ACOUSTIC 'BLACK HOLES' FOR FLEXURAL WAVES AND THEIR POTENTIAL APPLICATIONS

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1. INTRODUCTION

It is well known that when flexural vibrations (flexural elastic waves) propagate towards the edges of elastic plates of variable thickness gradually decreasing to zero (elastic wedges), they slow down and their amplitudes grow as they approach the edges. After reflection from the edge, with the module of reflection coefficient normally being equal to unity, the whole process repeats in the opposite direction [1,2]. If waves propagate obliquely from sharp edges towards wedge foundations, the corresponding gradual increase in local flexural wave velocity may cause total internal reflection of the incident flexural waves. The internally reflected flexural waves then reflect from the edge and experience internal reflection again, thus causing waveguide propagation along wedge edges - the so-called wedge acoustic waves (see, e.g. [1-5]).

Especially interesting phenomena may occur in the special case of wedge edges having cross sections described by a power law relationship between the local thickness h and the distance from the edge x: $h(x)=\varepsilon x^m$ (see Fig.1), where m is a positive rational number and ε is a constant [4-6]. In particular, for $m \geq 2$ - in free wedges, and for $m \geq 5/3$ – in immersed wedges, the flexural waves incident at an arbitrary angle upon a sharp edge can become trapped near the very edge and therefore never reflect back [5,6]. Thus, such structures materialize acoustic "black holes", if to use the analogy with corresponding astronomical objects which trap optical waves. In the case of localised flexural waves (also known as wedge acoustic waves) propagating along wedge edges of power-law profile the phenomenon of acoustic 'black holes' implies that wedge acoustic wave velocities in these structures become equal to zero [4,5]. This reflects the fact that the incident wave energy becomes trapped near the edge and wedge acoustic waves simply do not propagate.

As has been demonstrated theoretically by several authors, the principal possibility of the effects of zero reflection can exist also for wave phenomena of different physical nature. In particular, this may occur for underwater sound propagation in a layer with the sound velocity profile linearly decreasing to zero with increasing depth [7]. Similarly, the reflection may be absent for internal waves in a horizontally inhomogeneous stratified fluid [8] and for particle scattering in quantum mechanics [9]. For seismic interface waves propagating in soft marine sediments with power-law shear speed exponent equal to unity it is the wave velocity that may be equal to zero [10], exactly as in the case of the above-mentioned wedge acoustic waves [4,5]. One must note, however, that, whereas the conditions providing zero wave reflection can be hardly found in real ocean layers or atomic potentials, wedges of arbitrary power-law profile are relatively easy to manufacture. Thus, elastic solid wedges give the unique opportunity to realise the above-mentioned zero-reflection effects normally associated with 'black holes' and to use them for practical purposes.

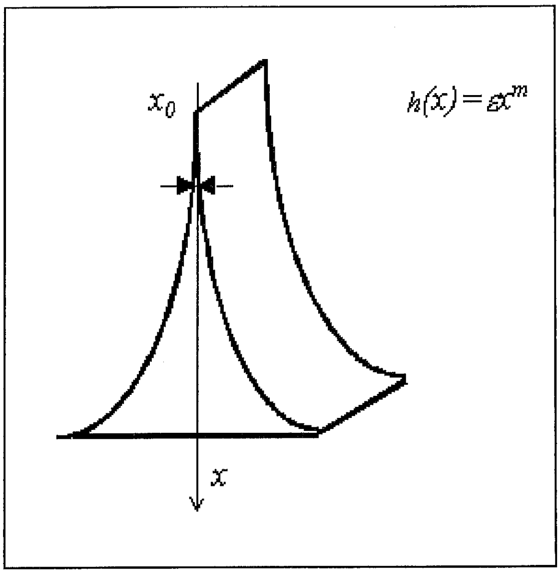

Fig. 1. Elastic wedge of power-law profile

The unusual effect of power-low profile on flexural wave propagation in elastic wedges has attracted some initial attention in respect of their possible applications as vibration absorbers. It was Mironov [6] who was the first to point out that a flexural wave does not reflect from the edge of a square-shaped wedge in vacuum ($m = 2$), and even a negligibly small attenuation may cause all the wave energy to be absorbed near the edge. Unfortunately, because of the deviations of manufactured wedges from the ideal power-law shapes, largely due to ever-present truncations of the wedge edges, real reflection coefficients in such homogeneous wedges are always far from zero [6]. Therefore, in reality such wedges can not be used as vibration absorbers.

However, the situation can be radically improved by modifying the wedge surfaces. In the present paper, we demonstrate that the presence of thin absorbing layers on the surfaces of wedges of power-law profile can drastically reduce the reflection coefficients. Thus, the combination of the power-law profile and of a thin absorbing layer can result in very effective damping systems for flexural vibrations.

2. OUTLINE OF THE THEORY

To explain the basic physics of the phenomenon let us consider the above-mentioned simplest case of a plane flexural wave propagating in the normal direction towards the edge of a free wedge described by a power law. Since flexural wave propagation in such wedges can be described in the geometrical acoustics approximation [1-6], the integrated wave phase Φ resulting from the wave propagation from an arbitrary point x on the wedge medium plane to the wedge tip ($x = 0$) can be written in the form:

$$\Phi = \int_0^x k(x)dx \quad . \tag{1}$$

Here $k(x)$ is a local wavenumber of a flexural wave in contact with vacuum: $k(x) = 12^{1/4} k_p^{1/2}(\varepsilon x^m)^{-1/2}$, where $k_p = \omega/c_p$ is the wavenumber of a symmetrical plate wave, $c_p = 2c_t(1-c_t^2/c_l^2)^{1/2}$ is its phase velocity, and c_l and c_t are longitudinal and shear wave velocities in a wedge material, and $\omega = 2\pi f$ is circular frequency. One can easily check that the integral in eqn (1) diverges for $m \geq 2$. This means that the phase Φ becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above mentioned ideal wedges represent acoustic "black holes" for incident flexural waves.

Real fabricated wedges, however, always have truncated edges. And this adversely affects their performance as potential vibration absorbers. If for ideal wedges of power-law shape (with $m \geq 2$) it follows from eqn (1) that even an infinitely small material damping, described by the imaginary part of

$k(x)$, is sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower integration limit in (1) must be changed from 0 to a certain value x_0 describing the truncation, thus resulting in the amplitude of the total reflection coefficient R_0 being expressed in the form [6]

$$R_0 = \exp(-2\int_{x_0}^{x} \mathrm{Im}\, k(x)dx) \quad . \tag{2}$$

According to eqn (2), for typical values of attenuation in the wedge materials, even very small truncations x_0 result in R_0 becoming as large as tens per cent [6].

In efforts to improve the situation, let us now consider covering the wedge surfaces by absorbing thin layers of thickness δ (e.g. by polymeric films). Note in this connection that the idea of applying absorbing layers for damping flexural vibrations of uniform plates is not new and has been successfully used since the 50-ies (see, e.g. [11-13]). The new aspect of this idea, which is discussed in the present paper, is the use of such absorbing layers in combination with a specific power-law geometry of a plate of variable thickness (a wedge) to achieve maximum damping. In what follows we consider only one possible film-induced attenuation mechanism – the one associated with in-plane deformations of the film under impact of flexural waves. Such deformations occur on the wedge surfaces as a result of the well known relationship between flexural displacements u_z and longitudinal displacements u_x in a plate: $u_x = -z(\partial^2 u_z/\partial x^2)$. Not specifying physical mechanisms of the material damping in the film material, we assume for simplicity that it is linearly dependent on frequency, with non-dimensional constant ν being the energy loss factor, or simply the loss factor.

To consider the effect of thin absorbing films on flexural wave propagation in a wedge in the framework of geometrical acoustics approximation one should analyse first the effect of such films on flexural wave propagation in plates of constant thickness. The latter problem can be approached by different ways. For example, it can be solved using the non-classical boundary conditions taking into account the so-called "surface effects" [14,15]. Alternatively, the energy perturbation method developed by Auld [16] can be used. However, the simplest way is to use the already known solutions for plates covered by absorbing coatings of arbitrary thickness obtained with regard to vibration damping in such plates [11-13].

In particular, for a plate of thickness h covered by visco-elastic layers of thickness δ on one of the surfaces the following expression for the additional loss factor ξ can be obtained (see, e.g. [12]):

$$\xi = \frac{\nu}{1+(\alpha_2\beta_2(\alpha_2^{\,2}+12\alpha_{21}^{\,2}))^{-1}} \quad . \tag{3}$$

Here ν is the loss factor of the material of the visco-elastic layer, $\alpha_2 = \delta/h$, $\beta_2 = E_2/E_1$, and $\alpha_{21} = (1+\alpha_2)/2$, where E_1 and E_2 are respectively theYoung's moduli of the plate and of the visco-elastic layer. Assume now that the plate is covered by visco-elastic layers on both surfaces and consider the limiting case of $\alpha_2 = \delta/h << 1$. Then, assuming that $\alpha_2\beta_2 << 1$, one can arrive from eqn (3) to the following simplified expression:

$$\xi = 6\alpha_2\beta_2\nu = 6(\delta/h)(E_2/E_1)\nu \tag{4}$$

Using eqn (4), one can write down the expression for $k(x)$, that takes into account the effects of both thin absorbing films and the wedge material on the imaginary part of a flexural wavenumber, Im $k(x)$, for a wedge of non-linear shape described by the local thickness $h(x)$:

$$\mathrm{Im}\, k(x) = \left[\frac{12^{1/4} k_p^{\;1/2}}{h^{1/2}(x)}\right]\left[\frac{\eta}{4}+\frac{3}{2}\frac{\delta}{h(x)}\frac{E_2}{E_1}\nu\right] . \tag{5}$$

Considering the wedge as being of the square shape, i.e. $h(x) = \varepsilon x^2$, substituting eqn (5) into eqn (2) and performing the integration, one can obtain the following analytical estimate of the resulting reflection coefficient R_0:

$$R_0 = exp(-2\mu_1 - 2\mu_2), \qquad (6)$$

where

$$\mu_1 = \frac{12^{1/4} k_p^{1/2} \eta}{4\varepsilon^{1/2}} \ln\left(\frac{x}{x_0}\right), \qquad (7)$$

$$\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{1/2} \nu\delta}{4\varepsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^2}\left(1 - \frac{x_0^2}{x^2}\right). \qquad (8)$$

In the absence of the absorbing thin film ($\delta = 0$ or $\nu = 0$, and hence $\mu_2 = 0$), the expressions (6)-(8) reduce to the results obtained in [6], where we have corrected the typographical misprint). If the absorbing film is present ($\delta \neq 0$ and $\nu \neq 0$), it brings the additional reduction of the reflection coefficient that depends on the film loss factor ν and on the other geometrical and physical parameters of the wedge and of the film.

3. NUMERICAL EXAMPLES

Let us choose for illustration purposes the following values of the film parameters: $\nu = 0.2$ (i.e., consider the film as being highly absorbing), $E_2/E_1 = 2/3$ and $\delta = 5\mu$. Let the parameters of the square-shaped wedge be: $\varepsilon = 0.05\ m^{-1}$, $\eta = 0.01$, $x_0 = 1.5\ cm$, $x = 50\ cm$ and $c_p = 3000\ m/s$. Then, for the frequency $f = 10\ kHz$ it follows from eqns (6)-(8) that in the presence of the absorbing film $R_0 = 0.017$ (i.e. *1.7 %*), whereas in the absence of the absorbing film $R_0 = 0.513$ (i.e. *51.3 %*). Thus, in the presence of the absorbing film the value of the reflection coefficient is much smaller than for a wedge with the same value of truncation, but without a film. Even relatively high absorption in the wedge material ($\eta = 0.01$) only slightly reduces the reflection coefficient from unity corresponding to the case of ideal wedge made of non-dissipative material.

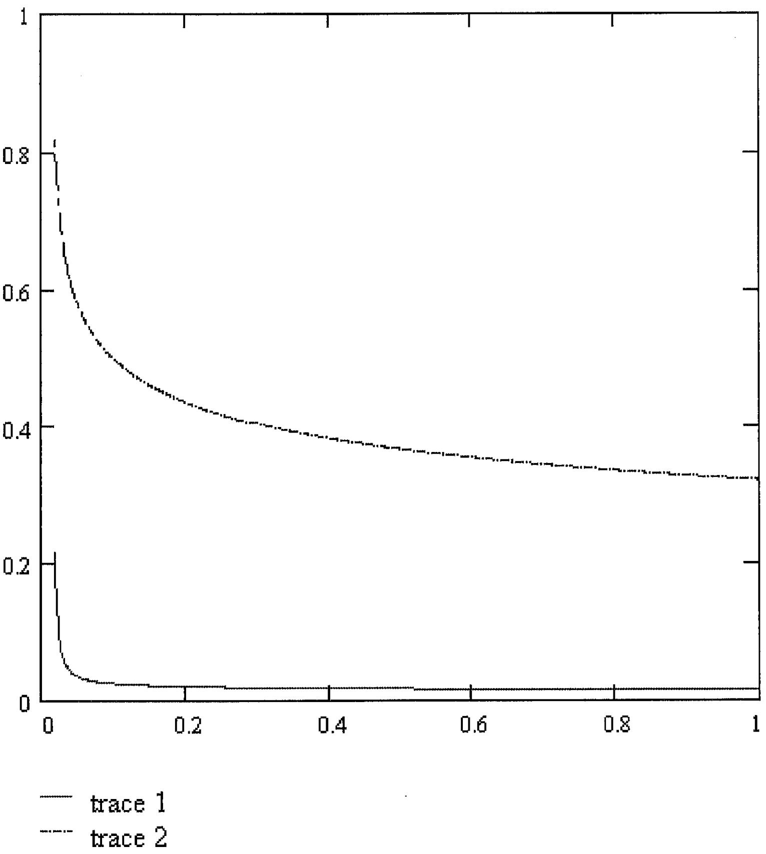

Fig.2. Reflection coefficient R_0 as a function of the total wedge length x (in *m*): traces 1 and 2 correspond to the values of the film material loss factor ν equal to 0.2 and 0.02 respectively.

It is interesting to compare the reflection coefficient of flexural waves for a wedge covered by a thin absorbing film with the one for the same waves reflecting from the blunt edge of a homogeneous plate made of the same material and having the thickness ($h = 1.3$ *cm*) equal to the initial local thickness h_{ini} of the wedge considered in the above example ($h_{ini} = 1.3$ *cm* at $x = 50$ *cm*). Calculations show that in the case of such a homogeneous plate $R_0 = 0.832$, i.e. the reflection coefficient for the plate is still very large and only slightly differs from the reflection coefficient for the plate without an absorbing film ($R_0 = 0.834$). Thus, the effect of thin absorbing film causes in this case only a slight reduction of the reflection coefficient from its value defined by energy losses in a plate. Obviously, it is both the unusual geometrical properties of a square-shaped wedge in respect of wave propagation and the effect of thin absorbing layer that result in a very efficient way of suppressing flexural vibrations.

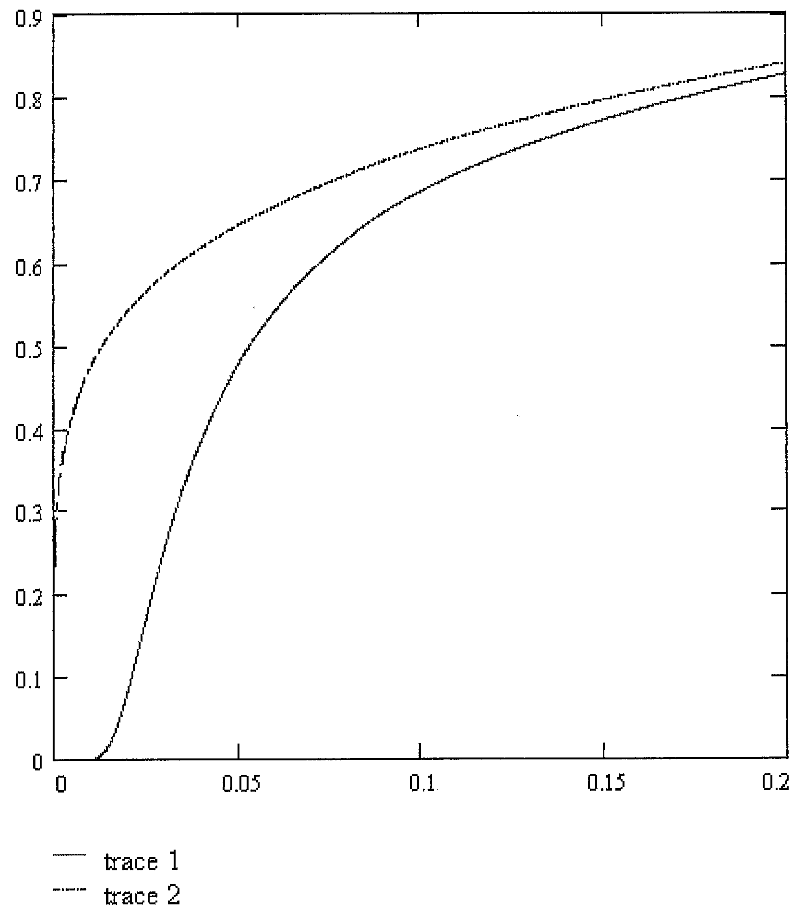

Fig. 3. Effect of wedge truncation x_0 (in *m*) on the reflection coefficient R_0 : traces 1 and 2 correspond to the values of the film material loss factor ν equal to 0.2 and 0 respectively.

Note that almost all absorption of the incident wave energy takes place in the vicinity of the sharp edge of a wedge. Fig. 2 shows the reflection coefficient R_0 as function of the total wedge length x for the example considered above. It is seen that R_0 changes noticeably only in the close proximity of the wedge edge. In the example discussed above (with $\nu = 0.2$), almost 99% of the incident elastic energy is absorbed within the length of 3 cm near the truncated edge.

The influence of the length of the wedge truncation x_0 in this example on the reflection coefficient R_0 is shown on Fig. 3 for the values of film material loss factor ν equal to 0.2, and 0 (the latter corresponds to the absence of an absorbing film). One can see that the values of x_0 still retaining the reflection coefficients R_0 that are close to zero depend strongly on the film attenuation. The larger attenuation the larger values of truncation x_0 can be allowed.

Fig. 4 illustrates the frequency dependence of the resulting reflection coefficient R_0 in the example considered for the values of film material loss factor ν equal to 0.2, and 0.02.

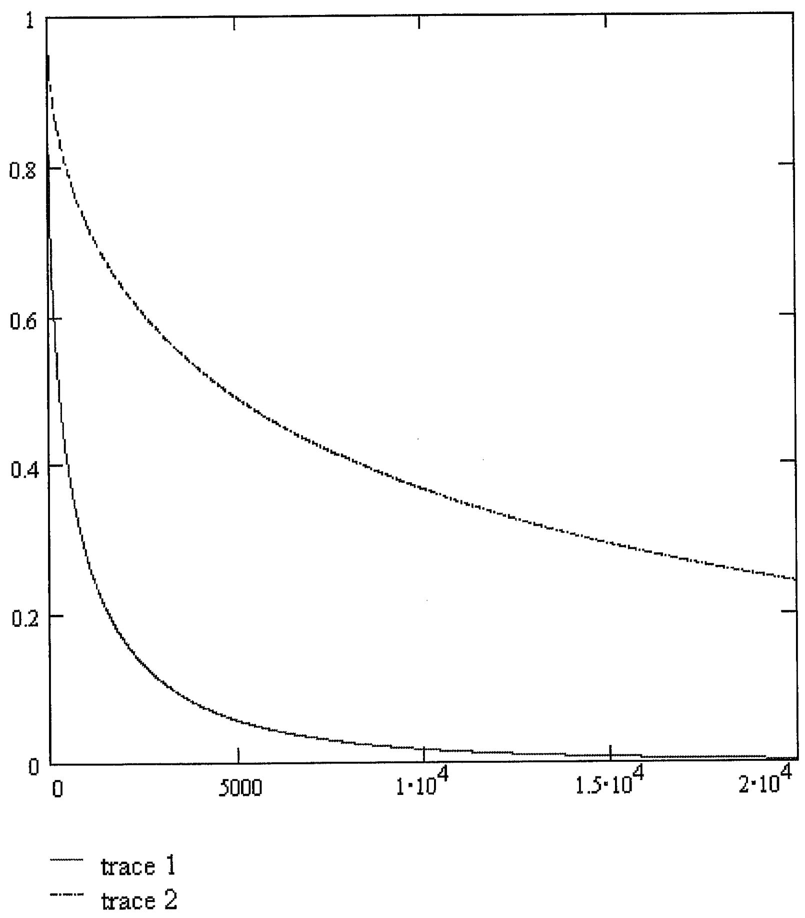

Fig. 4. Dependence of the reflection coefficient R_0 on frequency (in Hz): traces 1 and 2 correspond to the values of the film material loss factor ν equal to 0.2 and 0.02 respectively.

As one can see, in both cases the reflection coefficient reduces with the increase of frequency. However, in the case of the presence of the absorbing film such increase is much more rapid. For very low frequencies, when flexural wavelengths are comparable with the wedge total length x, the theoretical model considered is inaccurate since in this case one should take into account also non-propagating waves that may be excited at the wedge wide end. However, as follows from Fig.4, at low frequencies the reflection coefficients are expected to be relatively high even in the assumption that only propagating waves are reaching the wedge edge. This means that the wedges under consideration can act as effective absorbers only at relatively high frequencies (higher than 2-3 kHz), so that taking into account non-propagating flexural waves may prove being practically unimportant.

For more rigorous evaluation of flexural wave reflection in non-linear wedges covered by absorbing thin films, more elaborate physical models of wave energy absorption by thin films should be considered (e.g. the ones accounting for viscous friction corresponding to frequency dependence of the material attenuation proportional to the square of frequency, relaxation-type attenuation mechanisms, etc.). Although the corresponding estimates of the reflection coefficients may differ significantly for different models, the main outcome remains the same: all models end up with very large reductions in the values of R_0. Thus, it is the combination of the extraordinary wave propagation properties of a square-shaped wedge and the effect of thin absorbing layer that reduces dramatically the reflection coefficient (as compared to a free wedge and to a homogeneous plate covered by an absorbing film). This opens a very efficient way of suppressing flexural vibrations. The advantage of using the proposed wedge absorbers of flexural vibrations over traditional types of vibration absorbers lays in the fact that wedge absorbers are compact and light-weight (do not require additional masses to provide resonance absorption). Apart from this, they can be integrated into a construction on a design stage, as its inseparable parts.

4. CONCLUSIONS

Some preliminary results have been reported on the new type of vibration absorbers utilising the effect of acoustic 'black holes' for flexural waves propagating in elastic wedges of power-law profile. It has been demonstrated that the presence of thin absorbing layers on the surfaces of a square-shaped elastic wedge can significantly reduce the reflection of flexural waves from its truncated edge. Thus, the combination of the power-law profile of the wedge and the effect of thin absorbing layers can result in a very effective damping of flexural vibrations.

Despite the very encouraging preliminary theoretical estimates described above, further theoretical and experimental investigations are needed to validate the principle and to explore the most efficient ways of creating such 'wedge-like absorbers' of flexural vibrations.

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