MULTIPLE SCATTERING OF ACOUSTIC WAVES IN A BUBBLY LIQUID MEDIUM: COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS.

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1 INTRODUCTION

To be efficient, modern battleship must be undetectable targets and own the best means of target detection. Bow waves disrupt sonar detection. The ship wake has long life duration and is a detectable acoustic sign at several hundred meter of the ship. This paper presents a step towards a final objective of acoustic characterization of a ship wake. Bubbles, which provoke the deterioration of ship propellers with cavitations phenomenon [1], are also the essential element making up the ship wake. In this work, we investigate theoretical and experimental results on the interaction of an acoustic wave upon a bubble cloud in water. Relations are established between the bubbles radii and characteristic resonance frequencies. Obtained with an experimental set up described in this paper, we show results of sound damping change and sound velocity change which display prominently the sound dispersion in bubbly water. Theoretical results and experimental results of others papers will be compared with our results in the last section.

2 THEORETICAL STUDY

2.1 Resonance frequencies of a spherical air bubble in water

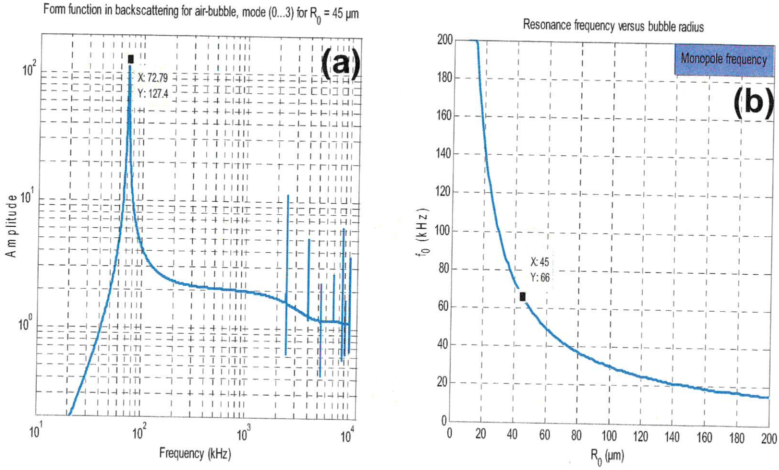

Figure 1: (a) Backscattering spectrum for R_0 = 45 µm, (b) Resonance frequencies vs. bubble radii.

In this part, being inspired by Sage, George and Überall works [2], we have determined the scattering function of a single spherical bubble in water as $f(\theta) = (1/ik)\sum_{n=0}^{\infty}(2n+1)A_n P_n(\cos\theta)$ (Eq1). We can observe resonance frequencies (Figure 1) of an air bubble in water for a bubble radius of 45 µm and for n = 0...3. We distinguish the resonance high peak (monopole resonance, n=0) at very low frequency about 72 kHz. In this paper, only the monopole resonance is studied. We have deduced, of Eq1, a relation between the monopole frequency f_0 and the equilibrium bubble radius R_0 as $f_0 = 0{,}01391.c_w/2\pi R_0$ (Eq2), where c_w is the sound velocity in water. The figure 2 shows the monopole resonance frequency change for bubble radii between 10 µm and 200 µm. Comparing these resonance frequencies to these obtained by the well known Minnaert frequency which is used in a large number of papers and books [3, 4, 5, 6], we observe that both results Eq2 and Minnaert frequency are in good agreement. We will use these results later, in the experimental part of this paper, to determine bubble sizes.

2.2 Multiple scattering and effective medium

Several multiple scattering models have been developed to describe the linear wave propagation through a host liquid medium containing bubbles. However we denote three pioneering approaches. First, the Foldy[7] approach which determines the total scattered field by the bubbly medium taking into account the interactions of each bubble with the incident field and with the scattered fields by each of others bubbles. Specifying the mean pressure in the bubbly liquid, a complex effective wavenumber is found as $k_m^2 = k^2 + 4\pi N f$ (Eq3), where f is the scattering function of one bubble, N the bubble density and k the wavenumber. This result has been revisited recently by Ye and Ding[8] and Henyey[9] including high orders of multiple scattering. Other works of Feuillade[10], also based on Foldy works, give an effective wavenumber including all orders of multiple scattering. Waterman and Truell[11] works, proposing an effective wavenumber which depend on scattering and backscattering functions, is in keeping with the Foldy approach. Medwin[12], reprised by Farmer and Vagle[13], has investigated a second approach which consists of the expression of the effective medium compressibility changes because of bubbles. The compressibility is made up of a part due to the bubble-free water and a part involving the change in volume of the bubbles. The Commander and Prosperetti[14] approach, recently re-examined by Kargl[15], uses a continuum theory. They have used a propagation equation which depends on the bubble size distribution function per unit volume, the average pressure field and the instantaneous bubble radius found using the radial motion equation of a bubble given by Keller [16]. Keller has considered that the multiple scattering effects are taken into account in the radial motion equation of a bubble. The effective wavenumber is given by $k_m^2 = k^2 + 4\pi\omega^2 \int_0^{\infty} (R_0 f(R_0)/(\omega_0^2 - \omega^2 + 2ib\omega))dR_0$ (Eq4), with ω_0 the bubbles resonance frequency and ω the exciting frequency and a damping term more explicated in Devin[4] paper. For all these effective medium models, the velocity V and the attenuation coefficient A of sound in the bubbly water are found as $V = \omega/\mathrm{Re}(k_m)$ (Eq5) and $A = 20(\log_{10} e)\mathrm{Im}(k_m)$ (Eq6). We have chosen to show results of this last model because it takes into account all multiple scattering effects without to investigate the scattered field by each bubble. We note that the others models presents slight differences with this one. Obtained for a bubble size distribution gaussian function giving an air concentration of 10^{-6} per unit water volume of the bubble central size of 45 µm (Appendix 1), we consider bubble radii between 30 and 60 µm which give bubble resonance frequencies between 50 and 100 kHz. Figure 2 shows that we obtain a strong sound damping in a frequency band around 40 to 100 kHz which is near the bubbles resonance frequencies band. There is a small difference between bubble resonance frequencies and the sound damping frequency band due to the fact that the biggest bubbles give the strongest damping. Other factor which increases the damping is the increasing of bubble density. Like that, we have, on this figure, the strongest damping around 68 kHz. It corresponds at the resonance frequency of bubbles of size near 45 µm which have the biggest density in the mixture. Figure 3 shows us respectively, in green area and in blue area, the sound velocity in mixture under and above sound speed in water representing by red dashes. Before 50 kHz, the change area of sound velocity in the mixture is under the sound speed in water. After 100 kHz, the change area is above the red dashes. Between 50 kHz and 100 kHz corresponding to resonance frequencies of bubbles in mixture, we remark that the change area of sound speed is larger but is under and above the sound speed in water. It is important to save that, when the bubble radius is the same in the mixture, at the bubble resonance frequency the sound speed in the mixture is equal to the sound speed in water.

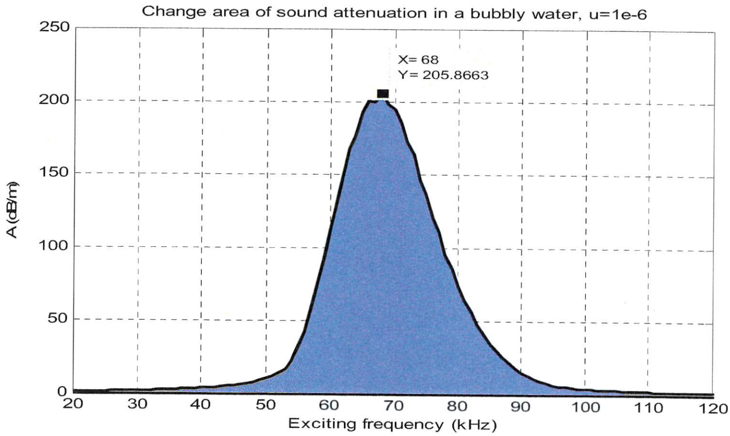

Figure 2: Change area of sound attenuation in a bubbly water vs. exciting frequency.

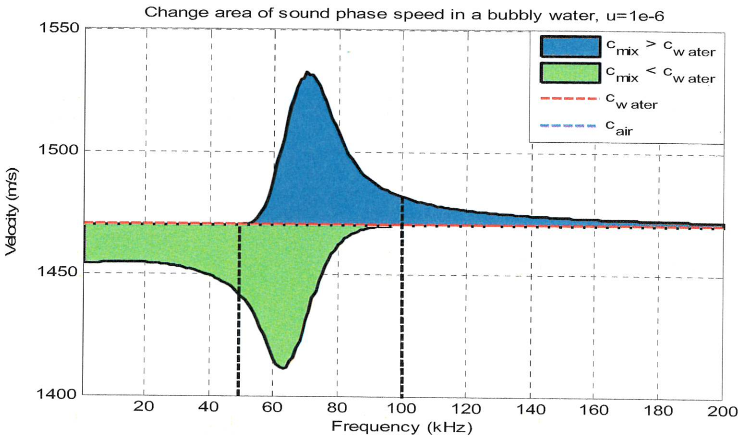

Figure 3: Change area of sound phase speed in a bubbly water vs. exciting frequency.

3 EXPERIMENTAL STUDY

3.1 Experimental set up and processing

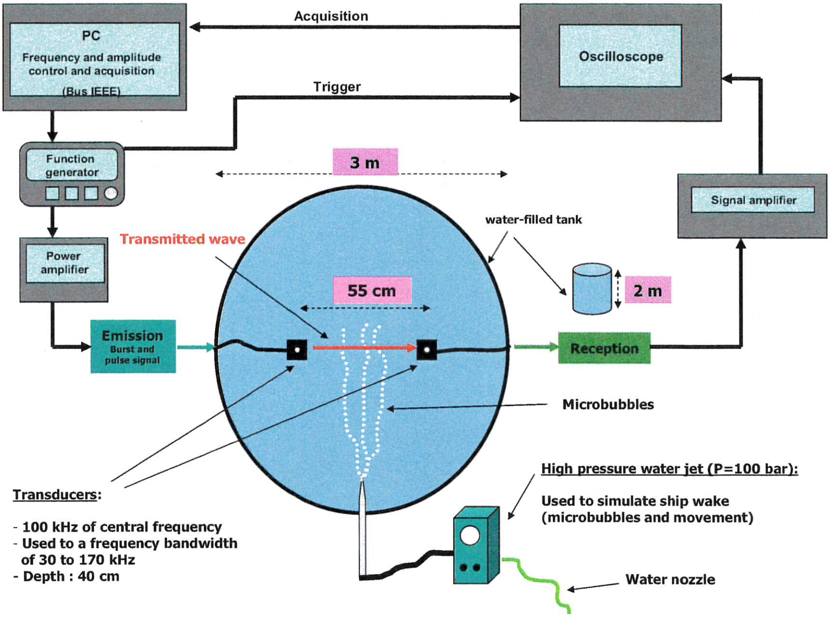

Figure 4: Experimental set up and processing for sound attenuation and phase speed measurement

Experiments are realised in a water-filled large cylindrical tank of 3 meters of diameter and 2 meters of height (Figure 4). To generate microbubbles and to simulate a ship wake, we use a high pressure water jet introduced underwater. With this generation technique, we do not know both the bubble density and the bubble size because we introduce randomly microbubbles during a limited duration of 30 second. Nevertheless, visual observations confirm the presence of microbubbles rising slowly toward the surface and going through the measurement acoustic field with a mean density decreasing with time. To transmit and receive acoustical waves, we have used two transducers of 100 kHz of central frequency. Their emission frequency bandwidth permits to use them between 30 kHz and 170 kHz. They are placed at a depth of 40 cm and the distance between them is of 55 cm. At the emission, we use a P.C. to control the exciting frequency given by a function generator HP 6320A and a power amplifier ENI of 50 dB. At the reception, we have a signal amplifier, an oscilloscope Lecroy triggered by the function generator and the P.C to control acquisition. For the damping measurements, we have transmitted a pulse signal for an excitation on the entire transducer frequency band. Signals are acquired, during several minutes with an acquisition period of 1 s, before, during and after the bubble jet. By Fourier transform on Matlab® software, we have plotted normalised spectrum evolution of acquired signals showing the damping change versus exciting frequency (Figure 5). For the velocity measurement, the emission is a sinusoidal burst signal of 20 cycles. With the PC, we have increased the exciting frequency at each acquisition between 30 and 170 kHz with a frequency quantum of 5 kHz. This frequency modulation is reiterated during every the measurement duration. We have controlled also the amplitude of the incident wave in order to have a measurable signal at each acquisition. To measure the phase speed, we have followed one point of received signals and we have calculated the velocity change ΔV by $\Delta V = d/t_f - d/t_i$ where d is the distance between the transducers, t_i the time of the initial position of a signal point and t_f the time of the final position of the same signal point.

3.2 Sound damping measurement in a bubbly water

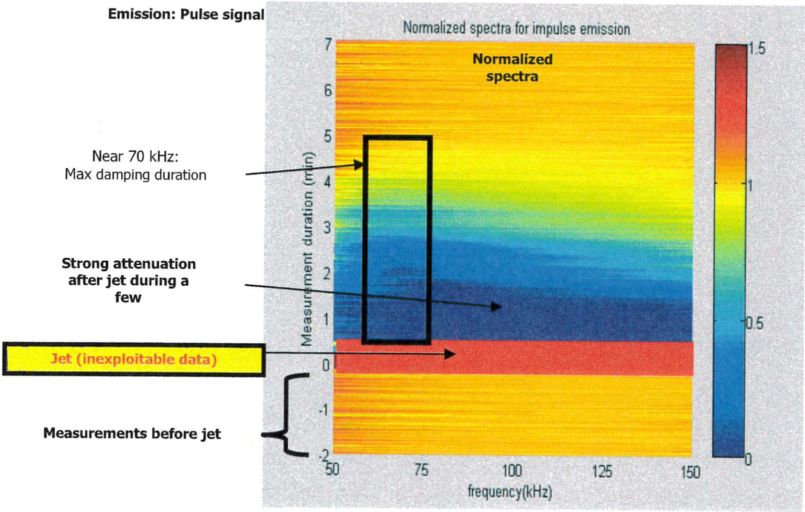

Figure 5: Signals normalized spectra vs. exciting frequency during measurement duration.

Figure 5 shows us the signal normalized spectra versus the exciting frequency during measurement duration. We observe after the bubble jet in red during some minutes a strong damping in blue. The amber colour denotes the normalised spectrum at level one i.e. without damping. We remark that it's around 70 kHz that damping duration is longer (2 at 3 minutes). An explanation is that frequencies around 70 kHz may be the resonance frequencies of bubbles having the biggest density in the mixture. If it's the case, monopole resonance excitation of bubbles absorbs acoustic wave energy. It would be in coherence with theoretical results presented in this paper and others theoretical or experimental results existing in many papers[5, 6, 10, 12, 17, 18]. Fairbank and Scully[17] show a result of pressure change around resonance frequencies of nitrogen bubbles around 20 to 40 µm of radius. Resonant bubbles around 70 kHz have radii around 45 µm (See section 2). However a classical bi-static measurement in transmission is not sufficient to estimate the mean bubble size because of the fact that more bubble is large more bubble produces damping. Thence another criterion of bubble size determination is necessary.

3.3 Sound phase speed measurement in a bubbly water

Figures 6a and 6b show phase speed changes vs. measurement duration for exciting frequencies between 30 and 75 kHz. Before bubble jet, measured sound velocity in the medium is near sound velocity in water. Because of bubble jet noise and a too strong damping of received signals just after bubble jet, the first measurable signals are around 2.5 minutes. Between 3 and 6 minutes of measurement, we observe dispersion effects of sound speed. Referring to sound speed in water (red dashes), we observe negative and positive sound speed changes. Algebraic sound speed changes increase (at 3.5 minutes: 1422 to 1472 $m.s^{-1}$) with exciting frequency increasing (30 to 75 kHz). After 6 minutes, dispersion effect disappears and sound speed joins a new valour of sound speed in water. This new valour is due to the fact that the host water-filled medium never goes back its initial hydrodynamic characteristics.

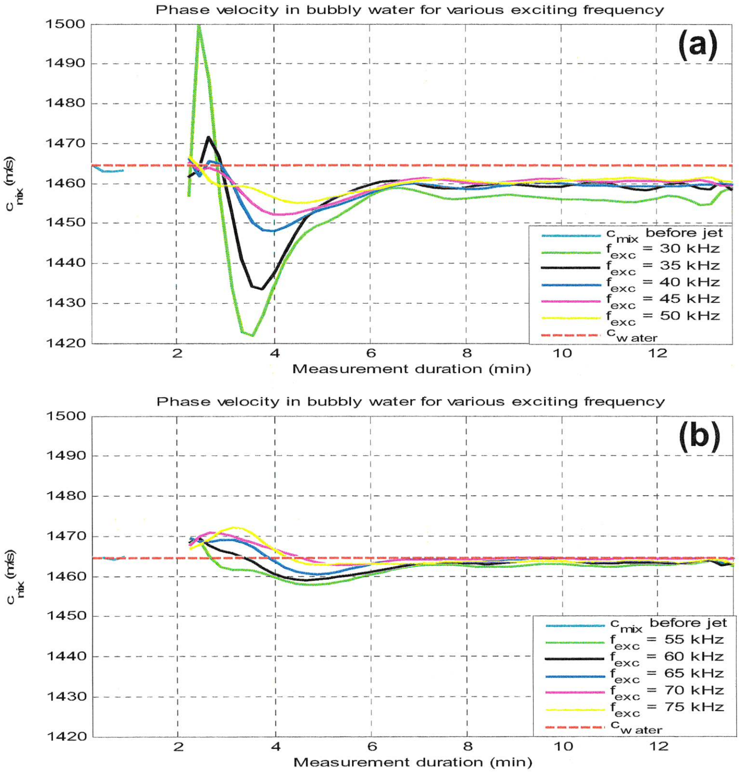

Figure 6: Sound phase speed change vs. measurement duration at different exciting frequencies between 30 to 50 kHz (a) and 55 to 75 kHz (b).

Figure 7 represents sound phase speed change vs. exciting frequency (30 to 170 kHz) at the time near 3.3 minutes (green line) and 8.2 minutes (black line) of measurement duration. Red dashes represent sound speed in water. We remark there is a cross section between these curves around 70 kHz of exciting frequency. At large effect dispersion moment, the green curve shows us the phase speed under and above phase speed in water respectively before and after 70 kHz. Interpreting with theoretical and experimental results of others papers[10, 12, 17, 19] which present also this cross section around bubble resonance frequency, we can estimate the presence of resonant bubbles around 70 kHz. Other remark, in Medwin[5, 12] papers, he explains that the asymptotic valour in very low frequency of sound phase speed permits to estimate bubble concentration. In Mobley[19] papers who uses encapsulated gas spheres, he shows that more bubble concentration is large more the asymptotic valour is low. In the present, we can not know the asymptotic valour in very low frequency because we were limited by the used transducer bandwidth. But the obtained results permit us to see that the asymptotic valour would be lower around 3.3 minutes than around 8.2 minutes of measurement. Consequently the bubble concentration would be larger around 3.3 minutes than around 8.2 minutes.

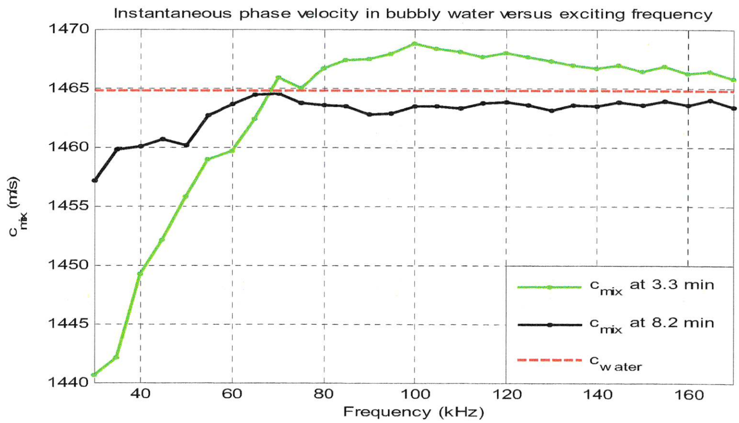

Figure 7: Sound phase speed change vs. exciting frequency at the time near 3.3 (green line) and 8.2 (black line) minute of measurement duration.

4 CONCLUSION

In this paper, we have presented some theoretical and experimental results on the sound propagation in a bubbly water. We have found some coherence between theories and experiments. First, the results show a strong sound damping around 70 kHz into the mixture. The velocity results show around 70 kHz a velocity cross section in the mixture with the sound velocity in water and sound dispersion effects. With both present results of sound damping and sound velocity in the bubbly medium and in agreement with theoretical and experimental results of others papers, we can claim that the bubble which have the larger density in the mixture have a resonant frequency of 70 kHz and thence a radius of 45 µm (see section 2).

5 ACKNOWLEDGMENTS

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7 Appendix

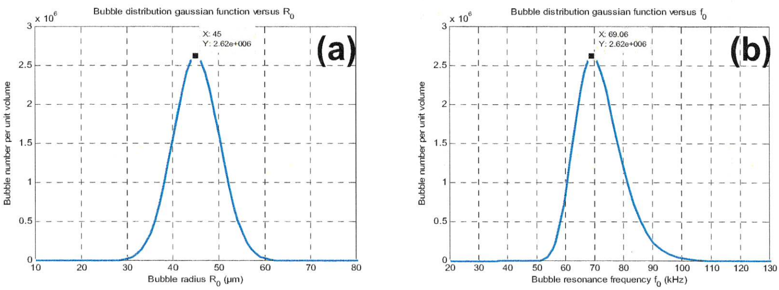

Figure 8: Bubble size distribution Gaussian function vs. bubble radius (a) and vs. bubble resonance frequency (b).