the time interval of arrival.

There are four possibilities of transitions between geometric and diffuse reflections:

1. diffuse-diffuse: handled as with radiosity,
2. geometric –diffuse: incoming particle energy is multiplied with the degree of diffusivity of the receiving surface and summed up to the diffuse patch energy,
3. geometric –geometric: incoming energy is transferred to a new created particle of the computed direction of reflection emitted from the receiving patch,
4. diffuse- geometric: same procedure as 3., incident energy is defined by the form factor.

Generalized, also diffraction with any angle and distance dependency may be introduced without any change of algorithm and computation time.

Immission by diffuse emission from patches is handled with convenient form factors, immission by particles (if last reflection was geometric) is performed by detectors around the receiving points.

5. SOLUTION OF THE PROBLEM OF THE ORDER OF EXECUTION – "SOUND PARTICLE LOGISTICS"

With both QPBT and the sound-particle-radiosity algorithm, re-distribution of sound energy is executed in a time iterative process getting along with a finite number of, generalized speaking, "**energy carriers"** EC (and computer storage). This requires an efficient "synchronizing management" or "sound particle logistics". As it was pointed out in discussing the mirroring procedure (fig.6.), energy should only be distributed further if all energy from earlier places is arrived, so: "the oldest EC first". A sorting according the exact age, however, is not necessary. At any surface patch (like in railway stations) a certain tolerance time interval Δt is necessary (prop. to a Δr in the SMIS). It might be only a bit un-efficient to catch first a bit "younger" pyr. (Then in the worst case, re-distribution from that had to be repeated.) So, it is sufficient, to find one of the ECs in the same time interval (line n_z in fig.14) by which all ECs may be sorted. As another (combining) sorting criterion for the origin and direction of the ECs a column n_s may be introduced. Thus, generalized, a matrix of ECs (resp. their places in computer memory) my be imaginated, s. fig.14. In these terms, the matrix may be worked off line by line, from bottom to top, finding just the lowest non-empty line in the actual moment and working it off in an arbitrary sequence - which is trivial compared with the starting question of an "absolutely oldest EC". So, both of the problems formulated in 2.4. are solved in a simple way: finding the oldest and finding the next neighbour (s.3.1.) Any "dynamical storing" as it was suspected earlier is avoidable.

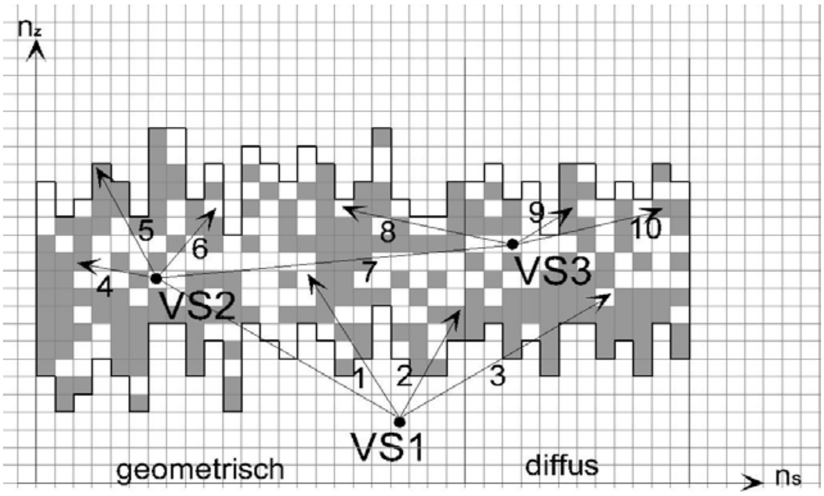


Fig.14.: energy re-distribution shown in a matrix of virtual sources (VS, symbolically);
column number n_s = function of place of origin in the MIS space and of the direction,
left part (symbolically): cells of geometrically reflected energy,
right part: cells of diffusely reflected or diffracted energy;
line number n_z= number of time interval modulo maximum number;
grey: with energy occupied cells;
re-distribution from VS1 by arrows 1-3 is first performed; only when reaching line of VS2, re-distribution from VS2 may be performed;
for example by arrow 7 from geometrically to diffusely reflected energy, by arrow 8 vice-versa.

Storing of the huge amount of simultaneously existing data of energy carriers is also simplified decisively by the fact, that once a line of the matrix is worked off (empty), it is re-usable by the data of newly generated ECs (as never "older" ECs may deliver energy to this line); during mirroring (s. fig.6), distances to the original source increase maximum by two room diagonals R_{max}. The travelled paths with reflecting or scattering by just one. Thus, the required storage is finite and can be re-used in a rotational exchange (line number in storage: n mod n_{max} with n= time interval number, $n_{max} = R_{max} / \Delta r$ =maximum number, Δr = chosen radius interval, for security, however, there should be reserved $2n_{max}$ lines.).

But are there not too many empty cells in the matrix such that searching for "occupied energy carriers" wastes time ? A statistical analysis showed: no, the degree of occupation of the matrix is in the order of $m / n_{max} = \Lambda / R_{max}$ which is not too small (order 10^{-1}). ($\Lambda = m \cdot \Delta r$, m= "time or distance resolution factor", depending of patch size, $R_{max} = n_{max} \cdot \Delta r$; this follows from the following argument: From any cell of the matrix energy is transferred to another over a maximum distance of n_{max} lines, however, according to one free path length, on average only over m line, hence, after a full set of transitions from m cells, the degree of occupation is m/n_{max}.) So, it will not pay in most cases, to use any dynamical storing. Otherwise they may be used pointers to concatenate occupied cells resp. to spare computation time for scanning empty cells. Astonishingly, a rigid working off of the matrix is possible.

Here, another reason for a convex sub-division of the room is remarkable: With sub-division, the maximum occurring path length reduces to the room diagonal of the smallest sub-space – which will be considerably smaller than of the whole room.

6. POSSIBILITIES FOR THE INTRODUCTION OF DIFFRACTION

Within still the basic approach of room acoustics, hypotheses are:

- Basic elements of diffraction are edges (mainly inner edges,
  other environments have to be composed by several ones)
- at an diffraction event, rays are splitted up into many new ones (where their number q is
  free and determinable by the user according accuracy criteria);
- rays, once diffracted, remain incoherent, afterwards again energies are added up.
- different edges near by each other have to be handled one after the other independently – (which is certainly a quite questionable: in cases of interference sometimes negative energies had to be added up);
- wanted is a diffraction model as a module to be introduced in the mirroring and radiosity model.

This was the basis of the earlier diffraction models by the author [9,10] as sketched in the introduction. Their algorithmic problem was, how to detect near-by passing of edges. A solution were "flag-wall" transparent, small narrow surfaces, attached at any inner wedges in the direction of their bisectors of angles. However, many contradictions arose (too wide interfering flags etc.). These some artificial methods may now be dropped. Subdivision of the room into convex sub-rooms is already a good pre-condition as the most important diffracting edges are already identified and marked, transparent inner surfaces serve as detectors (s. fig.4.). Also, edges lying within pyrs, are clearly detected.

So, there are mainly two possibilities now:

1. the edge-orientated model:
within the pyr model, there are created special beams – "prisms"- originating from edges (rather than from points): edges or their segments lying in an incident pyr (or precedent prism) serve as secondary sources.

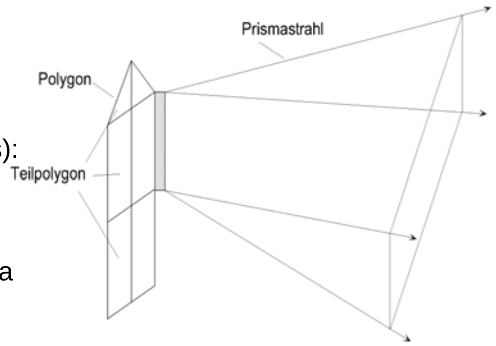
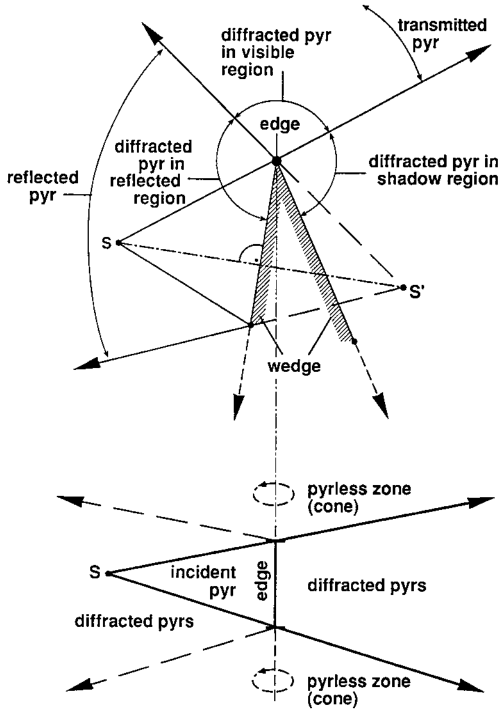


Fig. 15 prism beam emitting from the part of an edge of a polygon divided in some patches; this special beam should have four edges and surfaces two triangles, two squares (the beam is actually infinite to the right)

Convenient distance and angle dependent functions may be overtaken from the Uniform geometrical Theory of Diffraction (UTD[8]) – an assymptotic high-frequency approach meanwhile widely used in the field of electromagnetics (s.fig.16a).

Advantage: the Fermat principle of smallest detours (and incident =diffracted angle around the edge, s. fig. 16b) would be obeyed.



Disadvantages: a new shape and special cases hardly consistent with the pyr model and not at all with the radiosity method are established.
A principal, actually excluding reason against the UTD (and other approaches strictly derived from wave theory) is: They take also the flanking walls as a boundary condition into account. This, however, is a contradiction against the approach here, where diffraction is to be handled by one module, reflections at walls by another resp. the mirror image source model.
Further general problems are: the future distance to any receiving points are not known up to their detection, forgoing distances, too;
also: how far is the influence of the pre-history ?

Fig.16 zones of reflected and diffracted pyrs at an inner wedge of a room;
a) (above) from the side; the diffracting edge is perpendicular to the drawing plane,
S=source, S´= mirror image source
b) (below) the same from above

2. a surface -orientated model
fits much better into the model of subdividing rooms by transparent walls and into the radiosity model. Small patches near inner (and even outer) edges are established on flanking transparent surfaces (the same as those in fig. 4.), or even also on normal reflecting surfaces (s. fig. 17). From the centres of those may originate pyrs or rays – as from other diffusely reflecting patches also.

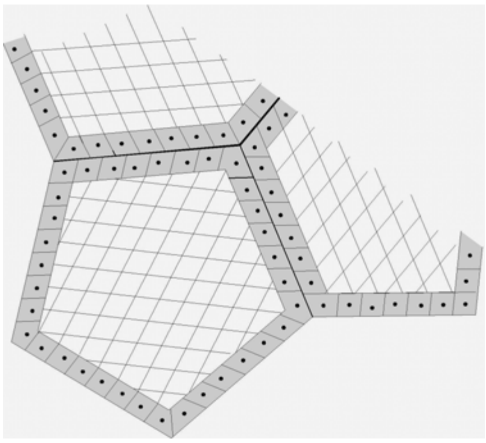


Fig. 17 patches on transparent surfaces (sub-dividing the room, s.fig.4.) and others (here 3 surfaces projected into one plane);
grey: the edge neighbouring patches, their centres (points) serve as secondary sources for pyrs or rays.

Surface sub-dividing techniques exist. They may be controlled in that way, that “patches of same diffraction strength” near edges get the desired size. Thus, diffraction modules utilizing the by-pass-distance from edges generating angle dependant deflection functions may be introduced easily. To those belong the self-derived sound particle diffraction models [9,10], especially the first being directly dependant from the by-pass distance and not from the distance to the receiving point. The only difference to mirroring patches is, that the virtual source lies ***on*** instead of behind them and that (with QPBT) ***many*** (q) instead of one secondary pyr has to be emitted; with the radiosity-sound-particle model there is not at all any difference to diffusely emitting patches. Patches also on non-transparent surfaces might be consequent as, according the Babinet principle, also their edges cause diffraction.

7. RESULT AND GENERAL PERFORMANCE OF THE QPBT AND THE SPR-ALGHORITHM

Finally, the general behavior, i.e. the increase of computation times, of the different methods should be roughly discussed and compared - where the QPBT and SPR-algorithm behave in principle same way. Best discussed is the number of energy carriers M (mainly proportional to the computation time) as a function of the order of reflection or scattering L, s. fig. 18.

With the classical MISM, M is an drastically (exponentially) increasing function of L (in fig. 18 a straight line as log(M) is plotted); with its qualitative improvement, the pyr method (with built-in visibility check) M is (by physical reasons) only proportional to L^3. But with introduction of any scattering M would again explode. On the other side, radiosity gets along with a constant number of K_0^2 paths of energy interchange. With the re-unification effect by quantization (either, with QPBT in MIS-cells, or with the SPR on surface patches) the number of energy carriers and, hence, computation time would be decisively reduced (in fig. 18 arrows directed downwards) to a constant number anyhow pre-defined by the user and is in the order of the number of patches (in the order of 100…10000 K_0^2) resp. number of cells in the SMIS or number of cells in the matrix of energy interchange (fig.14). Re-unification will happen more frequently with increasing reflection order L as the number of MIS increases with L^2 whereas the number of MIS cells of same order remains constant.

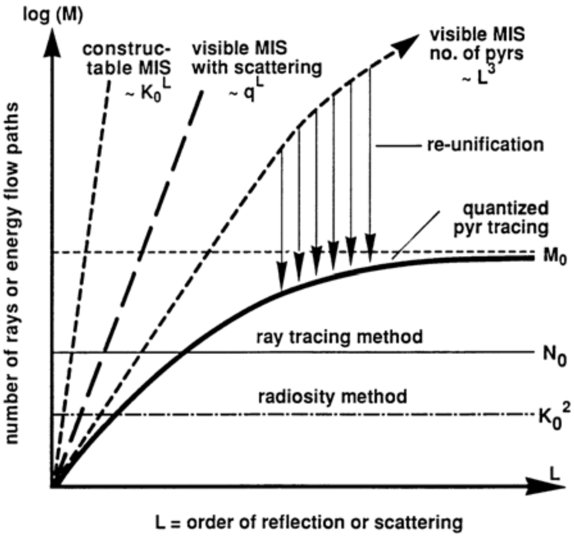


Fig. 18: Comparison of different numerical methods with respect to the growth of the number of energy carriers
K_0= number of walls (polygons),
q= splitting factor with scattering or diffraction,
N_0= number of emitted sound particles,
M_0= maximum and optimum number of pyrs

So, QPBT, although originating from mirroring, converges smoothly against only a re-distribution of energies like with radiosity. The goal of a single uniform algorithm is reached.

8. ESTIMATION OF MEMORY SPACE AND COMPUTATION TIMES

Deciding are the size of the surface patches (let the typical dimension be b) and the splitting factor q for scattering. The number of patches per original surface will then be proportional b^{-2}, and the number of paths of energy interchange (prop. the square of the number of surfaces K_0), prop. b^{-4}. In addition, the length of the time resp. distance intervals in a room should be in the same order as b, so their number with a given room size is prop. b^{-1}. So, the whole number of cells (and hence places in computer memory) is then prop. b^{-5}. The number K_0 of plane polygons of typical (rather simple) rooms is in the order of 100. With typical wavelengths around 70cm (at 500Hz) and $b \le \lambda / 2$, b is in the order of a few decimeter, the number prop. b^{-4} will then be in the order of 10^8, for b^{-5} in the order of 10^{10}, respectively. So with each pyr data set consuming some single bytes, 1…1000 GB (!) RAM will be required, dependent on room resolution (dependent on mean frequency and wavelengths). This may be reduced by sub-division in convex sub-spaces by the order of 10 – and with the QPBT further (as not all pyr data need to be stored all the time).

Conventional ray tracing (with in the order of 10000 rays and reflection order 10-20) takes only some minutes today´s PCs. Presupposed a single pre-computation of the huge form factor matrix (taking up to some hours), it is expected that SPR will take not significantly longer as conventional ray tracing. QPBT will take longer, but need less memory. But also this may be reduced.

9. SUMMARY

Both algorithms allow the introduction of scattering – not only some diffuse reflections but even diffractions with well-defined angle-dependent functions in any combination – by introducing “transparent walls” and edge zones from where pyrs or sound particles are re-distributed – without explosion of computation time. All this may be performed for all frequency bands simultaneously. However, some tuning of convenient functions and a lot of programming work has still to be done….

Another weak point is the question of frequency dependence. What would be the lower, what the upper frequency limit ? With that, prognosis of sound propagation in complex environments like cities or auditoria including reflection and diffraction in any combination may be performed in future more efficiently and accurately as before.

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