STATISTICAL PROPERTIES OF THE PHASE SHIFT BETWEEN ACOUSTIC PRESSURE AND PARTICLE VELOCITY

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1. INTRODUCTION

Statistical properties of phase difference between acoustic pressure and particle velocity for the 630-Hz tone and underwater ambient noise with various rates of coherence have been studied. Examination of statistical characteristics of tame derivative of pressure/velocity phase difference has been done and such quantities as related mean values, standard deviations, probability as well as their dependence on time and frequency bandwidth have been calculated. It has been that statistical of the phase difference derivative can be successfully used to detect low signal against the noise background at short average time [1,2].

2. RESULTS OF EXPERIMENTS

The paper discusses statistical characteristics of the quantity which measure a velocity of the phase shift variation. Its components can be written as the following time-derivatives:

$$\varepsilon_x(t) = d(\Delta\varphi_x(t))/dt, \qquad \varepsilon_y(t) = d(\Delta\varphi_y(t))/dt, \qquad \varepsilon_z(t) = d(\Delta\varphi_z(t))/dt, \qquad (1)$$

$\varepsilon_x(t), \varepsilon_y(t), \varepsilon_z(t)$ are measured in rad/sec and indicate rotating frequency of phase shift $\Delta\varphi_x(t)$, $\Delta\varphi_y(t)$, $\Delta\varphi_z(t)$ variation.

Naturally, each of $\varepsilon_x(t), \varepsilon_y(t), \varepsilon_z(t)$ can be positive or negative indicating "right" or "left" polarization of the particle velocity vector.

The procedure of the shallow-water experiment was as follows:

The bottom-mounted combined sensor system containing acoustic pressure receiver (scalar sensor) and 3-component particle velocity receiver (vector sensor) was at the depth of 30m. The combined receiver of the sensor system was 1.5m above the silt bottom. A continuous tone source was about 2 km apart from the sensor system, at the depth of 20m. The depth of radiation point was 80m. In autospectrum of the acoustic pressure, tone signal of f_s=630Hz exceeded the ambient underwater noise by 3-6dB. The focus was on tone signal f_s=630Hz and ambient underwater noise in different frequency bands around f_{N1}=440Hz and f_{N2}=501Hz. There was no any additional signal in the frequency bands around f_{N1} and f_{N2}.

Processing of the experimental data was as follows; outputs from channels P(t), $V_i(t)$ (i=x,y,z) were filtered with a digital filter with midband frequency f_0 and variable band B_0. Then the obtained data samples $P(f_0,t)$, $V_i(f_0,t)$ were exponentially integrated over different time intervals T. Phase shift samples $\Delta\varphi_i(f_0,t)$ were obtained from:

$$\Delta\varphi_i(f_0,t) = arctn\left\{ \mathrm{Im}(PV_i^*) / \mathrm{Re}(PV_i^*) \right\} \qquad (2)$$

where $P= P(f_0,t)$, $V_i^* = V_i^*(f_0,t)$ is conjugate to $V_i(f_0,t)$, Re, Im denote real and imaginary parts of PV_i^* process (i=x,y,z).
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Then differences between $\Delta\varphi_i(f_0,t_n)$ and $\Delta\varphi_i(f_0,t_{n+1})$ at the given frequency f_0 were computed, $\Delta(\Delta\varphi_i(f_0,t_n))= \Delta\varphi_i(f_0,t_{n+1})- \Delta\varphi_i(f_0,t_n)$, $\Delta t=t_{n+1}-t_n<T_0/2$, where T_0 is the wave period of signal or noise of interest. It was assumed: $\Delta(\Delta\varphi_i)=d(\Delta\varphi_i)$ and $\Delta t=dt$ because of their small values. In such a manner we obtained massives of instantaneous derivatives $d(\Delta\varphi_i(t))/dt$ (i=x,y,z) at a given frequency f_0 and composed the probability density functions (pdfs) of ε_i (i=x,y,z) (see Eq.(1)). Accumulation (average) time was taken: T=1,2,3,5,10,15,30 sec. While composing pdfs, the bin width was taken 2deg/sec.
Statistical analysis of $\varepsilon_i(t)$ (i=x,y,z) behavior was as follows:

1. probability density functions for signal $\rho_{i,s}$ and noise $\rho_{i,N1,2}$ were composed;
2. mean values $\langle\varepsilon_i\rangle = \left\langle \frac{d(\Delta\varphi_i(t))}{dt} \right\rangle$ for signal and noise were estimated;
3. standard deviations for signal $\sigma_{i,s}$ and noise $\sigma_{i,N1,2}$ were computed;
4. probability densities of mean values $<\varepsilon_{i,s}>$ and $<\varepsilon_{i,N1,2}>$ were obtained;
5. statistical properties of the phase shift derivative for tone signal and noise were explored while expanding the band of analysis. The following frequency intervals B_0=1.5,3,6,12,24,49Hz surrounding f_s=630Hz were taken at average times T=1,2,3,5,10,15sec. $\rho_{i,S+N}$; $\rho_{i,N1,2}$; $\left\langle \frac{d(\Delta\varphi_i(t))}{dt} \right\rangle_{s,N1,2}$; $\sigma_{i,S+N}$; $\sigma_{i,N1,2}$ were studied as function of $\sqrt{B_0T}$ (i=x,y,z).

The Table 1 shows $<\varepsilon_{i,N1,2}>$, $<\varepsilon_{i,S}>$, $\sigma_{i,N1,2}$, $\sigma_{i,S}$ and probability densities of mean values for noise and signal $\rho_{i,N1,2}$, $\rho_{i,S}$ at T=1,2,3,5,10,15,30sec, Δf=3Hz.
As seen from the Table 1, while averaging over T=1sec, mean value of the derivative for the signal $<\varepsilon_{x,S}>$=0.06deg/sec, but, while averaging over T=30sec, $<\varepsilon_{x,S}>$=0.00deg/sec, i.e. mean value of the velocity of the phase shift variation tends to a certain small limit with increasing average time.
Standard deviation $\sigma_{x,S}$=4.21deg/sec at T=1sec; $\sigma_{x,S}$=0.15deg/sec at T=30sec. So, $\sigma_{x,S}$ is decreasing with increasing average time.
As seen from the Table 1, for $<\varepsilon_{x,S}>$ (at T=1sec) it takes t=100sec to change phase shift $<\Delta\varphi_{x,S}(t)>$ by 6°. During the same time interval and under the same conditions $<\Delta\varphi_{x,N1,2}(t)>$ for noise at f_{N1} and f_{N2} vary by -169° and -204° respectively. $<\Delta\varphi_{x,S}(t)>$ averaged over T=10sec changes by -2° during this time. $<\Delta\varphi_{x,N1,2}(t)>$ averaged over 10sec change by -26° and 3° at f_{N1} and f_{N2} respectively. Therefore, phase shift between acoustic pressure P(t) and x-component of the particle velocity $V_x(t)$ for the signal is more slowly changing than that for the noise under the same conditions. At the same time, standard deviation for the noise is about 10 times greater than that for the signal.
So, mean values of the velocity of phase shift variation $<\varepsilon_{i,S}>$ and $<\varepsilon_{i,N1,2}>$ are functions of the averaging time T.
Figs.1-3 show curves composed on the base of the Table 1. Fig.1 shows $<\varepsilon_{i,S}>$ and $<\varepsilon_{i,N1,2}>$, mean values of $\frac{d(\Delta\varphi_i(t))}{dt}$. As evidences from these curves, in the case of signal the mean velocity of the phase shift variation $<\varepsilon_{i,S}>$ is close to zero at any average time T, beginning from T=1sec.

Consequently, the mean value of the phase shift for the signal at a certain average time T is:

$$\left\langle \Delta\varphi_{z,S}(t) \right\rangle_T \approx \Phi_0 = const \quad (3)$$

When $\Phi_0=0$, one can state at a measurement point a plane wave arrival from a single discrete source. The same is not true for $<\Delta\varphi_{Z,S}(t)>_T$, because $\rho_{Z,S}<1$ at T=30sec.
For the noise (Fig.1) $<\varepsilon_{i,N1,2}>$ >> $<\varepsilon_{i,S}>$, so $<\Delta\varphi_{i,N1,2}(t)>_T \neq$const. This descrepency can be taken as a criterion to distinguish the signal from the noise at T<10sec.

Fig.2 shows standard deviation levels $\sigma_{i,S}$ and $\sigma_{i,N1,N2}$ in dB as function of average time T. The levels were calculated from the expression $N(\sigma_i)=20lg(\sigma_{i,T}/\sigma_{i,T=1}),(i=x,y,z)$. As seen from Fig.2, the levels $N(\sigma_{i,S})$ at T=30 sec are 30 dB lower than that at T=1sec, however for both average times they possess the same dependence on average time. The levels $N(\sigma_{x,N1,2})$ and $N(\sigma_{y,N1,2})$ depend on time

the similar way. Table 1 evidences that absolute values of $\sigma_{i,S}$ (for i=x,y) is more than 10 times smaller than corresponding absolute $\sigma_{i,N1,N2}$.
Fig.3 shows that $\rho_{i,S}$ (for x,y) at T=1-10 sec tends to 1.0, whereas $\rho_{i,N1,2}<0.4$.

TABLE 1

X							
T, s	**1**	**2**	**3**	**5**	**10**	**15**	**30**
Noise, f_{N1}=440 Hz							
$\langle\varepsilon_{x,N1}\rangle$	-1,69	-0,41	0,015	-0,22	-0,26	-0,2	-0,1
$\sigma_{x,N1}$	44,44	20,38	12,83	7,86	4,15	2,55	1,11
$\rho_{x,N1}$	0,03	0,08	0,14	0,21	0,32	0,47	0,77
Noise, f_{N2}=501 Hz							
$\langle\varepsilon_{x,N2}\rangle$	-2,04	0,04	-0,22	0,39	0,03	-0	-0
$\sigma_{x,N2}$	42,73	17,69	11,66	9,81	4,0	2,5	0,91
$\rho_{x,N2}$	0,04	0,1	0,15	0,24	0,44	0,57	0,79
Signal, f_S=630 Hz							
$\langle\varepsilon_{x,S}\rangle$	0,06	-0,06	-0,05	-0,03	-0,02	-0	0
$\sigma_{x,S}$	4,21	2,13	1,35	0,823	0,43	0,3	0,15
$\rho_{x,S}$	0,29	0,46	0,61	0,81	0,9	1	1
Y							
T, s	**1**	**2**	**3**	**5**	**10**	**15**	**30**
Noise, f_{N1}=440 Hz							
$\langle\varepsilon_{y,N1}\rangle$	5,25	8,47	4,75	1,8	0,73	-1,2	-0,2
$\sigma_{y,N1}$	76	48,84	41,19	26,53	17,13	15,4	5,39
$\rho_{y,N1}$	0,02	0,03	0,05	0,09	0,14	0,21	0,29
Noise, f_{N2}=501 Hz							
$\langle\varepsilon_{y,N2}\rangle$	-3,82	-1,35	1,13	-1,67	-1,54	0,14	-0,2
$\sigma_{y,N2}$	74,37	54,11	41,98	26,74	15,85	11,9	4,53
$\rho_{y,N2}$	0,02	0,04	0,07	0,1	0,19	0,24	0,45
Signal, f_S=630 Hz							
$\langle\varepsilon_{y,S}\rangle$	0,31	-0,04	-0,05	-0,04	-0,02	-0	0
$\sigma_{y,S}$	6,9	3,13	2,18	1,34	0,68	0,47	0,24
$\rho_{y,S}$	0,18	0,31	0,43	0,60	0,88	0,96	0,99
Z							
T, s	**1**	**2**	**3**	**5**	**10**	**15**	**30**
Noise, f_{N1}=501 Hz							
$\langle\varepsilon_{z,N1}\rangle$	6,5	2,51	5,31	1,02	-0,81	-0,1	-0
$\sigma_{z,N1}$	78,29	51,81	39,28	26,57	13,82	4,78	2,09
$\rho_{z,N1}$	0,03	0,05	0,07	0,14	0,24	0,35	0,57
Noise, f_{N2}=501 Hz							
$\langle\varepsilon_{y,N2}\rangle$	3,42	-1,82	0,15	-1,74	-1,79	-1	-0,3
$\sigma_{y,N2}$	80,15	51,06	41,25	26,62	13,10	10,70	6,09
$\rho_{y,N2}$	0,03	0,07	0,06	0,09	0,17	0,25	0,37
Signal, f_S=630 Hz							
$\langle\varepsilon_{z,S}\rangle$	0,21	0,12	-0,73	0,25	0,17	0,14	0
$\sigma_{z,S}$	32,42	19,18	14,43	9,29	5,36	4,6	1,12
$\rho_{z,S}$	0,07	0,13	0,17	0,25	0,34	0,48	0,77

Statistical characteristics of ε_X, ε_Y, ε_Z for pure noise in different frequency bands as well as for noise mixed with the signal f_S=630Hz were also studied. The following frequency interval widths were

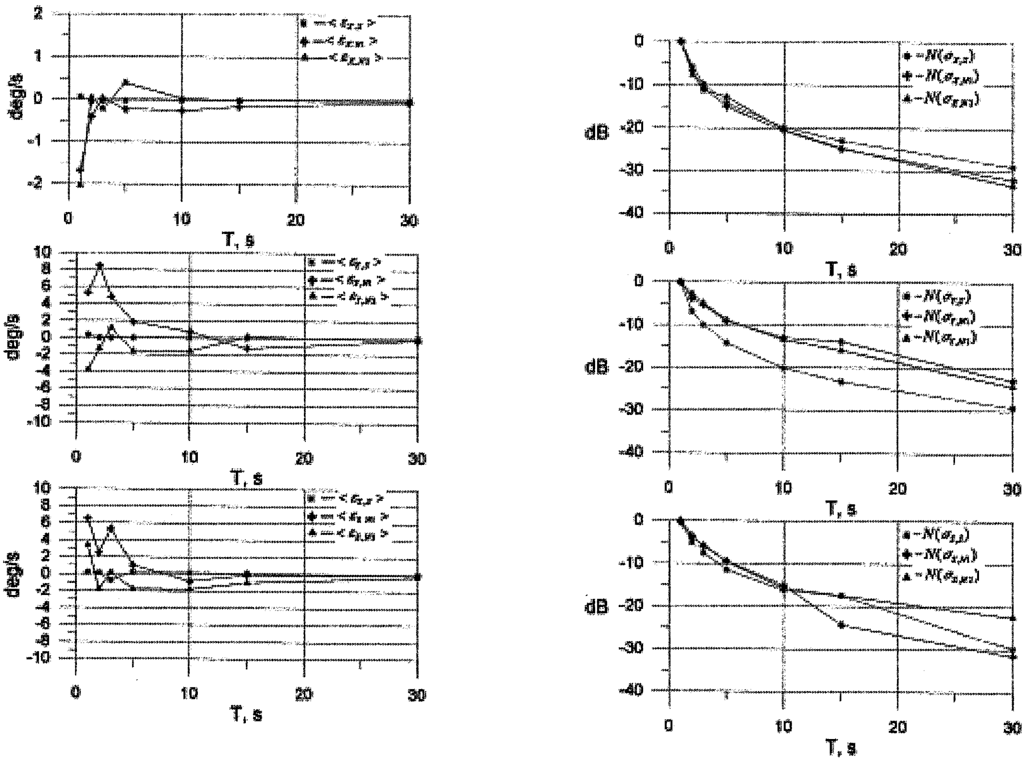
deg/s
T, s
dB

Fig. 1 Mean values of the phase shift derivatives $<\varepsilon_{i,S}>$ and $<\varepsilon_{i,N1,2}>$ as function of average time T (i=x, y, z)

Fig. 2 Standard deviation levels $\sigma_{i,S}$ and $\sigma_{i,N1,2}$ as function of average time T (i=x, y, z)

taken B_0=1.5, 3, 6, 12, 24, 49Hz. Exponent averaging was done over T_0=1, 2, 3, 5, 10 ,15 sec. While composing probability density functions, the bin was taken 2deg/sec. Statistical characteristics as function of $\sqrt{B_0T}$ were analyzed. Represent standard deviation for the frequency band where the signal f_S=630Hz is mixing with noise as $\sigma_{i,S+N}$, and that for the pure noise as $\sigma_{i,N1}$, $\sigma_{i,N2}$ (i=x,y,z); f_{N1}=440Hz and f_{N2}=501Hz are midband frequencies.

Since $\sigma_{i,S+N}$ and $\sigma_{i,N1,N2}$ significantly vary with $\sqrt{B_0T}$, the standard deviation levels $N(\sigma_{i,S+N})$ and $N(\sigma_{i,N1,N2})$ in dB were considered.

Fig.4 show $N(\sigma_{i,S+N})=20\lg\{\sigma_{i,S+N,BoT}/\sigma_{i,S+N,BoT=1.5}\}$ and $N(\sigma_{i,N1,N2})=20\lg\{\sigma_{i,N1,N2,BoT}/\sigma_{i,N1,N2,BoT=1.5}\}$ in double logarithmic scale (i=x,y,z). Here $\sigma_{i,S+N,BoT}$, $\sigma_{i,N1,N2,BoT}$ are standard deviations at current variable B_0T, $\sigma_{i,S+N,BoT=1.5}$, $\sigma_{i,N1,N2,BoT=1.5}$, are that for the fixed B_0T=1.5. Horizontal axis of the plot is scaled in 20lg $\sqrt{B_0T}$ in dB.

As seen from the Fig.4, both $\sigma_{i,S+N}$ and $\sigma_{i,N1,N2}$ decrease like $\sqrt{B_0T}$ with increasing B_0T. Note, that absolute standard deviations significantly differ from each other, e.g., $\sigma_{x,S+N}$=12.60deg/sec, $\sigma_{x,N1}$=80.89deg/sec, $\sigma_{x,N2}$=73.28deg/sec.

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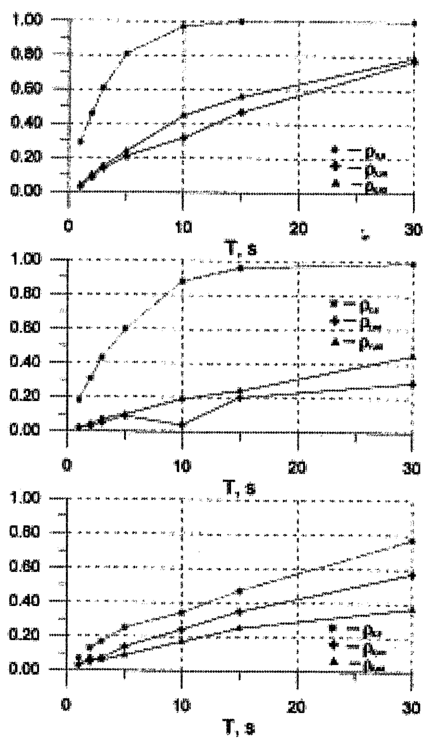
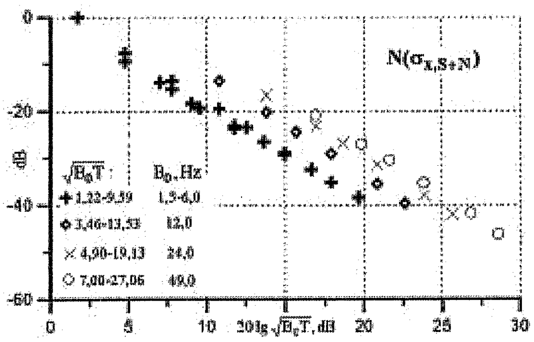


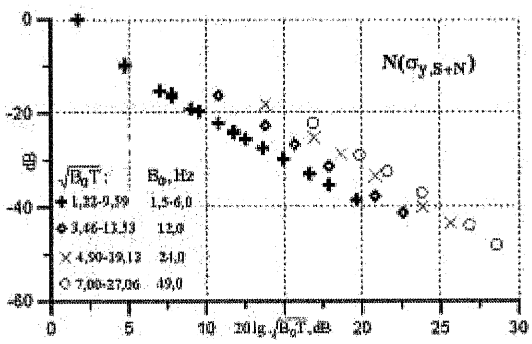
Fig. 3 Probability density functions $\rho_{i,S}$ and $\rho_{i,N1,2}$ of mean values $<\varepsilon_{i,S}>$ and $<\varepsilon_{i,N1,2}>$ as function of average time T (i=x, y, z)

Fig.4 show that $\sigma_{y,S+N}$ and $\sigma_{x,S+N}$ for the signal depend on $\sqrt{B_0T}$ the same way, however, $\sigma_{y,N1,N2}$ decreases in another manner (since the noise along y-axis is less coherent than along the x-axis).

Fig.5 evidence that expanding the frequency region of analysis around the signal until B_0=6Hz does not affect the statistical characteristics of ε_x, ε_y, ε_z, but at B_0>6Hz, probability characteristics of the signal with noise tend to that of the noise.

It is also clear if one consider the probability densities for noise with signal (Fig.5). Probability of the mean value $<\varepsilon_{x,S+N}>$ is 0.98 at $\sqrt{B_0T}$ =5. Corresponding probability density for the noise is less than 0.3. Note, that the noise along the x-axis is more coherent that along the y-axis.





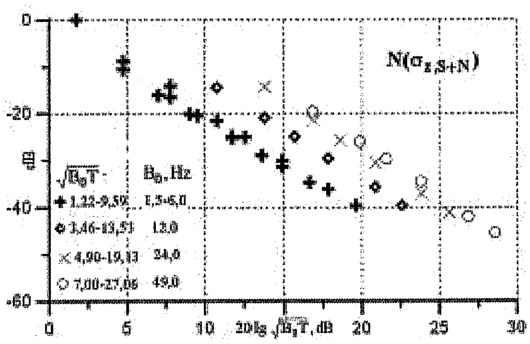


Fig. 4 The level of standard deviation N($\sigma_{i,S+N}$) as a function of $\sqrt{B_0 T}$ in the double logarithmic scale, in dB (i=x, y, z). The level is counted from $\sigma_{i,S+N,BoT=1.5}$ for i=x-12.60 deg/sec, i=y-20.94 deg/sec, i=z-11.57 deg/sec

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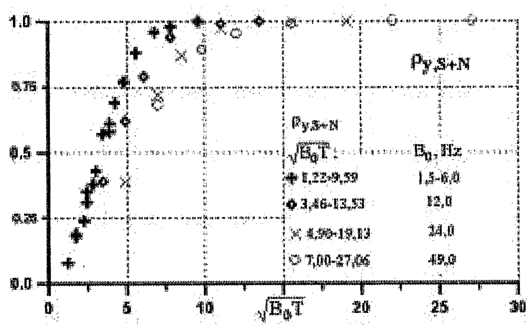

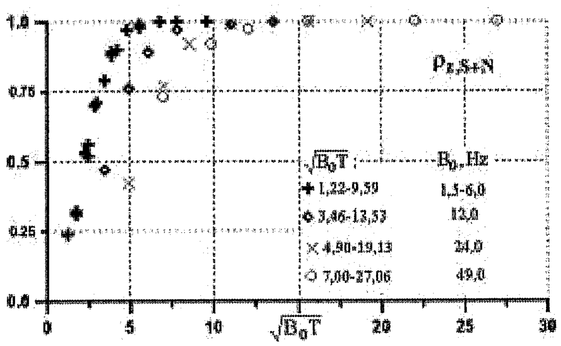

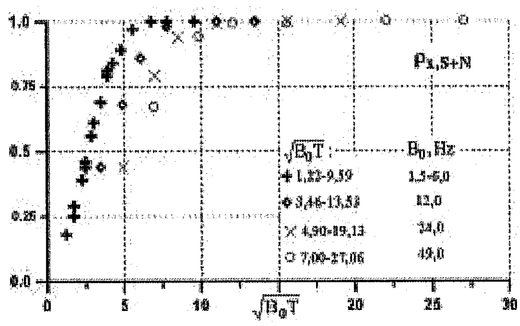

Fig. 5 Probability density function $\rho_{i,S+N}$ of the mean value $<\varepsilon_{i,S+N}>$ as a function of $\sqrt{B_0 T}$, (i=x, y, z)

3. CONCLUSIONS

In the frequency band around the signal, phase shifts $<\Delta\varphi_x>$, $<\Delta\varphi_y>$, $<\Delta\varphi_z>$ are stable and their derivatives are small. In the given case at $\sqrt{B_0 T}$ =5, the mean value probability of $<\varepsilon_{x,S+N}>=0$ is 0.9. While B_0 increases, probability of the mean value $<\varepsilon_{x,S+N}>$ decreases. In the cases of variously coherent noise, B_0 increasing causes increasing of mean value $<\varepsilon_{i,N1,2}>$ probability when $\sqrt{B_0 T}$ >5. In such a manner, it is possible to detect a signal against the noise background analyzing statistical characteristics of ε_x, ε_y, ε_z at short average (observation) times T.
Studying of statistics of phase-shift relationships between acoustic pressure P(t) and particle velocity vector $\vec{V}(t)$ is likely to provide an information useful while analyzing complicated sound fields at short observation times.

4. REFERENCE

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