

# ACOUSTIC BLACK HOLES: A NEW APPROACH TO VIBRATION DAMPING IN LIGHT-WEIGHT STRUCTURES

VV Krylov      Loughborough University, Loughborough, Leicestershire LE11 3TU, UK.  
EP Bowyer      Loughborough University, Loughborough, Leicestershire LE11 3TU, UK.

## 1 INTRODUCTION

'Acoustic black holes' for flexural waves in plates have been introduced and investigated mainly during the last decade<sup>1-6</sup>. They can absorb almost 100% of the incident wave energy, which makes them attractive for damping structural vibrations. The main principle of this effect is based on a linear or higher order power-law-type decrease in velocity of the incident wave with propagation distance to almost zero accompanied by efficient energy absorption in the area of low velocity via small pieces of inserted absorbing materials. The required gradual reduction in wave velocity with distance can be easily achieved by changing the plate local thickness according to a power law, with the power-law exponent being equal or larger than two. This principle has been applied to achieve efficient damping of flexural waves in plate-like structures using both one-dimensional 'acoustic black holes' (power-law wedges with their sharp edges covered by narrow strips of absorbing materials) and two-dimensional 'acoustic black holes' (power-law-profiled pits with small pieces of absorbing materials attached in the middle). The key advantage of using the acoustic black holes for damping structural vibrations is that it requires very small amounts of added damping materials, in comparison with traditional methods, which is especially important for vibration damping in light-weight structures used in aeronautical and automotive applications.

The present paper provides a brief review of the theory of acoustic black holes and of the recent experimental work carried out at Loughborough University on damping structural vibrations using the acoustic black hole effect. Experimental investigations have been carried out on a variety of plate-like and beam-like structures containing one- and two-dimensional acoustic black holes. The results of the experimental investigations demonstrate that in all of the above-mentioned cases the efficiency of vibration damping based on the acoustic black hole effect is substantially higher than that achieved by traditional methods.

## 2 THEORETICAL BACKGROUND

### 2.1 One-dimensional Acoustic Black Holes for Flexural Waves

Let us first consider a general case of one-dimensional wave propagation characterised by the distance  $x$  in an ideal medium with power-law dependence of wave velocity  $c$  on  $x$  as  $c = ax^n$ , where  $n$  is a positive rational number and  $a$  is a constant. One can express the geometrical acoustics solution for the complex amplitude  $U(x)$  of a wave propagating from any arbitrary point  $x$  towards zero point (where  $c = 0$ ) as

$$U(x) = A(x)e^{i\Phi(x)}, \quad (1)$$

where

$$\Phi = -\int_x^0 k(x)dx = \int_0^x k(x)dx \quad (2)$$

is a total accumulated phase, and  $A(x)$  is a slowly varying amplitude. Since  $k(x) = \omega/c(x) = \omega/ax^n$ , one can see from Eqn (2) that the phase  $\Phi$  becomes infinite if  $n \geq 1$ . This means that under these circumstances the wave never reaches the edge. Therefore, it never reflects back either, i.e. the

wave becomes trapped, thus indicating that the above mentioned ideal medium with a linear or higher power-law profile of wave velocity can be considered as 'acoustic black hole' for the wave under consideration.

For the first time this phenomenon has been described in 1946 by Pekeris<sup>7</sup> for sound waves in a stratified ocean, for a layer with sound velocity profile linearly decreasing to zero with increasing depth. Later on, several other authors have predicted the possibility of the effects of zero reflection for wave phenomena of different physical nature: in particular, for internal water waves in a horizontally inhomogeneous stratified fluid<sup>8, 9</sup>, as well as for particle scattering in quasi-classical approximation of quantum mechanics<sup>10</sup>. Mironov<sup>11</sup> has predicted a practically important possibility of zero reflection of flexural waves from a tip of an ideal quadratic wedge. Note that a quadratic wedge provides the above-mentioned linear decrease in flexural wave velocity towards a sharp edge. And, whereas the conditions providing a linear or higher-order decrease in wave velocity can be rarely found in a real ocean environment, elastic wedges of arbitrary power-law profile are relatively easy to manufacture. Thus, elastic solid wedges give a unique and very convenient opportunity to materialise the above-mentioned zero-reflection effects associated with 'black holes' and to use them for practical purposes.

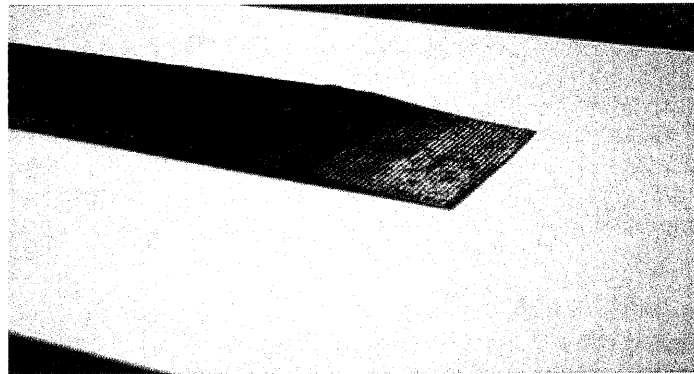


Figure 1. Manufactured steel wedge of power-law profile materializing a one-dimensional acoustic black hole for flexural waves in plates.

To understand the phenomenon of acoustic black holes for the case of flexural waves one should consider the simplest one-dimensional case of plane flexural wave propagation in the normal direction towards the edge of a free elastic wedge (see Figure 1) described by a power-law relationship  $h(x) = \varepsilon x^m$ , where  $m$  is a positive rational number and  $\varepsilon$  is a constant. Since flexural wave propagation in such wedges can be described in the geometrical acoustics approximation<sup>12</sup>, the integrated wave phase  $\Phi$  resulting from the wave propagation from an arbitrary point  $x$  located in the wedge medium plane to the wedge tip ( $x = 0$ ) can be expressed by the above-mentioned Eqn (2). In this case though  $k(x)$  is a local wavenumber of a flexural wave for a wedge in contact with vacuum:  $k(x) = 12^{1/4} k_p^{1/2} (\varepsilon x^m)^{-1/2}$ , where  $k_p = \omega/c_p$  is the wavenumber of a symmetrical plate wave,  $c_p = 2c_l(1 - c_t^2/c_l^2)^{1/2}$  is its phase velocity, and  $c_l$  and  $c_t$  are longitudinal and shear wave velocities in a wedge material, and  $\omega = 2\pi f$  is circular frequency. Again, one can easily see that the integral in Eqn (2) diverges for  $m \geq 2$ . This means that the phase  $\Phi$  becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above mentioned ideal wedges represent acoustic 'black holes' for incident flexural waves.

Real fabricated wedges, however, always have truncated edges. And this adversely affects their performance as 'black holes'. If for ideal wedges of power-law shape (with  $m \geq 2$ ) it follows from Eqn (2) that even an infinitely small material attenuation, described by the imaginary part of  $k(x)$ ,

would be sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower integration limit in Eqn (2) must be changed from 0 to a certain value  $x_0$  describing the length of truncation. Therefore, for typical values of attenuation in such materials as steel, even very small truncations  $x_0$  result in the reflection coefficients  $R_0$  being as large as 50-70 %, which makes it impossible to use such wedges as practical vibration dampers.

It has been found though that the situation for real wedges (with truncations) can be drastically improved<sup>1-4</sup> via increasing wave energy dissipation in the area of slow wave velocity (near the sharp edges) by covering wedge surfaces near the edges by thin absorbing layers (films), e.g. by polymeric films. The simplest way of analyzing this problem is to use the already known solutions for plates covered by absorbing layers of arbitrary thickness obtained by different authors with regard to the description of damped vibrations in such sandwich plates. Using this approach, one can derive the corresponding analytical expressions for the reflection coefficients of flexural waves from the edges of truncated wedges covered by absorbing layers<sup>3,4</sup>. Calculations show that, if an absorbing layer is present, it brings a very substantial reduction of the reflection coefficient, sometimes down to 1-3 %, which constitutes the 'acoustic black hole effect'<sup>3,4</sup>. This means that the combined effect of power-law geometry of a wedge and of a thin absorbing layer makes such systems viable and attractive for practical applications. The same principle of combining power-law geometry with thin absorbing layers can be applied also to beams of power-law profile<sup>6,13</sup>.

The first experimental observation of the 'acoustic black hole effect' for a wedge of quadratic profile has been described in Ref. 5. The system under investigation consisted of a steel wedge of quadratic shape covered on one side by a strip of absorbing layer located at the sharp edge. Measurements of point mobility for such a system undertaken in the frequency range of 100-6500 Hz have shown that, in agreement with the theory, a very significant reduction of resonant peaks (up to 20 dB) has been observed in a wedge covered by an absorbing layer, in comparison with the uncovered wedge or with the reference plates of constant thickness. This has confirmed that vibration damping systems utilising the acoustic black hole effect are efficient and suitable for practical applications.

## 2.2 Two-dimensional Acoustic Black Holes for Flexural Waves

Two-dimensional acoustic black holes for flexural waves, such as protruding cylindrically symmetrical indentations (pits) drilled in a regular thin plate of constant thickness (see Figure 2), have been first proposed and investigated in Ref. 6 using geometrical acoustics approach in Hamiltonian formulation. Earlier, a similar approach was applied to analysing Rayleigh surface wave propagation across smooth large-scale surface irregularities<sup>14,15</sup>. The analysis shows that, in the case of symmetrical pits of power-law profile with  $m \geq 2$ , a number of rays that are close enough to a direct ray, including a direct ray itself, will deflect towards the centre of the pit, approaching it almost in the normal direction. Since the central area of the pit covered by absorbing acts as an efficient absorber for flexural waves, all such 'captured' rays can be considered as fully absorbed rays that have taken away part of the energy of the incident wave.

The above-mentioned two-dimensional acoustic black holes (power-law pits) can be placed at any point of a plate or any other plate-like or shell-like structure. The effect of such black hole is in eliminating some rays, intersecting with the black hole, from contributing to the overall frequency response function of a structure, which may result in substantial damping of some resonant peaks in the frequency response function. To amplify the vibration damping effect of two-dimensional acoustic black holes one can place ensembles of several black holes distributed over the structure (e.g. periodic arrays of black holes), if this does not compromise its main functions, e.g. its rigidity.

Experimental investigation of two-dimensional acoustic black holes has been first described in Ref. 16 for a power-law pit located in one of the foci of an elliptical plate, with an electromagnetic shaker being located in another focus. In such a plate, all radiated flexural wave rays were focused in the area of black hole, which amplified the damping efficiency. Experimental investigations of two-

dimensional acoustic black holes placed in arbitrary locations of rectangular plates was first reported in Ref. 17.

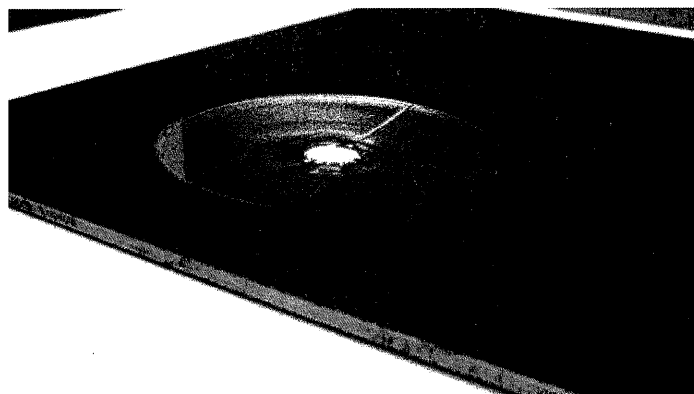


Figure 2. Circular indentation of power-law profile materialising a two-dimensional acoustic black hole for flexural waves in plates.

One of the most important advantages of the above-mentioned one-dimensional and two-dimensional acoustic black holes as dampers of structural vibrations is that they are efficient even for relatively thin and narrow strips of attached absorbing layers. The reason for this is that wave energy dissipation takes place mainly in a very narrow area near sharp edges. This is in contrast with the traditional techniques employing covering the whole surfaces of structures by relatively thick layers of absorbing materials<sup>18, 19</sup>. And this important feature of the acoustic black holes can be very attractive for many practical applications, especially for those involving light-weight structures used in aeronautical and automotive engineering.

### 3 SOME RECENT EXPERIMENTAL INVESTIGATIONS

#### 3.1 Rectangular Plates with Attached Wedges of Power-law Profile

Examples of typical structures that have been recently investigated at Loughborough included plates or beams bounded by the attached power-law wedges, with the addition of small amounts of absorbing material at their sharp edges<sup>20, 21</sup>. The above-mentioned wedges-like structures materialise one-dimensional acoustic black holes for flexural waves placed at the edges of basic rectangular plates vibrations of which are to be damped. The experiments have demonstrated that power-law wedges covered by narrow strips of thin absorbing layers, when attached to edges of rectangular plates, are much more efficient dampers of structural vibrations than traditional rather thick layers of absorbing materials covering entire plate surfaces.

In Ref. 21, the effects of deviations of real manufactured wedge-like structures from ideal elastic wedges of power-law profile on damping flexural vibrations have been investigated experimentally. Namely, the effect of mechanical damage to wedge tips has been studied, including tip curling and early truncation, as well as the placement of absorbing layers on different wedge surfaces. In particular, it has been found<sup>21</sup> that a tip damage (curling), resulting in a wedge with an extended sharp edge, is not detrimental for its damping performance. On the contrary, the extended wedge provided a very efficient damping. Note that similar experimental results have been obtained independently by Bayod<sup>22</sup> who investigated vibration damping in a power-law wedge extended at the sharp edge to form a thin plate of constant thickness, which was made specifically to overcome difficulties associated with manufacturing of very sharp wedges.

### 3.2 Plates Containing Ensembles of Two-dimensional Acoustic Black Holes

Other important structures that have been investigated were rectangular plates with tapered indentations (pits) of power-law profile drilled inside the plates<sup>23, 24</sup>. In the case of quadratic or higher-order profiles, such pits materialise two-dimensional acoustic black holes for flexural waves. To make two-dimensional acoustic black holes even more efficient, ensembles of several (up to six) black holes have been used (see Figure 3). Note that two-dimensional black holes and their ensembles offer an important advantage in comparison with the case of one-dimensional acoustic black holes (wedges of power-law profile). Namely, the potentially dangerous sharp edges of power-law wedges can be eliminated.

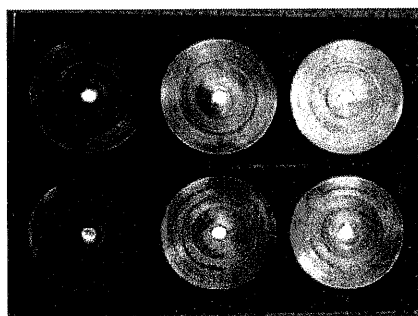


Figure 3. Manufactured steel plate containing an array of six two-dimensional acoustic black holes<sup>23, 24</sup>.

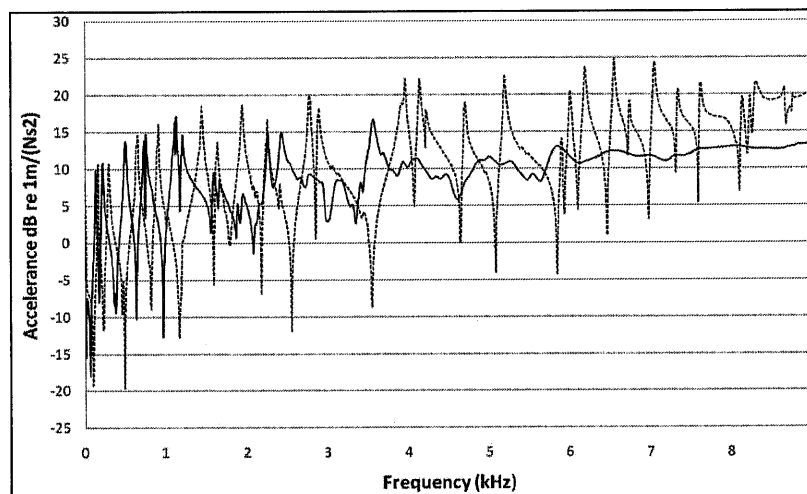


Figure 4. Measured acceleration for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers (solid line), as compared to a reference plate (dashed line)<sup>23</sup>.

Figure 4 shows the measured frequency response for a plate containing six power-law profiled circular indentations with added pieces of damping layers in comparison with the response of a

reference plate of the same size<sup>23</sup>. As it can be seen, the damping at resonant peaks caused by acoustic black holes is quite substantial, up to 15 dB at higher frequencies.

Figure 5 shows the measured distribution of vibration amplitudes over the surface of the above-mentioned rectangular plate with six power-law profiled indentations at the resonant frequency of 4.75 kHz (b), in comparison with the distribution for a reference plate of the same dimensions (a) and at the same frequency<sup>24</sup>. Measurements in this case have been carried out using a scanning laser vibrometer. It can be clearly seen that the presence of acoustic black holes changes the modal shape drastically. In the plate with black holes, the energy of vibration is concentrated in the locations of black holes, where the level of vibration amplitudes is of the same order and even larger than that for a reference plate. And this is where the energy of vibration is absorbed most efficiently. However, outside the black holes the vibrations are almost zero, which illustrates the overall substantial damping observed in Figure 4. Measurements of the total acoustic power radiated by the same plate with six acoustic black holes<sup>24</sup> demonstrate its substantial reduction, which is achieved due to the overall reduction of the vibration amplitudes in such a plate, in spite of the local increases in vibration levels in the areas of black holes.

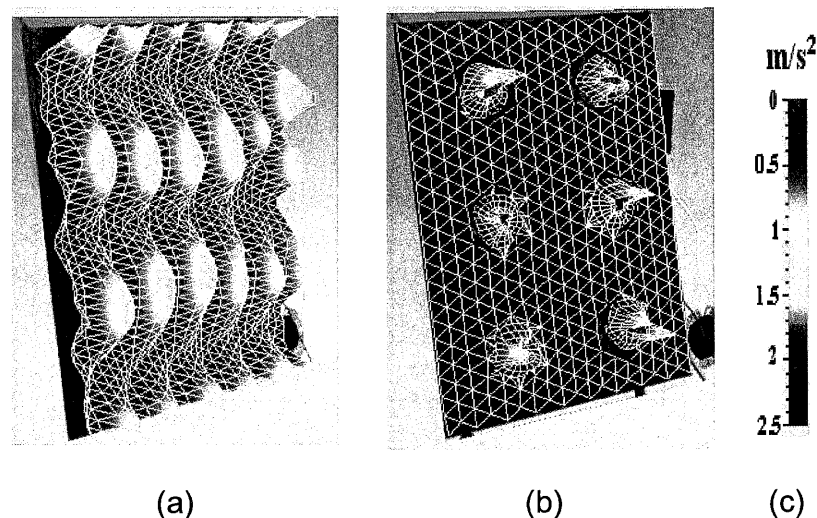


Figure 5. Measured modal shapes for the resonant peak at 4.75 kHz for the plate containing six indentations with damping layers<sup>24</sup>: (a) Modal response of the reference plate, (b) Modal response of the plate with six indentations, (c) Amplitude of response, key.

### 3.3 Some Other Acoustic Black Hole Configurations

Among other promising types of acoustic black-hole geometry that have been studied recently were slots of power-law profile made inside rectangular plates<sup>25</sup>. As was mentioned above, one of the problems faced by one-dimensional black holes formed by power-law wedges attached to edges of structures is having the sharp wedge tip exposed on the outer edge. One of the solutions to this problem is the two-dimensional black holes described in the previous section. Slots of power-law profile placed inside structures represent another possible solution that moves the power-law wedges inside a plate, so that they form edges of power-law slots within the plate. Different configurations of such slots in plates have been manufactured and tested experimentally. It has been demonstrated that slots of power-law profile located within plates materialise a specific type of quasi-one-dimensional acoustic black holes for flexural waves and represent an effective method of damping structural vibrations. The maximum damping achieved on a steel plate with a slot was about 11 dB.

Following the idea first proposed in Ref. 5, experimental investigations into damping of flexural vibrations in turbofan blades with trailing edges tapered according to a power-law profile have been carried out<sup>26</sup>. Four samples of model fan blades have been manufactured. Two of them were then twisted, so that a more realistic fan blade could be considered. All model blades were excited by an electromagnetic shaker, and the corresponding frequency response functions have been measured. The results have shown that the fan blades with power-law tapered edges have the same pattern of damping that can be seen for plates with attached wedges of power-law profile, when compared to their respective reference samples. The obtained results demonstrate that power-law tapering of trailing edges of turbofan blades can be a viable method of reduction of blade vibrations.

In Ref. 27, experimental investigations of one- and two-dimensional acoustic black holes made in composite plates and panels have been carried out. The addition of acoustic black holes resulted in further substantial increase in damping resonant vibrations, in addition to the already noticeable inherent damping due to large values of the loss factor for composites (0.1 - 0.2). Note that, due to large values of the loss factor for composite materials used, no additional layers of absorbing material were required, as expected.

## 4 CONCLUSIONS

The results of the theoretical and experimental investigations briefly described in this paper demonstrate that the efficiency of vibration damping based on the acoustic black hole effect is substantially higher than that achieved by traditional methods. The key advantage of using the acoustic black hole effect for damping structural vibrations is that it requires very small amounts of added damping materials, which is especially important for damping vibrations in light-weight structures used in aeronautical and automotive applications.

The main disadvantage of this method of damping is the requirement to introduce power-law wedges or indentations into structures to be damped, which may compromise their rigidity or integrity. Therefore, this method is limited to some specific structures and applications where structural rigidity and integrity are either not so important or suitable for improvement.

## REFERENCES

1. V.V. Krylov, Laminated plates of variable thickness as effective absorbers for flexural vibrations, Proceedings of the 17th International Congress on Acoustics, Rome, Italy, 2-7 September 2001, Ed. A. Alippi, Vol. 1, 270-271 (2001).
2. V.V. Krylov, Acoustic 'black holes' for flexural waves and their potential applications, Proceedings of the Institute of Acoustics Spring Conference, Salford, UK, 25-27 March 2002 [CD-ROM].
3. V.V. Krylov and F.J.B.S. Tilman, Acoustic 'black holes' for flexural waves as effective vibration dampers, Journal of Sound and Vibration, 274, 605-619 (2004).
4. V.V. Krylov, New type of vibration dampers utilising the effect of acoustic 'black holes', Acta Acustica united with Acustica, 90(5), 830-837 (2004).
5. V.V. Krylov and R.E.T.B. Winward, Experimental investigation of the acoustic black hole effect for flexural waves in tapered plates, Journal of Sound and Vibration, 300, 43-39 (2007).
6. V.V. Krylov, Propagation of plate bending waves in the vicinity of one- and two-dimensional acoustic 'black holes', Proceedings of the ECCOMAS International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPdyn 2007), Rethymno, Crete, Greece, 13-16 June 2007 [CD-ROM].
7. C.L. Pekeris, Theory of propagation of sound in a half-space of variable sound velocity under condition of formation of a shadow zone, J. Acoust. Soc. Amer., 18, 295-315 (1946).

8. S.I. Badulin, L.S. Tsimring and V.I. Shrira, Trapping and vertical focusing of internal waves in the pycnocline due to horizontal inhomogeneities of the stratification and currents, *Dokl. Akad. Nauk SSSR*, 273(2), 459-463 (1983).
9. S.I. Badulin and V.I. Shrira, On the irreversibility of internal wave dynamics due to wave-trapping by mean flow inhomogeneities. 1. Local analysis, *Journal of Fluid Mechanics*, 251, 21-53 (1993).
10. L.D. Landau and E.M. Lifshits, *Quantum Mechanics: Non-Relativistic Theory*, Pergamon Press, Oxford-New York (1977).
11. M.A. Mironov, Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval, *Soviet Physics – Acoustics*, 34, 318-319 (1988).
12. V.V. Krylov, Conditions for validity of the geometrical-acoustics approximation in application to waves in an acute-angle solid wedge, *Soviet Physics – Acoustics*, 35(2), 176-180 (1989).
13. V. Kralovic and V.V. Krylov, Damping of flexural vibrations in tapered rods of power-law profile: Experimental studies, *Proceedings of the Institute of Acoustics*, 29(5), 66-73 (2007).
14. V.V. Krylov, Transmission of a Rayleigh wave across smooth large-scale surface irregularities, *Soviet Physics – Acoustics*, 34, 613-618 (1988).
15. S.V. Biryukov, Yu.V. Gulyaev, V.V. Krylov and V.P. Plessky, *Surface Acoustic Waves in Inhomogeneous Media*, Springer, Berlin, Heidelberg (1995), Chapter 9.
16. V. Georgiev, J. Cuenca, M.A. Molerón-Bermúdez, F. Gautier, L. Simon and V.V. Krylov, Numerical and experimental investigation of the acoustic black hole effect for vibration damping in beams and elliptical plates, *Proceedings of the European Conference on Noise Control "Euronoise 2009"*, Edinburgh, UK, 26-28 October 2009 [CD-ROM].
17. E.P. Bowyer, D.J. O'Boy, V.V. Krylov and F. Gautier, Experimental investigation of damping flexural vibrations using two-dimensional acoustic 'black holes', *Proceedings of the International conference on Noise and Vibration Engineering (ISMA 2010)*, Leuven, Belgium, 20-22 September 2010 (Ed. P. Sas, B. Bergen), pp. 1181-1192 (2010).
18. M. Heckl, L. Cremer and E. Ungar, *Structure Borne Sound*, 2nd Edition, Springer, Berlin, Heidelberg (1988).
19. D.J. Mead, *Passive Vibration Control*, Wiley, Chichester (1998).
20. D.J. O'Boy, V.V. Krylov and V. Kralovic, Damping of flexural vibrations in rectangular plates using the acoustic black hole effect, *Journal of Sound and Vibration*, 329, 4672-4688 (2010).
21. E.P. Bowyer, D.J. O'Boy, V.V. Krylov and J.L. Horner, Effect of geometrical and material imperfections on damping flexural vibrations in plates with attached wedges of power law profile, *Applied Acoustics*, 73, 514-523 (2012).
22. J.J. Bayod, Experimental study of vibration damping in a modified elastic wedge of power-law profile, *Journal of Vibration and Acoustics*, 133, 061003 (2011).
23. E.P. Bowyer, D.J. O'Boy, V.V. Krylov and F. Gautier, Experimental investigation of damping flexural vibrations in plates containing tapered indentations of power-law profile, *Applied Acoustics*, 74(4), 553-560 (2013).
24. E.P. Bowyer and V.V. Krylov, Sound radiation of rectangular plates containing tapered indentations of power-law profile, *Proceedings of Meetings on Acoustics (POMA)*, 18, pp.030002-13 (2013).
25. E.P. Bowyer, V.V. Krylov and D.J. O'Boy, Damping of flexural vibrations in rectangular plates by slots of power-law profile, *Proceedings of the conference 'Acoustics 2012'*, Nantes, France, 23-27 April 2012, pp. 2187-2192 (2012).
26. E.P. Bowyer, J.M. Lister, V.V. Krylov and D.J. O'Boy, Experimental study of damping flexural vibrations in tapered turbofan blades, *Proceedings of the conference 'Acoustics 2012'*, Nantes, France, 23-27 April 2012, pp. 2201- 2206 (2012).
27. E.P. Bowyer, P. Nash and V.V. Krylov, Damping of flexural vibrations in glass fibre composite plates and honeycomb sandwich panels containing indentations of power-law profile, *Proceedings of Meetings on Acoustics (POMA)*, 18, pp.030004-13 (2013).