NONSTATIONARY SOUND SCATTERING FROM SEA WATER MICROINHOMOGENEITIES

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1. INTRODUCTION

Sea water usually contains different microinhomogeneities in the form of gaseous or vapor bubbles, solid particles of different types, etc. Especially the sea media is characterized by the existence of biological objects such as zoo- and phyto plankton. Simultaneously in such type deep sea layers the gaseous bubbles can take place as product of plankton life [1]. Also the great quantity of bubbles is situated near the sea surface. The detection of bubbles and the measurement of the distribution function of bubbles on their sizes are conducted by different authors with the aid of different methods [2-14].

One of the first that considered the distribution of bubbles in sea water was Blanchard and Woodcock work [2], in which the optical method was proposed to evaluate concentration and sizes of bubbles in a layer of sea water on depth 10 cm, formed at breaking of wind surface waves. According to their results, the concentration of bubbles in such layer is 300-1000 bubbles/m^3, and the radiuses R of bubbles are in an interval R = 0.01-0.02 cm. They made the attempt to investigate process of bubbles formation in sea water during a rain and a snow.

The optical methods were used also in papers [5,6]. In the paper [5] the trap of bubbles for registration of them optically by photo camera was offered. This method was used for study of bubbles distribution on the sizes in conditions of opened ocean depending on depth and wind speed (down to 13 m/c) [5]. It was established that the depth, on which bubbles penetrate, depends on a degree of wind excitement. At wind speed smaller 13 m/c the majority of bubbles places at depths from 1.5 m to 8 m. It was shown that the function of bubbles distribution of on radiuses $g(R)$ has rather sharply expressed maximum at $R=7\cdot10^{-3}$ cm and only small quantity of bubbles has radius $R<3\cdot10^{-3}$ cm.

In the paper [6] the optical method of bubbles registration was also used for an establishment of a kind of function of distribution $g(R)$, and as against Kolobayev's work [5] Johnson and Cook [6] used dipped photo camera for registration of data directly in a point of immersing. According to work [6] bubbles were registered down to depth 4 m at wind speed from 8 to 13 m /s. The smallest registered bubbles radius was about $1.7\cdot10^{-3}$ cm, the function of distribution had a maximum at the radius $R\sim4\div5\cdot10^{-3}$ cm.

Other direction in development of experimental methods of the research of bubbles distribution on the sizes is an acoustic method, based both on measurements of resonant attenuation of sound in water with bubbles [3-4], and on measurements of the level of a sound backscattering [8-11]. The method of a sound backskattering was used in works [10, 12, 13] also for research of space bubbles distribution and their influence to intensity sound scattering.

It should be noted, that in work [8-11] it was not found out of a maximum of function of bubbles distribution on the sizes $g(R)$ in a water down to radiuses $R\sim2\cdot10^{-3}$ cm. Function of distribution *g(R)* grew at reduction of radius under the law $g \sim R^{-n}$. The specified results are in the obvious contradiction with results of works [5, 6], based on optical methods of bubbles registration.

It is necessary to note, that the acoustic methods are indirect methods and field of a sound wave are based on the certain proposals about character of bubbles dynamics. The main assumption, underlying of methods of work [3-4, 10-13], consists of selectivity of attenuation and sound scattering by resonance bubbles. But as it is shown in works [8, 9] the result can also strongly depends on the function of bubbles distribution on the sizes. In this connection it is obviously important to consider both various methods of acoustic spectroscopy of bubbles and results obtained with their help. In this paper the methods of acoustical spectroscopy of gaseous bubbles using nonstationary sound scattering are presented. Also the possibilities of their separation from others nonresonance nuclei are considered. The experimental values of distribution function of gaseous bubbles in uplayer of the sea are presented.

We are discussing the nonstationary sound scattering by different microinhomogeneities of sea water using parametric arrays [8, 9, 14]. Its allowed us to apply the wide frequency band analysis of backscattering coefficient in order to obtain the necessary information.

Besides, the possibilities of others methods of acoustical spectroscopy are discussed [15, 16]. Particularly, the stationary nonlinear sound scattering is compared with the nonstationary linear and nonlinear sound scattering. Both the theory and experimental results are presented.

2. SOUND SCATTERING IN WATER WITH INHOMOGENEITIES

To describe sound scattering in a medium with microinhomogeneities, they introduce the concept scattering cross section of unit volume σ_S (or volume scattering coefficient m_V) which is a defined in a single scattering approximation (Born approximation) by the equation

$$\sigma_s \equiv m_V = \frac{2}{\pi}\frac{1}{c\tau\theta(\omega)^2}\left|\frac{P_s(\omega)}{P_i(\omega)}\right|^2, \qquad (1)$$

where P_i and P_S are pressure amplitudes in the incident and scattered waves, respectively, $I_i \sim P_i^2$ and $I_S \sim P_S^2$ are the intensity of the incident and scattered sound, θ is the half width of the directivity diagram, τ is the pulse length, *c* is the sound velocity, $\omega=2\pi f$ and *f* is the frequency.

Let us write the expressions for the scattering cross section of different types of microinhomogeneities. The cross section of sound scattering from resonant bubbles $\sigma_S^{(b)}$ is equal to

$$\sigma_s^{(b)}(R_\omega) = \frac{\pi R_\omega^3 g^{(b)}(R_\omega)}{2\delta_\omega}, \qquad (2)$$

where R_ω is the resonance radius of bubble at the frequency ω, $R_\omega = \left[3\gamma P_0/\rho\right]^{1/2}/\omega$, δ is the damping constant for resonant bubble oscillation [4, 8], *g(R)* is the distribution function associated with concentration (i.e. the quantity of bubbles in a unit volume) by the relation $N = \left[4\pi/3\right]\int R^3 g^{(b)}(R)dR$, P_0 is hydrostatic pressure; ρ is fluid density; and γ is adiabatic constant. Formula (2) together with (1) is the basis for the remote acoustic spectroscopy of bubbles [8-11].

The cross section of sound scattering from nonresonant inclusions (solid particles, zoo- and phytoplankton, nonresonant bubbles) is equal to

$$\sigma_s^{(s)} = Dk^4, \qquad D = \left[\frac{\rho c^2}{9\rho\left(c\right)^2}\right]^2 \int_0^\infty R^6 g^{(s)}(R)dR \approx \left[\frac{\rho c^2}{9\rho\left(c\right)^2}\right]^2 \bar{R}^3\eta^{(s)}, \qquad (3)$$

where k is the wave number, $k=\omega/c$, the primes refer to the inclusions and the final value characterizes the Gaussian distribution function $g^{(s)}(R)$, where $\bar{R}$ is the mean size of the inclusion. Formulae (3) together with (1) allow the determination of the volume concentration of nonresonant inclusions $\varphi^{(s)}$. Here the frequency dependence of $\sigma_s^{(s)}$ is $\sigma_s^{(s)}(\omega) \sim \omega^4$.

3. THE MAIN EQUATIONS OF BUBBLES DYNAMICS

The analysis of sound scattering by bubbles and another inclusions in liquids should be begin from formulation of the main equations of bubble dynamics under the action of sound. Let a bubble with $R << \lambda = c/f$ is situated in the sound field so that we can write the following approximations for radial oscillations of the bubble

$$\rho\left[R\dot{U}\left(1-\frac{2U}{c}\right)+\frac{3}{2}U^2\left(1-\frac{4U}{3c}\right)\right]+P_\infty(t)+P_0-P_R(t)-\frac{R}{c}\left(1-\frac{U}{c}\right)\dot{P}_R+\frac{R\dot{P}}{c}=0, \quad (4)$$

$$P(t)=P(\infty,t)=P_0+P_\infty(t), \qquad \dot{P}\equiv dP/dt, \;\; U\equiv \mathrm{v}_{R(t)}, \;\; R(t)=R(t)[1+z(t)], \;\; |z|<<1. \quad (5)$$

The Eq.(4) is similar to well-known Herring-Flynn's equation for the bubble dynamics under the sound action. From Eqs.(4)-(5) one can write in the second approximation the following equations:

$$z=z^{(1)}+z^{(2)} \qquad \ddot{z}^{(1)}+2\mu\dot{z}^{(1)}+\omega_0^2 z^{(1)}=f(t) \quad (6)$$

$$\ddot{z}^{(2)}+2\mu\dot{z}^{(2)}+\omega_0^2 z^{(2)}=-\left[z^{(1)}\ddot{z}^{(1)}+\frac{3}{2}\dot{z}^{(1)2}-2\mu\dot{z}^{(1)}z^{(1)}-\omega_0^2\frac{3\gamma+1}{2}z^{(1)2}\right] \quad (7)$$

$$f(t)=-\omega_0^2\left\{\tilde{P}(t)+R\dot{\tilde{P}}(t)/c\right\}, \qquad \tilde{P}(t)=\frac{P_\infty(t)}{3\gamma P_0}, \;\; \omega_0^2=\frac{3\gamma P_0}{\rho R^2} \quad (8)$$

These equations are the base for theoretical consideration of nonstationary bubbles dynamics.

4. NONSTATIONARY OSCILLATIONS OF BUBBLES

In most cases various types of inhomogeneities occur in the water medium simultaneously and to separate the contribution to sound scattering of ones from that of the others by the frequency dependence alone does not seem possible. Let us show the possibility of separating of sound scattering by resonant microinhomogeneities of the bubble type from the scattering by other nonresonant microinhomogeneities using nonstationary acoustic spectroscopy [8, 9]. The main point of the method consists in the following. Resonant gas bubbles have a rather high quality factor so the stationary bubble oscillations reach the steady state at resonance not instantaneously but during some time, which depends on the quality Q. Using acoustic pulses of different length τ, one can determine the contributions of resonant bubbles to the general sound scattering by different microinhomogeneities through the increment of the scattered signal depending on τ. Further the theory of such nonstationary response of bubbles is presented for different regimes (linear and nonlinear) of sound scattering by bubbles.

4.1. Nonstationary Linear Oscillations under the Action of a Single Pulse

Let us calculate the nonstationary (transient) oscillations of bubbles under the action of a single sound pulse. Let a pulse of length τ and frequency ω be incident on a bubble

$$P_\infty(t)=P_m \mathrm{Re}\left\{e^{-i\theta}e^{-i\omega t}\right\}\times[\eta(t)-\eta(t-\tau)], \qquad \theta=\omega\tau/2, \quad (9)$$

where $\eta(t)$ is the Heaviside function. The problem above considered can be solved using the method of integral transformations such as Fourier transformation. So that one can write

$$z_p = \frac{f_p}{\omega_0^2 - p^2 - i2p\omega}, \quad f_p = \frac{f_m}{2\pi i(p-\omega)}\left[e^{i(p-\omega)\tau} - 1\right]e^{-i\tau}, \quad f_m = \frac{\omega_0^2 P_m}{3\gamma P_0} \tag{10}$$

Then using inverse Fourier transformation one can obtain the equation for nonstationary oscillations of bubble in the form [9]:

$$z(t) = f_m \operatorname{Re}\left\{ e^{-i\tau}\left[\frac{e^{-i\omega t}(1-ikR)}{\omega^2-\tilde{\omega}_0^2}\left[\eta(t)-\eta(t-\tau)\right] + \tilde{\Psi}(t) - \tilde{\Psi}(t-\tau)e^{-i\omega\tau}\right]\right\},$$

$$\tilde{\Psi}(t) = \eta(t)\frac{e^{-\delta t}}{2\omega_0}\left[\frac{e^{-i\omega_0 t}}{\tilde{\omega}_0-\omega}(1-ik_0R) + \frac{e^{i\omega_0 t}}{\tilde{\omega}_0^*-\omega}(1+ik_0R)\right], \quad \tilde{\omega}_0 = \omega_0(1-i\delta) \tag{11}$$

At Figure 1 the amplitude of nonstationary oscillations of bubbles calculated according to equations (11) at different frequencies is shown. It is seen that at the resonance the stationary value of amplitude is appeared only after the time duration of several sound periods $\tau^* = 1/\delta\omega \equiv Q/\omega$, which depends on the quality factor Q [8].

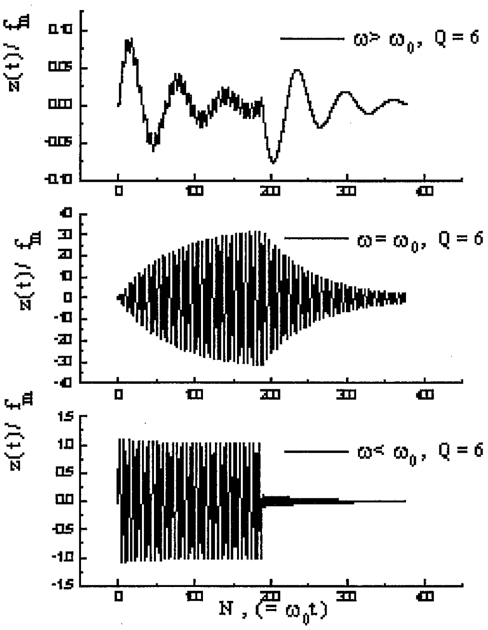

Figure 1. The amplitude of nonstationary oscillations of bubbles at different frequencies is shown:

1) $\omega > \omega_0$*, 2)* $\omega \sim \omega_0$*, 3)* $\omega < \omega_0$*.*

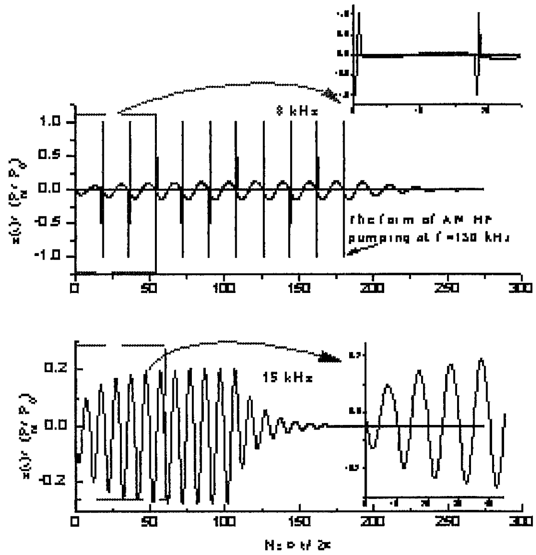

Figure 2. Coherent low-frequency (8 and 25 kHz) oscillations of bubble by AM HF sound pulses at the pumping frequency 150 kHz.

4.2. Coherent Linear Oscillations under the Action of AM HF Sound Pulses

Let a pulse of length $\tau = K \cdot \Delta t$ consists of short elementary δ-type pulses be incident on a bubble, so that appropriate equations can be written in the form [15,16]

$$P_z(t) = P_m \operatorname{Re}\left\{\sum_{k=0}^{K}\left[\eta(t-k\Delta t) - \eta(t-k\Delta t - T)\right]e^{-i\omega t}\right\}\left[\eta(t)-\eta(t-\tau)\right], \quad K = \frac{\tau}{\Delta t}, \quad T = \frac{2\pi}{\omega} \tag{12}$$

$$z_p = \frac{f_p}{\omega_0^2 - p^2 - i2p\omega} \quad f_p = \frac{f_m}{\omega}\sum_k e^{-i(p-\omega)k\Delta t}, \quad \omega_0\Delta t = 2\pi, \quad \omega\Delta t = 2\pi n \tag{13}$$

It should be noted that conditions (13) points out that only at the value of the ratio of high frequency to resonance frequency, $\omega/\omega_0 = n$, the effective coherent pumping is possible. Then using inverse Fourier transformation one can obtain the equation for nonstationary coherent linear oscillations under the action of AM HF sound pulses in the form:

$$z(t) = -B_l \frac{1}{3\gamma}\frac{\omega_0}{\omega}\frac{P_m}{P_0}\sin(\omega_0 t),\quad B_l = -\sum_{k=0}^{K} e^{-\delta\omega_0(t-k\tau)}\eta(t-k\tau), \tag{14}$$

$$B_l \approx \frac{1}{2\gamma\delta}[\Gamma(t)-\Gamma(t-\tau)]\quad \Gamma(t) = \eta(t)[1-e^{-\omega_0\delta t}],\qquad \frac{\omega}{\omega_0} = n. \tag{15}$$

Thus, one can obtain at the resonance the following equation for the amplitude

$$z_m = \frac{1}{3\gamma\delta}\frac{\omega_0}{\omega}\frac{P_m}{P_0},\quad \frac{\omega}{\omega_0} = n = 1,2,3,\ldots, \tag{16}$$

which is smaller then the common linear value at the same frequency:

$$z_m = \frac{1}{3\gamma\delta}\frac{P_m}{P_0}, \tag{17}$$

but it is bigger in n times then the common nonresonant value (without coherent pumping).

At Figure 2 the amplitude of coherent low frequency (8 and 25 kHz) oscillations of bubble by AM HF sound pulses at the pumping frequency 150 kHz is shown. It is seen, that at $\omega/\omega_0 = n$ the effective coherent pumping is possible. So that the coherent pumping allows us to obtain the possibility of effective spectroscopy, which takes place only for the resonant objects such as bubbles.

4.3. Nonstationary Nonlinear Oscillations Produced by Single Sound Pulse

The use of the nonlinear sound scattering for the problem of bubble diagnostics in liquid has been demonstrated in a number of works [7, 17, 18]. The problem of the nonlinear oscillations of bubble and sound scattering by a bubble in liquid was most likely solved for the first time by Zabolotskaya and Soluyan in the paper [18]. This problem was solved for the case of stationary sound scattering. This paper considers nonlinear nonstationary oscillations of bubbles and nonstationary sound scattering by bubbles of different size distributed in liquid.

Let us consider the nonlinear sound scattering by the bubble in detail in the quadratic nonlinearity approximation of the equation of state and the bubble movement equation. Let a biharmonical pulse of length τ with frequency $\omega_j,\ j = 1,2$ be incident on a bubble

$$P_i[t] = \mathrm{Re}\left\{\sum_{j,k} P_j e^{-i\omega_j t(-1)^k} e^{-i\varphi}\right\}[\eta(t)-\eta(t-\tau)],\qquad j,k = 1,2,\ldots n \tag{18}$$

The problem above considered can be solved using the method of Fourier transformation. So that one can write formula for the spectral components:

$$z^{(1)}_q = \frac{\omega_0^2[1-ik_qR]}{Q_{\omega_0,q}}\sum_{j,k}\tilde P_j D_{q,jk},\quad Q_{\omega_0,q} = \omega_0^2 - q^2 - i2q\delta,\quad \Omega = \omega_1-\omega_2, \tag{19}$$

$$D_{q,jk} = \frac{1}{2\pi i(q-\omega_j(-1)^k)}\left[e^{i(q-\omega_j(-1)^k)\tau}-1\right]e^{-i\varphi},\quad \tilde P_j = \frac{P_j}{3\gamma P_0},\quad \Omega \ll \omega_2 \approx \omega, \tag{20}$$

$$z^{(2)}_q = \frac{\omega_0^4[1-ik_qR]}{Q_{\omega_0,q}}\sum_{jk,lm}\frac{\omega_l^2+\frac{3}{2}[q-\omega_l(-1)^m]\times\omega_l(-1)^m+\omega_0^2}{Q_{\omega_0,q-\omega_l(-1)^m}Q_{\omega_0,\omega_l(-1)^m}}D_{q-\omega_l(-1)^m,jk}\tilde P_j\tilde P_k. \tag{21}$$

Then using inverse Fourier transformation one can obtain the equation for nonstationary nonlinear oscillations of bubble in the form [16]:

$$z^{(2)}_\Omega(t) \propto \omega_0\frac{\tilde P_1\tilde P_2}{Q_\Omega}[1-e^{-\delta\omega_0 t}]e^{-i\Omega t}[\eta(t)-\eta(t-\tau)], \tag{22}$$

$$Z_\Omega = -\frac{[1 + \gamma \cdot x^2(3\gamma + 1)]x^4}{Q_\Omega Q_1 Q_2}, \quad z^{(2)}\big|_{t>>1/\delta_\Omega \Omega} \to Z_\Omega \frac{P_1 P_2}{Q_\Omega}, \quad x = \frac{\omega_0}{\omega}, \quad \gamma = \frac{\Omega}{\omega}, \tag{23}$$

$$Q_\Omega = x^2 - \gamma^2 - i x^2 \delta_\Omega, \quad Q_1 = x^2 - 1 - i\delta_1, \quad Q_2 = x^2 - (1 - 2\gamma) - i\delta_2. \tag{24}$$

At a large length of pulses $\tau > \tau_\Omega^*$, $\tau_\Omega^* = 1/\delta_\Omega \Omega$, the known formula for amplitude of bubble oscillation at difference frequency in a steady state is obtained [17, 18] $z^{(2)} = Z_\Omega P_1 P_2 / Q_\Omega$. Essentially another result is obtained at a small length $t < t_\Omega^*$, then at frequencies $\Omega \sim \omega_0$ we obtain the estimation

$$z^{(2)}{}_\Omega(t) \sim Z_\Omega \frac{P_1 P_2}{Q_\Omega}\left(1 - e^{-\delta_\Omega \Omega t}\right)e^{-i\Omega t}\theta(t) \approx Z_\Omega \frac{P_1 P_2}{Q_\Omega} e^{-i\Omega t}\theta(t)\frac{t}{\tau_\Omega^*} \to 0. \tag{25}$$

Thus it is shown that similar linear case nonlinear oscillations of bubbles has transient part during of which oscillations increase from a small values and the time of nonstationary depends on attenuation decrement.

5. NONSTATIONARY ACOUSTIC SPECTROSCOPY

The above considered phenomena of nonstationary linear and nonlinear oscillations of bubbles can be put as a basis of nonstationary acoustic spectroscopy.

5.1. Nonstationary Linear Acoustic Spectroscopy

Let us calculate the cross section of sound scattering from resonant bubbles under nonstationary conditions. It is convenient to express the field scattered by the bubble in terms of the scalar potential φ_s, connected with sound scattering pressure P_s by the known relation $P_s = \rho\, \partial\varphi_s / \partial t$. Defining the intensity of the scattered field on a single bubble by the formula $I_s[\mathbf{r}, \mathbf{r}', t, t'] = \rho \left|\langle \partial\varphi_s / \partial t \cdot \nabla\varphi_s \rangle\right|$, where the angular brackets mean time averaging and where r and r' are the coordinates of the scattered signal in the reception point and in the position of bubble, respectively, one can in the Born approximation calculate the scattering cross section σ_s inside the pulse volume due to the bubbles or another inclusions by the formulae

$$\sigma_s[\mathbf{r}, t] = \frac{2}{\rho}\frac{1}{c\tau^2 I_i}\left\langle \int_{\Delta V} I_s[\mathbf{r}, \mathbf{r}', t, t'] g[R, \mathbf{r}'] dR \right\rangle = \sigma_s^{(b)} + \sigma_s^{(s)} \tag{26}$$

Introducing the function $W = [\bar{P}_s / P_i]/\sqrt{t}$, one can obtain the following dependence [8]

$$W^2(t) = \frac{\rho c \tau^2}{2}\left[\sigma_s^{(b)} F(t/t^*) + \sigma_s^{(s)}\right], \quad F(t/t^*) = 1 - \frac{1 - \exp[-t/t^*]}{t/t^*}, \tag{27}$$

which allows us to define the bubble distribution function as [8, 9, 16]

$$g^{(b)}(R_\omega) = \frac{4\delta_\omega [W_\infty^2 - W_0^2]}{\rho^2 c \tau^2 R_\omega^3}, \quad W_\infty = W|_{t>>t^*}, \; W_0 = W|_{t<<t^*}, \tag{28}$$

$$F(t/t^*) \approx \begin{cases} 0, & t << t^* \\ 1, & t >> t^* \end{cases}, \quad t^* = \frac{1}{\delta_0 \omega_0}. \tag{29}$$

Thus, the bubble size distribution function can be defined by the formula (28) using the data on backscattering of acoustic pulses of long and short length. At the same time it should be noted that the cross section of scattering by other resonant inclusions (of solid particles type and so on) can be defined by the formula

$$\sigma_\omega^{(s)} = \frac{2}{\rho c \tau^2} W_0^2. \tag{30}$$

To diagnose bubbles in water, the form of the function $W(\tau)$ is of interest which includes the time the oscillations reach the steady state at resonance τ^* as a parameter

$$W(t) = \left\{W^2(0) + F(t/t^*)\left[W^2(\infty) - W^2(0)\right]\right\}^{1/2}. \qquad (31)$$

The function $W(t)$ changes smoothly from the value $W(0)$ at $\tau << \tau^*$ to the value $W(\infty)$ at $t >> t^*$, so the nonstationary time can be determined from the form of the function $W(t)$ and then the bubble quality factor of resonant frequency [8, 9] can be calculated.

5.2. Nonstationary Coherent Acoustic Spectroscopy

Let us calculate the cross section area of sound scattering from resonant bubbles under the action of AM HF pulses. It can be written in the form [15, 16]

$$\sigma_{\omega_0}^{(b)}(t) = \frac{R_r^2 R g(R)}{2\delta(\omega_0, R)} F(t/t_0) \qquad R = R_r n \qquad (32)$$

so that the bubble distribution function can be defined as

$$g^{(b)}(R) = \frac{4\delta}{\pi \rho^2 c \pi R_r^2 R F(t/t^*)|_{t/t^*>>1}} \frac{\left\langle |P_{\omega_0}|^2 \right\rangle}{\left\langle |P_i|^2 \right\rangle}, \qquad R = R_r n \qquad (33)$$

where $\left\langle |P_{\omega_0}|^2 \right\rangle$ is the mean quadrate of sound pressure scattered by bubbles with the transformation of the frequency. It is interesting noted that nonstationary function $F(t/t^*)$ is the same for different cases of linear sound scattering.

5.3. Nonstationary Nonlinear Acoustic Spectroscopy of Bubbles

Let us consider a nonlinear effects in the sound field far from bubble. The pressure in outgoing spherical wave is defined as $P(r,t) = (R/r) P(t - (r - R)/c)$, $r > R$. The pressure at bubble surface $P_R(t)$ one can define in approximation of incompressible liquid from Rayleigh equation. Finally in the second order one obtain

$$P(r,t) = \frac{\rho R^3}{r}\left[\ddot{\xi} + 2\xi\ddot{\xi} + 2\dot{\xi}^2\right]_{t-(r-R)/c}, \qquad r > R \qquad (34)$$

Cross section of nonlinear stationary scattering at single inclusion of radius R for difference frequency Ω one can define from the equation:

$$\sigma_{N\Omega}^{(2)}(R) = \frac{R^2}{9}\left(\frac{\omega}{\omega_r}\right)^4\left(\frac{R}{R_r}\right)^4\left[(3\gamma + 1)\left(\frac{R}{R_r}\right)^2 - 1\right]^2 \frac{|K_1 K_2^* P_1 P_2^*|^2}{|q_1 q_2^* q_\Omega|^2}, \qquad (35)$$

where K is the adiabatic compressibility of bubbles, which is equal to $K \approx 3\gamma / P_0$. Cross section of nonlinear nonstationary scattering one can define from such equation:

$$\sigma_N^{(2)}(R,t) = \sigma_{N\Omega}^{(2)}(R_r)\left[1 - e^{-t/t^*}\right]^2. \qquad (36)$$

Integrating over the distribution function $g(R)$ one can obtain the following equation at the difference frequency $\Omega = \omega_1 - \omega_2$:

$$\sigma_N^{(2)}(R,t) = \sigma_{Nr}^{(2)} + \sigma_{N\Omega}^{(2)} F(t/t^*), \qquad (37)$$

where

$$\sigma_{Nr}^{(2)} = \frac{\pi^2(3\gamma + 2)^2 \omega^4 R_r^3 g(R_r)}{2\delta_r(\delta_r^2 + \Omega^2)} \frac{|KP_1|^2}{9} \qquad (38)$$

$$\sigma_{N\Omega}^{(2)} = \frac{\pi^2}{2} \frac{|K_1 K_2^*|\omega^4}{2|K_\Omega|^2} \frac{R_\Omega^3 g(R_\Omega)}{\delta_\Omega} \frac{|K_1 P_1|^2}{9}, \quad K_\Omega = K(R, \Omega). \qquad (39)$$

It should be noted that function $F(t/t^{*})$ has the same form as in the equations for the nonstationary linear sound scattering [8, 9]. Thus from equations (37)-(39) one can conclude that at large pulses cross section of nonlinear scattering is defined by sum of

$$\sigma_{\Omega}^{(2)} = \sigma_{\Omega\omega}^{(2)} + \sigma_{\Omega\Omega\tau}^{(2)}, \qquad t >> t^{*} \qquad (40)$$

and at short pulses it is defined by

$$\sigma_{\Omega}^{(2)} = \sigma_{\Omega\omega}^{(2)}, \qquad t << t^{*}. \qquad (41)$$

In the case of monotonically changing distribution function *g(R)* cross section of nonlinear scattering at difference frequency is defined by bubbles resonating at pumping frequency, i.e.

$$\sigma_{\Omega}^{(2)} = \sigma_{\Omega\omega}^{(2)}, \qquad g(R) : R^{-n}. \qquad (42)$$

Then the addition of the nonstationary sound scattering is very small.

It should be noted that for the case of *g(R)* with maximum, when the pumping frequencies is appropriate resonance bubbles at dropping chain of distribution function one can obtain inverse relationship: $\sigma_{\Omega\Omega\tau}^{(2)} >> \sigma_{\Omega\omega}^{(2)}$. In this case it is necessary take into account effects of nonstationary nonlinear sound scattering. The existence of nonstationary nonlinear sound scattering points out the existence of maximum of *g(R)* with the value of R_{max} , obeying to $R_{\omega}<R_{max}<R_{\Omega}$. Then the cross section area of sound scattering from resonant bubbles of radius R_{Ω} can be defined as

$$\sigma_{\Omega}^{(2)} = \sigma_{\Omega\Omega\tau}^{(2)}, \qquad R_{\omega} < R_{\max} < R_{\Omega}. \qquad (43)$$

Thus in this case in contrast of previous case of monotonically changing distribution function *g(R)* the acoustical spectroscopy of bubbles with $R_{\Omega}>R_{max}$ is possible.

6. BUBBLES SPECTROSCOPY BY PARAMETRIC ACOUSTIC SOURCES

Recently parametric acoustic sources (PS) have found wide application in the ocean acoustics owing to their unique properties - broadband operation, high directivity in a wide frequency range and also the absence of lateral lobes in the directivity diagram. Due to the indicated properties, the task of determining the sizes and distribution of gas bubbles and other inclusions can be solved by acoustic spectroscopy methods with the application of nonstationary scattering of acoustic pulses of different duration and frequency [8, 9].

It should be emphasized that thanks to the broadband of parametric acoustic sources operation one succeeds radiating pulses of small length (up to one period) without a tail at different frequencies. It is just owing to this important circumstance that we can realize nonstationary acoustic spectroscopy of the medium and study the quality of the oscillations of bubbles and other resonant inclusions from the nature the scattering reaches the steady state.

Figure 3 shows the function $W(\tau)$ obtained experimentally in sea water at depth of about 3 m for acoustic signals with different frequencies. It is seen that the value of this function decreases in accordance with formula (27) with the decrease of τ. The circlets in Figure 3 denote the values corresponding to the duration of $\tau^{*} = Q/\omega$. So the bubble quality factor Q can be determined which proves to be close enough to the values calculated from theoretical formulae [8, 9].

7. BUBBLES DIAGNOSTICS NEAR THE SURFACE BY NONLINEAR REFLECTING PULSES

Along with the traditional use of parametric sources for spectroscopy of the medium, we studied the possibility of using nonlinear scattering. The crux of method is as follows [14, 16].

In case when we use "inverted" echo sounder, sea surface can be used for acoustic wave front converting at its reflection from the surface. Then studying acoustic pulses with different carrier frequencies f_1 and f_2 one can create nonlinear interaction between the 1-st and the 2-nd pulses whose fore front is separated from those of the second by the time interval Δt. At $\Delta t = 0$ interaction occurs between the direct pulse with frequency f_1 and the converted one with frequency f_2, starting from the depth $z_{min} = c\tau_2/2$ up to $z_{max} = c\tau_1/2$. At $\Delta t \neq 0$ the value z_{min} changes as $z_{min} = c(\tau_2 + \Delta t)/2$. Thus we can investigate the nonlinear interaction at different depths. The result of the nonlinear interaction of acoustic pulses is nonlinear oscillations of bubbles and other nonlinear resonant inclusions (if they are present). That finally leads to the generation of secondary frequencies equals to $mf_1 \pm nf_2$, where $m, n = 0, \pm 1, \pm 2,$ etc. The presence of those frequencies points to the presence of the bubbles.

It should be noted that in this method the ordinary stationary nonlinear scattering is enough for the separation of resonance inclusions from another. However for the sure it is need to use the nonstationary nonlinear scattering. In this case we can use the equations (37) and (39) for the calculation of the bubbles concentration.

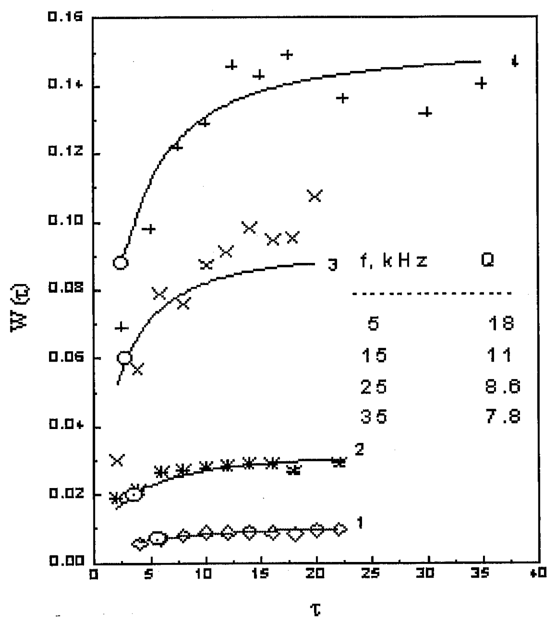

Figure 3. The function $W(\tau)$ at different frequencies: 1 - f=5 kHz, 2 - 15 kHz, 3 - 25 kHz, 4 - 35 kHz. Open circle marks the begining of stationary scattering. Here Q is the experimental values of quality factor for resonance bubbles.

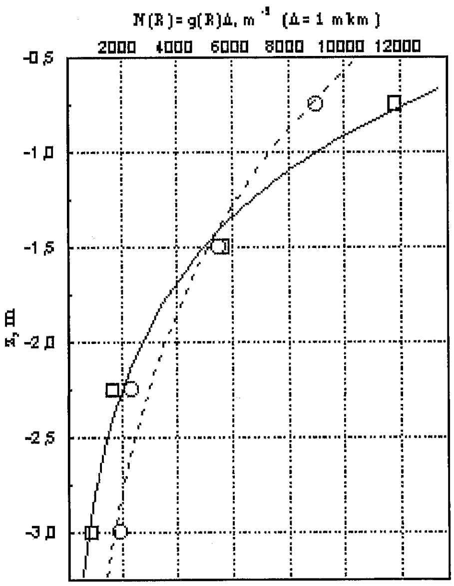

Figure 4. Bubbles distribution for $R=2\cdot10^{-3}$ cm. Method of nonlinear sound scattering by bubbles: $N=N_0\exp(-z/L)$.
Solid line - $N_0 = 1.3\cdot10^4$ m^{-3}, $L=1.4$ m;
dash line - $N_0 = 3\cdot10^4$ m^{-3}, $L=0.8$ m.

To check the above assumptions we have conducted the experiments at a sea. Figure 4 presents the results of measurements of bubbles by using of nonlinear scattering coefficient at the difference frequency 16 kHz as function of depth. The dependence is connected with bubble distribution near the sea surface. It is shown that near the surface the powerful bubble layer takes place which decays on exponential law with the characteristic thickness L~1 m. Different curves 1 and 2 denote different measurements at the same place (shallow water, the Japan Sea). It is should be noted that the value of bubble concentration agrees with the values obtained by using nonstationary linear scattering with applying of parametric sources [8, 9].

8. SOME EXPERIMENTAL RESULTS FOR BUBBLE DISTRIBUTION IN SEA WATER

We shall consider experimental results, received by the mentioned above method using parametric sources. Figure 5 presents the averaged bubble size distribution function *g(R)* at different depths obtained in subtropical waters of the North Pacific at the same conditions of sea state of 3-4 for Beaufort scale and at the wind velocity of 8-10 m/s. From Figure 5 it is clear that a bubble layer is well seen up to depth about 15 meters and the thickness of this layer depends on the radius R (resonance frequency). Besides it is clear, that these experimental data could be approximated roughly by the power dependence $g(R) = AR^{n}$. Attention should be called to the fact that the power index n changes with depth, decreasing, as a rule. The coefficient *A* also depends on depth *z* decreasing by the exponential law *A*~exp*(-z/L)* , where the parameter *L* is confined within the limits of 0.6÷1.0 m.

Very much frequently in oceanological and applied hydroacoustic researches there is a problem on, how fast the track from a disturbance disappear in a sea water, for example, a wake behind a vessel. At Figure 6 the changes of distribution function of bubbles in a wake behind a small boat are shown. One can be seen, that even without a special processing techniques the wake of the boat was observed in sea water up to 5 minute after passing one. The thickness of a subsurface layer of bubbles L depends on the sea surface state and the wind speed. As a rule, the thickness of the main subsurface layer of bubbles does not exceed 15 meters for typical marine conditions.

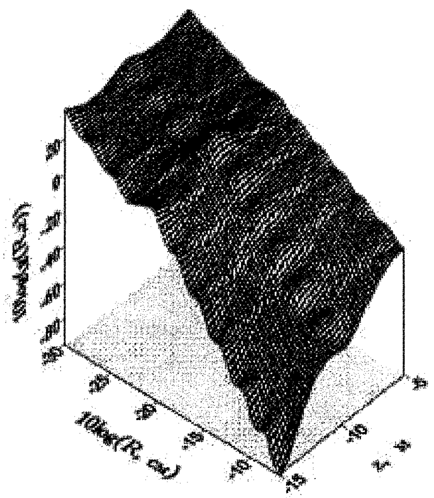

Figure 5. Distribution function g(R, z) for bubbles in subarctic waters of the North Pacific.

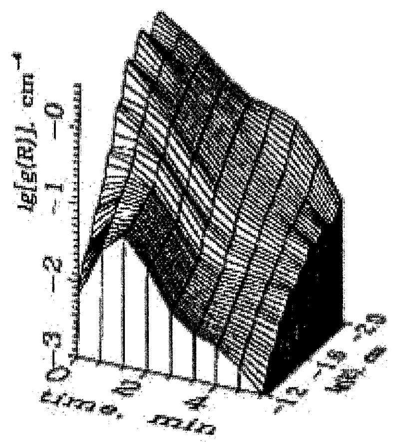

Figure 6. The changes of *g(R)* in a wake behind a small boat in the Japan Sea.

The visualization of bubbles in an active layer of the sea using parametric sources application for a sound scattering at various frequencies is represented very interesting and powerful instrument for ocean investigations. At Figure 7 and Figure 8 the structure of a bubbles layer at various frequencies 150 kHz and 20 kHz is submitted at various speed of a wind from 8 up to 15 m/s. The structure a bubble layer is of columnar type, which is connected to formation of Langmuir cells by surface waves [10, 12, 13, 19]. It is necessary to pay attention to considerable depth of entrainment of sub-surface bubbles, which one exceeds the characteristic size of orbital motion of particles in a surface gravity wave.

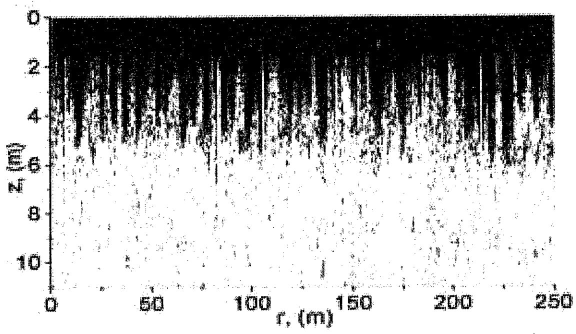

Figure 7: Acoustic scattering from a curtain of bubbles in an active layer of the sea at frequency 150 kHz, R=20 mkm, the wind velocity 8-10 m/c.

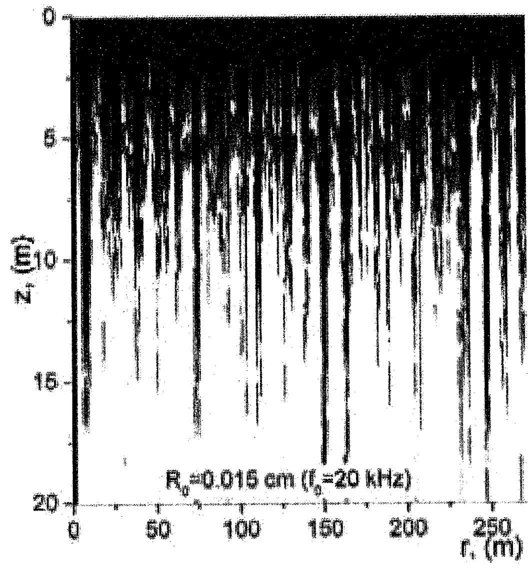

Figure 8: Acoustic scattering from a curtain of bubbles at frequency 20 kHz, R=150 mkm, the wind velocity up to 15 m/c. →

The last circumstance was found out early by Thorp [12, 13], but also by Crawford and Farmer [10] at their researches in the Atlantic Ocean with the help of sonars, radiating signals from below upwards. Such variability they connected to availability of bubble clouds distributed in subsurface layer of the sea, generated at breaking of surface waves, and seized by Langmuir's circulation in the cells [19].

9. CONCLUSIONS

We have developed a new theory of bubbles response to a real sound beam. Individual free bubble responses to pulsed incident waves were calculated in the second approximation. A large number of single-bubble responses were combined to produce time-domain echoes that realistically reflect the distribution of bubble sizes.

The developed theory was applied to determine the distribution function of bubbles on the sizes *g(R)* at different depths of subsurface water in North Pacific. It was shown that in an interval of the sizes from *0.1* cm up to *0.002* cm the function can be presented as $g(R,z) = Ae^{-z/L}R^{-n(z)}$, where the index $n(z) \cong 3{,}5 - 4$. Also it should be noted the considerable depth of entrainment of sub-surface bubbles, which one exceeds the characteristic size of orbital motion of particles in a surface gravity wave. The maximum depth of bubble penetration is 15 m with speed less than 11 m/c. The depth of the exponential drop in bubble concentration *L* is approximately equal to the root-mean square value of the surface wave heights , where *U* is the wind speed in m/s at a 10-m elevation above sea level and *L* is in meters $L \cong (2.5 \div 4.0) \cdot 10^{-3} U_{10}^{2.5}$. In subarctic water masses, bubble concentration is, as a rule, one or two orders of magnitude higher than that of subtropical waters.

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