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COMPARISON OF TWO MAIN MECHANISMS OF GENERATING GROUND VIBRATIONS BY ROAD VEHICLES

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1. INTRODUCTION

According to the survey of vibration nuisance from road traffic undertaken by the UK Transport Research Laboratory [1], among the most annoying mechanisms of traffic-induced ground vibrations are: 1) vehicles accelerating and braking - respectively 49.7% and 30.2% of the respondents, and 2) damaged or bumpy road surfaces - 39.4% of the respondents. While ground vibrations due to uneven road surfaces were investigated both analytically and numerically by several authors [2-5], there is no adequate theoretical research where vibrations from accelerating or braking vehicles were approached. The existing literature on elastic wave radiation by forces moving at nonconstant speed along a free surface is not directly applicable to the case of road-traffic-induced ground vibrations. For example, in the paper [6] the response has been considered of an elastic half space to a decelerating surface point load, directed normally to the surface. The load speed varied from superseismic to a value slightly less than the Rayleigh surface wave velocity. In contrast, road vehicle speeds are invariably much lower than Rayleigh wave velocities. Note in this connection, that trans-Rayleigh train speeds are a reality and have important implications for generation of ground vibrations [7].

In this paper both these main mechanisms of generating ground vibrations are investigated theoretically, the main attention being paid to generating ground vibrations by accelerating and braking vehicles. We consider vehicles travelling at constant speed on damaged or bumpy surfaces, and vehicles accelerating (decelerating) with a constant acceleration a from rest to a constant speed, or braking from a constant speed to a stop. In the case of uneven road surfaces excitation of the most important vehicle resonances is being taken into account. However, an accelerating or braking vehicle of mass M is modelled as a point horizontal traction force $F_x = aM$ applied to the ground and moving along with the vehicle.

It should be emphasised that the main mechanism of ground vibration generation by accelerating or decelerating vehicles is the action of horizontal traction forces which are applied from tyres to the ground only during the time of accelerating or braking (the contribution of normal load forces is relatively small for speeds and accelerations typical for road traffic), whereas for vehicles travelling along uneven or bumpy roads, the most important are vertical dynamic forces caused by the obstacles.

2. ACCELERATING AND BRAKING VEHICLES

In this section we briefly describe the main features of generating ground vibrations by accelerating and braking lorries and discuss some numerical results. Being interested only in low-frequency ground vibrations typical of traffic-induced mechanisms of generation and assuming that the main vehicle resonances [8,9] are not excited during horizontal movement, we model an accelerating or braking vehicle as a point load travelling along the surface either with acceleration or deceleration (Fig.1). The low-frequency approximation assumes that the shortest wave-lengths of the generated ground vibration spectra are larger than the dimensions of the vehicle.

If a vehicle is accelerated or decelerated at constant absolute value a, then the point load force applied from a vehicle to the ground surface is a horizontal traction force that moves along the x-axis with a vehicle and has the amplitude

$$F_x = aM . \qquad (1)$$

The related mechanical shear stresses applied to the ground surface during acceleration or deceleration, i.e. during the period of time from $t = 0$ to $t = v/a$, where v is the final or initial speed, are described respectively as follows:

$$T_{xz}(\rho, t) = -F_x\, \delta(x - at^2/2)\delta(y), \qquad (2)$$

and

$$T_{xz}(\rho, t) = F_x\, \delta(x - vt + at^2/2)\delta(y). \qquad (3)$$

Here T_{ij}, where $i,j = 1,2,3,$ are the components of a load stress tensor applied to the surface, $\rho = \{x, y\}$ is the surface radius-vector, and $\delta(z)$ is Dirac's delta-function.

The ground vibration field generated by accelerating or braking vehicles in an elastic half space, which we assume to be homogeneous and isotropic, should satisfy the elastic Lame' equation

$$(\lambda+2\mu)\, grad\, div\, \mathbf{u} - \mu\, rot\, rot\, \mathbf{u} - \rho_0 \partial^2 \mathbf{u}/\partial t^2 = 0 , \qquad (4)$$

and the boundary conditions on the ground surface taking into account only horizontal traction forces (2) and (3):

$$\begin{aligned} \sigma_{xz} &= 2\mu u_{xz} = - T_{xz}(\rho, t), \\ \sigma_{yz} &= 2\mu u_{yz} = 0, \\ \sigma_{zz} &= \lambda u_{nn} + 2\mu u_{zz} = 0. \end{aligned} \qquad (5)$$

Here $\mathbf{u}$ is the particle displacement vector with the components u_i ; λ and μ are the elastic Lame' constants; and $u_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ are the components of the linearised deformation tensor. Without loss of generality, we will limit our calculation of frequency spectra to the vertical component of ground vibration velocity $v_z = du_z/dt$ which is usually measured in experimental observations.

The solution to the problem (1)-(5) using the Green's function method (see, e.g., [10]) and taking into account only the contribution of generated Rayleigh surface waves results in the following expression for the vertical component of the surface velocity spectrum:

$$v_z(\rho,\Theta,\omega) = \left(\frac{2\pi}{k_R\rho}\right)^{1/2} \frac{\omega k_R^2[k_t^2\nu_l\nu_t - k_R^2 b(k_R)]\cos\Theta}{\pi\mu F'(k_R)k_t^2} \times$$

$$T_{xz}(\omega, k_R\cos\Theta)e^{-k_R\gamma\rho}e^{ik_R\rho - i3\pi/4} \qquad (6)$$

Here the following notations are used: $\rho = \rho(x,y)$ is the distance to the observation point; $k_R = \omega/c_R$ is the Rayleigh wave number, where c_R is the Rayleigh wave velocity; $b(k_R) = 2k_R^2 - k_t^2 - 2\nu_l\nu_t$, $\nu_{l,t} = (k_R^2 - k_{l,t}^2)^{1/2}$ are nonspecified expressions, where $k_{l,t} = \omega/c_{l,t}$ are the wavenumbers of bulk longitudinal and shear acoustic waves, c_l and c_t are their phase velocities; $F'(k_R)$ is the derivative $dF(k)/dk$ of the Rayleigh determinant $F(k) = (2k^2 - k_t^2)^2 - 4k^2\nu_l\nu_t$ taken at $k = k_R$; $\Theta = \cos^{-1}(x/\rho)$ is the observation angle; and $T_{xz}(\omega, k_R\cos\theta)$ is the Fourier transform of the load force ((2) or (3)) which has the following expressions for accelerating and braking vehicles respectively:

$$T_{xz}(\omega, k_R\cos\theta) = -(1/2\pi)aM\int_0^{v/a} e^{i\omega t - ik_R\cos\Theta(at^2/2)}dt, \qquad (7)$$

$$T_{xz}(\omega, k_R\cos\theta) = (1/2\pi)aM\int_0^{v/a} e^{i(\omega - k_R v\cos\Theta)t + ik_R\cos\Theta(at^2/2)}dt. \qquad (8)$$

In writing (6) we have taken account of attenuation of generated ground vibrations in the ground by replacing the wavenumber of a Rayleigh wave in an ideal elastic medium $k_R = \omega/c_R$ by the complex wavenumber $k_R' = k_R(1+i\gamma) = (\omega/c_R)(1+i\gamma)$. Here $\gamma << 1$ is a positive constant which describes the linear dependence of a Rayleigh wave attenuation coefficient on frequency ω. For different types of ground γ are in the range from 0.01 to 0.2 [11]. One can prove that (7) and (8), corresponding to accelerating and braking vehicles respectively, differ from each other only by the phase factor. Therefore, being interested only in amplitudes of ground vibrations $V(\rho,\omega) = |v_z(\rho,\omega)|$, in what follows we discuss the ground vibration spectra generated only by braking vehicles.

The complexity of the integrals (7) and (8) justifies an approximate analytical evaluation avoiding direct numerical integration. One of the simplest approximate approaches is based on neglecting the exact space location of the vehicle during acceleration or braking. For example, in the case of a braking vehicle the corresponding approximation means that (8) is replaced by

$$T_x(\omega, k_R\cos\theta) = T_x(\omega) = (1/2\pi)aM \int_0^{v/a} e^{i\omega t}dt = \frac{1}{2\pi} vM \frac{\sin(\omega v / 2a)}{\omega v / 2a} e^{i\omega v/2a} . \qquad (9)$$

One must keep in mind that most spectrum analysers used in environmental acoustics give readings averaged over certain frequency bands around the central frequency of interest - the octave, 1/2-octave and 1/3-octave spectra [12]. In further consideration we will use 1/3-octave spectra. The comparison of the 1/3-octave spectra calculated numerically for approximate and exact values of the Fourier integral for $T_x(\omega, k_R\cos\theta)$ shows that they are almost identical. Thus, for calculations involving averaging over 1/3-octave frequency band one can use the simpler formula (9).

Fig. 2 illustrates the behaviour of 1/3-octave spectra of ground vibrations (in dB relative to the reference level of 10^{-9} m/s) generated by a braking lorry with $M = 20000\ kg$ for three different values of deceleration: $a = 1$, 5 and $9\ m/s^2$ (curves *1*, *2*, and *3* respectively). The initial speed is $v = 10\ m/s$. Other parameters are the following: mass density of the ground ρ_0 is $2000\ kg/m^3$, velocity of longitudinal bulk waves - $c_l = 471\ m/s$, shear waves - $c_t = 272\ m/s$ and Rayleigh surface waves - $250\ m/s$ (this corresponds to a Poisson ratio of $\sigma = 0.25$). The constant of ground attenuation of Rayleigh waves was set as $\gamma = 0.05$. It follows from Fig.2 that amplitudes of generated ground vibrations at all frequencies increase with increase of a. The behaviour of generated ground vibration spectra for three different values of the initial speed v = 5, 10 and 20 m/s (curves *1*, *2*, and *3* respectively) is shown on Fig.3 for a = 5 m/s^2 and $\Theta = \pi/3$. It is seen that for frequencies higher than 4-5 Hz the spectra are almost independent of v.

In contrast to the case of generating ground vibrations by vehicles travelling on uneven roads, accelerating and braking vehicles generate ground vibrations dependent on the observation angle Θ with respect to the vehicle movement. If one calculates angular dependence of ground vibration velocity using the traditional definition of directivity patterns in acoustics (see, e.g., [13]) as function $V(\Theta)$ taken at one particular frequency (in practical measurements this means using a very narrow-band spectral analysers), then it turns out that the influence of the exponential in (8) results in dramatic changes of directivity patterns even for small change in central frequency of a radiated spectrum. Such a strong dependence of directivity pattern on frequency makes it impracticable to use a standard definition of directivity for the case considered. Instead, we calculate directivity patterns averaged in the 1/3-octave frequency range using both exact and approximate approaches ((6), (8) and (6), (9) respectively). The results show that calculations according to (6) and (8) averaged over 1/3-octave frequency band give for all central frequencies the angular functions which are very close to $\cos\Theta$ corresponding to the approximate approach

((6) and (9)). It is evident that there is no radiation at the angles $\Theta = \pi/2$ and $3\pi/2$, or in the directions perpendicular to the vehicle movement.

3. VEHICLES TRAVELLING ON DAMAGED OR BUMPY ROAD SURFACES

In the existing theoretical investigations of generating ground vibrations by vehicles travelling on uneven surfaces [2-5] the attention has been paid to statistically rough surfaces of rather good quality, whereas in many practical situations roads have localised uneven areas, such as pits resulting from surface damage or artificial bumps aiming to calm road traffic in some residential areas. Practical observations show that such pits and bumps may become sources of noticeable ground vibrations. In what follows we give a very brief description of the analysis of ground vibration generation by vehicles travelling over pits or bumps.

Typical mechanical model of a road vehicle travelling on uneven road possesses four degrees of freedom corresponding to four main resonance frequencies of low-frequency vibrations corresponding to body bounce and pitch, or to front- and rear-axle hops [4, 8, 9]. Frequencies of body bounce and pitch resonances are normally very low (in the range of 1-3 Hz). Axle-hop resonance frequencies are essentially larger (from 8 Hz to 12 Hz) and are therefore more important from the point of view of generating ground vibrations (we remind the reader that Rayleigh wave generation efficiency is higher at higher frequencies).

Keeping this in mind, we use a simplified model of a vehicle considering the carriage as immobile in vertical direction and taking into account only axle vibrations. This model consists of two identical vibrating systems each having one degree of freedom and comprising an axle mass m and two springs with constants K_1 and K_2 modelling respectively the elasticity of tyre and suspension (Fig.4). Axles are separated from each other by the distance L . We also assume that pit or bump cross-section in the plane $y = 0$ is described by the function $z_1 = f(x)$.

According to the model considered, the equation describing vertical displacements of each axle versus its static position z_2 has the form

$$m\partial^2 z_2/\partial t^2 + Q\partial z_2/\partial t + Kz_2 = K_1 z_1(vt), \qquad (11)$$

where $K = K_1 + K_2$ is a combined elasticity of tyre and suspension, and Q is a total damping coefficient. Assuming that damaged area of the road is small and its centre is located at $x = 0$ and $y = 0$, the related normal stress T_{zz} applied to the ground may be written in the form

$$T_{zz}(\rho,t) = K_1[(z_2(t) - Z_1(t) + z_2(t-L/v) - Z_1(t-L/v)]\delta(x)\delta(y), \qquad (12)$$

where $Z_1(t) = z_1(vt)$ and $Z_1(t-L/v) = z_1(vt-L)$ are the input functions for the front and rear axles respectively.

As before, the generated ground vibrations should satisfy the dynamic Lamé equation (4). But, instead of the boundary conditions (5), one should use the conditions

$$\begin{aligned} \sigma_{xz} &= 2\mu u_{xz} = 0 \\ \sigma_{yz} &= 2\mu u_{yz} = 0, \\ \sigma_{zz} &= \lambda u_{nn} + 2\mu u_{zz} = -T_{zz}(\rho, t). \end{aligned} \qquad (13)$$

Solving the problem (4), (11)-(13) by the Green's function method (using the appropriate component of the Green's tensor) and taking into account only generated Rayleigh waves one can derive the following expression for the vertical component of the surface vibration velocity spectrum:

$$v_z(\rho, \Theta, \omega) = \left(\frac{2\pi}{k_R \rho}\right)^{1/2} \frac{(-i\omega) k_R k_t^2 v_l}{2\pi\mu F'(k_R)} T_{zz}(\omega) e^{-k_R \gamma \rho} e^{ik_R \rho - i3\pi/4} . \qquad (14)$$

Here $T_{zz}(\omega)$ represents the Fourier transform of $T_{zz}(0,t)$ which can be obtained from (11) and (12). In particular, it follows from (11) that

$$z_2(\omega) = \frac{\omega_1^2 Z_l(\omega)}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\omega\alpha)^2}} exp[-i\, tan^{-1}(\frac{2\omega\alpha}{\omega_0^2 - \omega^2})] , \qquad (15)$$

where $\omega_0 = (K/m)^{1/2}$ is the hop resonance frequency, $\omega_l = (K_l/m)^{1/2}$ is the tyre "jumping" resonant frequency, $\alpha = Q/2m$ is a normalised damping coefficient, and $Z_l(\omega)$ is the Fourier transform (spectrum) corresponding to the pit or bump profile.

Obviously, if the spectrum $Z_l(\omega)$ is wide enough, i.e., if the vehicle speed is comparatively high, then, according to (15), the axle vibrations at the hop resonance are effectively excited and noticeable generation of ground vibrations takes place. One can show that in this case the force applied to the ground, $T_{zz}(\omega_0)$, is proportional to $m\omega_l^2(hl/v)cos(\omega_0 L/2v)$, where h is a characteristic height or depth of a localised obstacle and l is its characteristic length. Numerical calculations of generated ground vibration amplitudes for typical values of h , l, and vehicle parameters show that these amplitudes are generally higher than the amplitudes of vibrations generated by earlier considered accelerating and braking vehicles.

4. CONCLUSIONS

Generation of low-frequency ground vibrations by road vehicles has been considered for both vehicles accelerating (decelerating) with a constant acceleration and vehicles travelling at constant speed on damaged or bumpy surfaces.

The amplitudes of ground vibration spectra generated by accelerating and braking vehicles at medium and upper bands of the spectra are determined mainly by acceleration and are almost independent of the initial (final) vehicle speed. For low-frequency spectral bands, oscillations of ground vibration amplitudes versus both acceleration and initial speed may take place. These oscillations may be responsible for large statistical deviations of experimentally observed ground vibration levels.

Ground vibrations generated by vehicles travelling on damaged or bumpy surfaces depend strongly on the relation between the width of the frequency spectrum corresponding to the profile of a pit or a bump and the axle-hop resonance frequency. For typical obstacle and vehicle parameters, the amplitudes of ground vibrations due to pits or bumps are usually higher than the amplitudes of vibrations generated by accelerating and braking vehicles.

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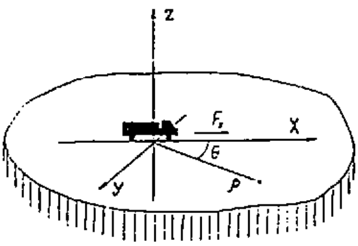

Fig. 1. Geometry of the problem

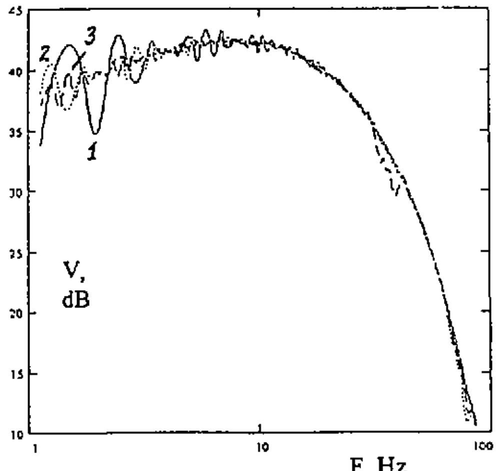

Fig.3. 1/3-octave spectra for different initial speeds

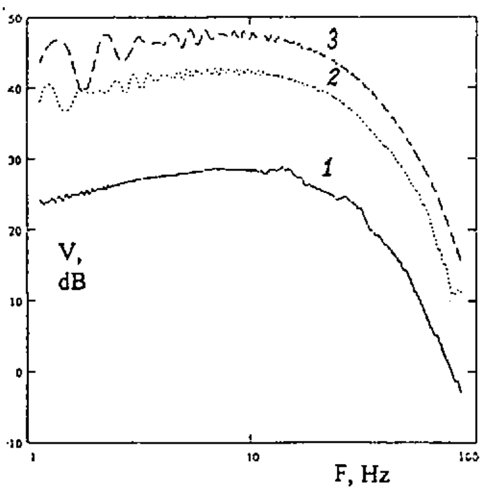

Fig.2. 1/3-octave spectra for different accelerations

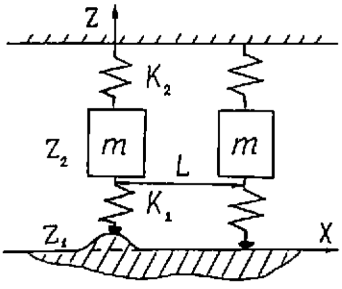

Fig.4. Mechanical model of a vehicle considering axle-hop resonances