GENERATION OF GROUND VIBRATIONS BY HIGH-SPEED TRAINS TRAVELLING ON SOFT SOIL

V. V. Krylov

Centre for Research into the Built Environment, The Nottingham Trent University,
Burton Street, Nottingham NG1 4BU, UK

1. INTRODUCTION

Measurements of railway-induced ground vibrations show that the increase in train speeds is normally accompanied by higher levels of generated ground vibrations. Recent theoretical investigations of ground vibrations from high-speed trains (Krylov 1994, 1995, 1996, 1998) contributed to understanding the reasons why an increase in train speeds is accompanied by higher levels of generated ground vibrations. It has been predicted during these investigations that especially large increase in vibration level should take place if train speeds v exceed the velocity of Rayleigh surface waves in the ground c_R. If this happens, a ground vibration boom takes place, similar to a sonic boom predicted by E. Mach more than a century ago.

Note that it took more than 50 years between the publication of the Mach's theory and the appearance of first supersonic aircraft generating a sonic boom. The distance from the first theoretical prediction of a ground vibration boom from high-speed trains (Krylov 1994) to its practical realisation was much shorter. As was reported by Dr C. Madshus at the recent Conference *"Ground Dynamics and Man-made Processes: Prediction, Design, Measurement"* (ICE, London, 20 November 1997), in October 1997 the research team from the Norwegian Geotechnical Institute (NGI) have observed the severe ground motions when train speeds exceeded the Rayleigh wave velocity in the supporting ground. The problem was experienced by the Swedish Railway Authorities (Banverket) when their West-coast main line from Gothenburg to Malmö was opened for the X2 high-speed train (Madshus 1997). The speeds achievable by the X2 train (up to 200 km/h) can be larger than Rayleigh wave velocities in this part of South-Western Sweden characterised by very soft ground. In particular, at the location near Ledsgärd the Rayleigh wave velocity in the ground was as low as 45 m/s, so that the increase in train speed from 140 to 180 km/h lead to about 10 times increase in generated ground vibration level. Thus, these first observations of railway-generated ground vibration boom indicate that the era of "supersonic" or (more precisely) "trans-Rayleigh" trains has begun.

In the present paper we describe the effects of soft soil on ground vibrations generated by high-speed trains, especially in trans-Rayleigh regime, and compare the resulting theoretical

predictions with the recent measurements of a railway-generated ground vibration boom on the West-coast Main Line in Sweden.

2. OUTLINE OF THE THEORY

There are several mechanisms of generating ground vibrations by railway trains. Among these mechanisms the most common and the most important one is a quasi-static pressure of wheel axles onto the track (Fig. 1,a) which is also responsible for railway-generated ground vibration boom. An important aspect of analysing the above mentioned wheel-axle pressure generation mechanism is calculation of the track deflection curve as function of the applied axle load. One can treat a track as an Euler -Bernoulli elastic beam of uniform mass m_0 lying on a visco-elastic half space $z > 0$. To describe vertical deflections of the beam one can use the dynamic equation (see, e.g., Fryba 1973; Belzer 1988):

$$EI\, \partial^4 w/\partial x^4 + m_0 \partial^2 w/\partial t^2 + \alpha w = T\delta\,(x\text{-}vt)\,. \tag{1}$$

Here w is the beam deflection magnitude, E and I are Young's modulus and the cross-sectional momentum of the beam, α is the proportionality coefficient of the equivalent (Winkler) elastic foundation, x is the distance along the beam, T is a vertical point force, v is its speed, and $\delta\,(x)$ is the Dirac's delta-function. The solution of (1) has the form

$$w(x\text{-}vt) = (T/8EI\beta^3\delta)exp(\text{-}\beta\delta|x\text{-}vt|)[cos(\beta\eta(x\text{-}vt) + (\delta/\eta)sin(\beta\eta|x\text{-}vt|)]\,, \tag{2}$$

where $\beta = (\alpha/4EI)^{1/4}$, $\delta = (1 - v^2/c^2_{min})^{1/2}$ and $\eta = (1 + v^2/c^2_{min})^{1/2}$; the term $c_{min} = (4\alpha EI/m_0^2)^{1/4}$ represents the minimal phase velocity of bending waves propagating in a system track/ground. To calculate forces applied from sleepers to the ground, e.g., for a sleeper located at $x = 0$, one should substitute eqn (2) into the following expression:

$$P(t) = T[2w(vt)/w_{max}^{st}](d/x_0^{st}), \tag{3}$$

where d is a sleeper periodicity, index "st" corresponds to the quasi-static solution of eqn (1), i.e., for $m_0\partial^2 w/\partial t^2 = 0$. In particular, w_{max}^{st} is the maximal value of $w(vt)$ in quasi-static approximation, and $x_0^{st} = \pi/\beta$ is the effective quasi-static track deflection distance. Since the factor $\delta = (1 - v^2/c^2_{min})^{1/2}$ is present in the denominator of the expression for $P(t)$ following from eqns (3) and (2), these forces increase as the train speed approaches the minimal track wave velocity.

As the next step, one has to derive the Green's function for the problem under consideration. To take into account the effect of layered geological structure, one can use an approximate Green's function which takes into account the effects of layered structure on the amplitude

and phase velocity of only the lowest order surface mode which goes over to a Rayleigh wave at higher frequencies. For simplicity, we also assume that the Poisson ratio σ of the layered ground and the mass density ρ_0 are constant. The corresponding approximate expression for the Green's function component $G^L_{zz}(\rho,\omega)$ describing approximately the effect of a layered structure on generated vertical component of ground vibration velocity has the form:

$$G^L_{zz}(\rho,\omega) = D^L(\omega)(1/\sqrt{\rho}) \, exp(ik_R^L\rho - \gamma k_R^L\rho), \qquad (4)$$

$$D^L(\omega) = (\pi/2)^{1/2}(-i\omega)q^L(k_R^L)^{1/2}(k_t^L)^2 exp(-i3\pi/4)/\mu^L(\omega)F_L'(k_R^L). \qquad (5)$$

Here $\rho = [(x-x')^2 + (y-y')^2]^{1/2}$ is the distance between the source (with current coordinates x', y') and the point of observation (with coordinates x, y), $\omega = 2\pi F$ is a circular frequency, $k_R^L = \omega/c_R(\omega)$ is the wavenumber of a lowest order Rayleigh mode propagating with frequency-dependent velocity $c_R(\omega)$; terms $k_l^L = \omega/c_l^L(\omega)$ and $k_t^L = \omega/c_t^L(\omega)$ are "effective" wavenumbers of longitudinal and shear bulk elastic waves at given frequency ω. In the model under consideration, the "effective" longitudinal $c_l^L(\omega)$ and shear $c_t^L(\omega)$ wave velocities as well as the corresponding "effective" shear modulus $\mu^L(\omega)$ are expressed in terms of frequency-dependent Rayleigh wave velocity $c_R(\omega)$ using the well known relations:

$$c_R(\omega)/c_t^L(\omega) = (0.87 + 1.12\sigma)/(1+\sigma), \qquad (6)$$

$$c_l^L(\omega)/c_t^L(\omega) = [(1 - 2\sigma)/2(1 - \sigma)]^{1/2}, \qquad (7)$$

$$\mu^L(\omega) = \rho_0[c_t^L(\omega)]^2. \qquad (8)$$

The term q^L is defined as $q^L = [(k_R^L)^2 - (k_t^L)^2]^{1/2}$, and the factor $F_L'(k_R^L)$ is determined according to the following relationship (Biryukov et al. 1995):

$$F_L'(k_R^L) = N(\sigma)(k_R^L)^3, \qquad (9)$$

where $N(\sigma)$ is a dimensionless function of the Poisson ratio σ (e.g., for $\sigma = 0.25$, the function $N(\sigma)$ takes the value -2.3.).

In writing (4) we have accounted for attenuation in soil by replacing $1/c_R(\omega)$ in the exponential of the Green's function by the complex value $1/c_R + i\gamma/c_R$, where $\gamma = 0.001 - 0.1$ is a constant describing the "strength" of dissipation of Rayleigh waves in soil (Gutovski & Dym, 1976).

The dependence of Rayleigh wave velocity on frequency, $c_R(\omega)$, is determined by the particular profile of the layered ground, characterised by the dependence of its elastic moduli

GENERATION OF GROUND VIBRATIONS BY HIGH-SPEED TRAINS

λ, μ and mass density ρ_0 on vertical coordinate z. In what follows we consider published values of the functions $c_R(\omega)$ and use their simple analytical approximation:

$$c_R(\omega) = (c_1 - c_2)exp(-\alpha'\omega/2\pi) + c_2 \ . \qquad (10),$$

Here c_1 and c_2 are values of $c(\omega)$ for $\omega = 0$ and $\omega = \infty$ respectively, parameter α' describes the "strengths" of dispersion (it depends on the characteristic layer thickness and on the difference between elastic moduli in the depth and on the surface of the ground).

To calculate ground vibrations generated by a train one needs superposition of waves generated by each sleeper activated by wheel axles of all carriages, with the time and space differences between sources (activated sleepers) being taken into account (Fig. 1,b). Using the Green's function this may be written in the form (Krylov & Ferguson, 1994)

$$v_z(x,y,\omega) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} P(x',y',\omega)G^L_{zz}(\rho,\omega)dx'dy', \qquad (11)$$

where $P(x',y',\omega)$ describes the space distribution of all load forces acting along the track in the frequency domain. This distribution can be found by taking a Fourier transform of the time and space dependent load forces $P(t, x', y'=0)$ applied from the track to the ground. For all sleepers, axles and carriages being taken into account this function has a form

$$P(t, x'y'=0) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n[P(t-(x'+nL)/v)+P(t-(x'+M+nL)/v)]\delta(x'-md)\delta(y'), \qquad (12)$$

where $\rho_m = [y_0^2 + (md)^2]^{1/2}$, N is the number of carriages, M is the distance between the centres of bogies in each carriage and L is the total carriage length. Dimensionless quantity A_n is an amplitude weight-factor to account for different carriage masses (for simplicity we assume all carriage masses to be equal, i.e., $A_n = 1$).

Eqns (11), (12) and (2) - (9) result in the following expression for the frequency spectra of vertical vibrations at $z=0$, $x = 0$ and $y = y_0$ generated by a moving train:

$$v_z(0, y_0, \omega) = P(\omega)D^L(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} [exp(-\gamma\omega\rho_m/c_R(\omega))/\sqrt{\rho_m}][1+exp(iM\omega/v)]\cdot$$
$$exp(i(\omega/v)(md+nL)+i(\omega/c_R(\omega))\rho_m) \ . \qquad (13)$$

The expression (13) is applicable to trains moving at arbitrary speeds. For "trans-Rayleigh trains", i.e., trains travelling at speeds higher than Rayleigh wave velocity in the ground, it

follows from (13) that maximum radiation of ground vibrations (a ground vibration boom) takes place if the train speed v and Rayleigh wave velocity $c_R(\omega)$ satisfy the relation

$$cos\Theta = 1/K = c_R(\omega)/v, \qquad (14)$$

where Θ is the observation angle. Since the observation angle Θ must be real $(cos\Theta \leq 1)$, the value of $K = v/c_R(\omega)$ should be larger than 1, i.e., the train speed v should be larger than Rayleigh wave velocity $c_R(\omega)$.

3. EFFECT OF TRAIN SPEED ON GENERATED GROUND VIBRATIONS

In this section we discuss some numerical results following from the above theory.

Figure 2,a shows the effect of train speed v on the amplitude of 1/3-octave spectral component of ground vibration velocity v_z (in dB relative to 10^9 m/s) generated by a TGV train travelling at speeds from 0 to 110 m/s (396 km/h) on a homogeneous ground ($\alpha' = 10$) with Rayleigh wave velocity $c_R = 125$ m/s. Note that the above mentioned train speeds are always lower than Rayleigh wave velocity in the case considered which represents typical medium-soft soil. Calculations were carried out using averaging of eqn (4) around the central frequency of 25 Hz. Train consisted of $N=5$ equal carriages with the parameters $L = 18.9$ m and $M = 15.9$ m. Since the bogies of TGV and Eurostar trains have a wheel spacing of 3 m and are placed between carriage ends, i.e., they are shared between two neighbouring carriages, one should consider each carriage as having one-axle bogies ($a = 0$) separated by the distance $M = 15.9$ m. Other parameters of track and ground were: $T = 100$ kN, $\beta = 1.28$ m^{-1}, $y_0 = 30$ m. One can see that, in agreement with numerous practical observations, the averaged level of vibrations increases slowly with the increase of train speed v (the sharp peak around $v = 17.5$ m/s relates to the sleeper passage frequency $f_p = v/d$ which for $f_p = 25$ Hz and $d = 0.7$ m gives exactly $v = 17.5$ m/s).

Fifure 2,b illustrates the effect of train speed v on ground vibrations generated by a TGV train travelling at the same speeds as on Fig. 2,a, but on very soft ground for which the recent observation of a ground vibration boom has been made by the team from the Norwegian Geotechnical Institute (see the Introduction). Since no detailed site information and experimental methodics related to this observation are available at the moment, we calculate ground vibration velocity v_z^{av} averaged over the whole frequency range of interest, 0-50 Hz, and use the reported low value of Rayleigh wave velocity ($c_R = 45$ m/s), assuming that the Poisson ratio of the ground σ is 0.25 and $\alpha' = 10$. To facilitate the comparison of the predicted increase in ground vibration level with the observed one we calculate the amplitudes of ground vibrations in linear units (m/s). One can see that the predicted amplitudes of

vertical vibration velocity of generated ground vibrations change from $2\,10^{-5}$ m/s at $v = 140$ km/h $(38.8\ m/s$; see vertical mark "*" on Fig. 2,b) to $16\,10^{-5}$ m/s at $v = 180\ km/h$ $(50\ m/s$; see vertical mark "**" on Fig. 2,b). Thus, the estimated 8 times increase in ground vibration level following from the above theory for the considered train speeds and Rayleigh wave velocity of the ground is in reasonable agreement with the 10 times increase recently observed experimentally (Madshus 1997).

4. CONCLUSIONS

Theoretical calculations of ground vibrations associated with a railway-generated ground vibration boom earlier predicted by the present author (Krylov 1994) show that for trains travelling on soft soils the expected increase in amplitudes of generated vibrations agrees with the recent experimental observations carried out by the Norwegian Geotechnical Institute for Swedish X2 high-speed trains operating on the line connecting Gothenburg and Malmö. This implies that railway-generated ground vibration boom is no longer an exotic effect of the future. It is a reality for high-speed railway lines crossing soft soils, and so are "supersonic" or "trans-Rayleigh" trains. The builders and operators of high-speed railways must be aware of possible consequencies of a ground vibration boom, so that it would be possible to undertake necessary remediation measures.

5. REFERENCES

BELZER A.I. (1988) "Acoustics of Solids", Springer-Verlag, Berlin, Heidelberg.

BIRYUKOV S.V., GULYAEV Yu.V., KRYLOV V.V. and PLESSKY V.P. (1995) "Surface Acoustic Waves in Inhomogeneous Media", Springer Series on Wave Phenomena, Vol. 20, Springer-Verlag, Berlin, Heidelberg.

FRYBA L. (1973) "Vibration of solids and structures under moving loads", Noordhoff, Groningen, The Netherlands.

GUTOVSKI T.G., DYM C.L. (1976) "Propagation of ground vibration: a review", Journ. of Sound and Vibration, 49, 179-193.

KRYLOV V.V. (1994) "On the theory of railway-induced ground vibrations", Journ. de Physique IV, 4, C5-769-772.

KRYLOV V.V. (1995) "Generation of ground vibrations by superfast trains", Applied Acoustics, 44, 149-164.

KRYLOV V.V. (1996) "Vibrational impact of high-speed trains. I. Effect of track dynamics", Journ. Acoust. Soc. Am., 100 (5), 3121-3134.

KRYLOV V.V. (1998) "Effect of track properties on ground vibrations generated by high-speed trains". Acustica-acta acustica, 84, No 1, 78-90 (1998).

KRYLOV V.V., FERGUSON C.C. (1994) "Calculation of low-frequency ground vibrations from railway trains", Applied Acoustics, 42, 199-213.

MADSHUS C. (1997) "Private communication".

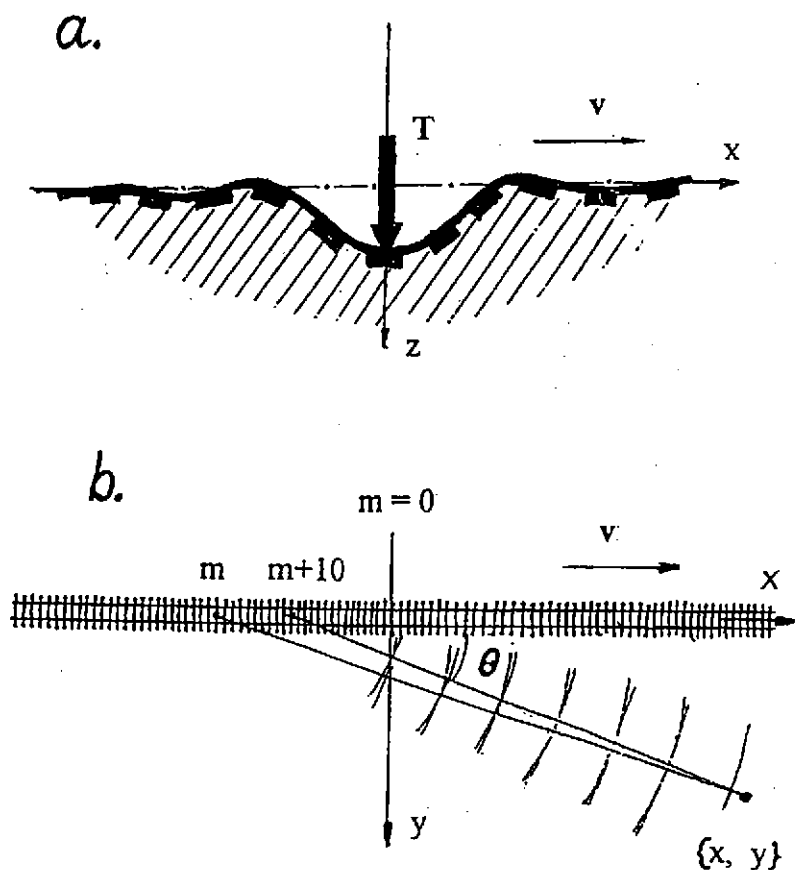GENERATION OF GROUND VIBRATIONS BY HIGH-SPEED TRAINS

Fig. 1. Geometry of the problem:
wheel-axle pressure generation mechanism - (*a*);
superposition of ground vibrations generated by
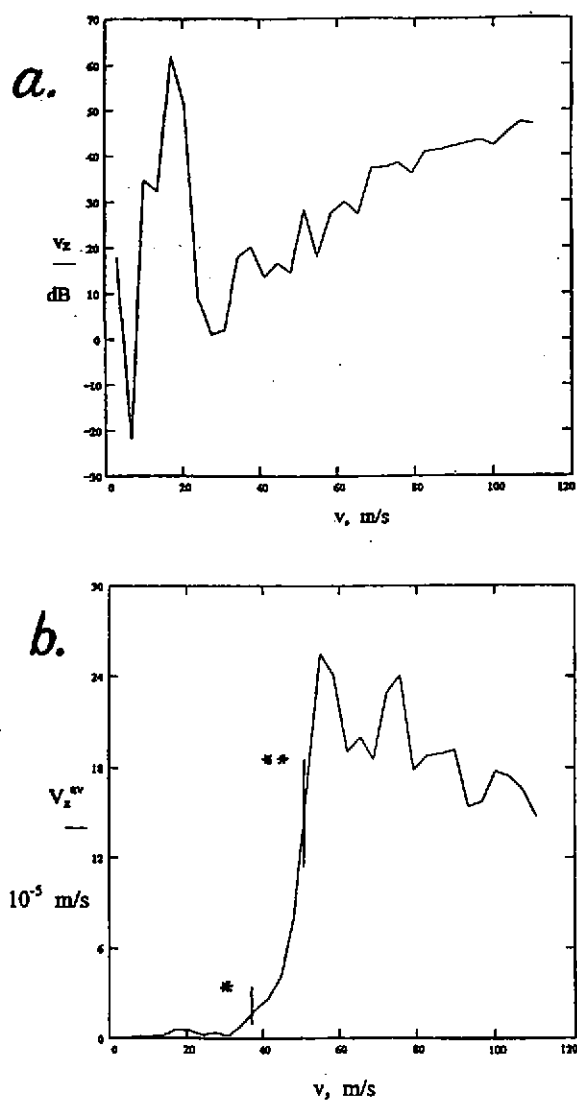different sleepers - (*b*)

GENERATION OF GROUND VIBRATIONS BY HIGH-SPEED TRAINS

Fig. 2. Effect of train speed v on generated ground vibrations