GENERATION OF GROUND VIBRATIONS BY VEHICLES CROSSING FLEXIBLE SPEED BUMPS

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1 INTRODUCTION

Speed bumps and speed humps are used widely in residential areas to enforce drivers to reduce vehicle speeds, which decreases the number and severity of road accidents. Unfortunately, this positive effect is achieved partly in expense of the increased level of traffic-induced ground vibrations and the associated structure-borne noise that can be very annoying to residents of nearby buildings [1]. Note that speed bumps differ from speed humps mainly in the length of the obstacle. A speed bump is shorter than a speed hump. A typical length of a speed bump is from 0.3 to 1 m, and their typical height is from 5 to 15 cm. This configuration dictates relatively comfortable crossing speeds of 10 km/h or less. Therefore, speed bumps are usually used on low-speed private roads and on some residential streets. Speed humps, having the same range of heights, are essentially longer than speed humps. Their typical length is from 2 to 4 m. For that reason, speed humps are usually used on residential streets where speed limits are up to about 35 km/h.

In addition to traditional rigid speed bumps made of asphalt or stones, flexible bumps made of reused rubber tyres or some polymeric materials have become popular over the last years. The advantage of flexible speed bumps is that they can be installed easily (see Fig. 1), and they can be quickly removed if not needed. Whereas ground vibrations from road traffic have been studied in a number of theoretical and experimental papers (see e.g. [1-11]), only a few of them paid attention specifically to ground vibrations generated by speed bumps [1, 6-8]. All these papers considered rigid speed bumps. Ground vibrations from flexible speed bumps received no attention so far.

In the present paper, ground vibrations generated by vehicles travelling on roads with installed flexible speed bumps are investigated theoretically. The main practical question here is whether flexible speed bumps, in addition to the above-mentioned advantage in installation, can also reduce the levels of generated ground vibrations, which would be natural to expect. To answer this question, a simple theoretical model of flexible speed bumps is proposed, and the analytical expressions for generated ground vibration spectra are derived. The obtained results are illustrated by numerical calculations and compared to the case of rigid speed bumps.

2 OUTLINE OF THE THEORY

2.1 Vehicle Interaction with Flexible Speed Bumps

We will model a flexible bump as a rigid base covered by a rubber or plastic layer of constant thickness h considered as a Winkler elastic foundation (Fig. 2). The approximate relationship between the vertical elastic displacement of this layer δ and the vertical force P applied to the layer from a vehicle tyre over the typical contact patch S can be expressed using the well-known constitutive equation relating the applied stress P/S with the elastic deformation δ/h (it should be noted that shear elasticity is not taken into account in this model):

$$\frac{P}{S} = E\frac{\delta}{h}. \qquad (1)$$

Here E is the Young's modulus of the material of the elastic layer. After a simple rearrangement, this equation can be rewritten in a more convenient form that will be used in further derivations:

$$P = K_L \delta \,, \qquad (2)$$

where $K_L = SE/h$ is the equivalent stiffness of the virtual spring supporting the moving contacting tyre and associated with the elasticity of the layer. For simplicity, we will neglect the gradual decrease in the layer thickness h at the bump's edges, assuming that this decrease is abrupt

Fig. 1. View of a typical flexible speed bump installed on a residential street (courtesy of Wikimedia Commons).

In what follows we give a brief description of the mechanism of vehicle interaction with flexible bumps necessary for understanding the process of generating ground vibrations. It is assumed that both wheels of the front axle of a vehicle hit bumps simultaneously. This process is then repeated for the rear wheel axle. Thus, for modelling purposes bumps can be considered as two-dimensional obstacles extended to infinity in *y*-direction (perpendicular to *x*-direction indicating the direction of travel). Despite the two-dimensional bump geometry, the problem of wave generation by vehicles interacting with such bumps is three-dimensional. At low frequencies typical for vehicle-generated ground vibrations, the resulting action of two wheels interacting with the bump can be modelled as a single vertical force applied to the ground surface at $x = 0, y = 0$.

A typical mechanical model of a road vehicle travelling over speed bumps has four degrees of freedom corresponding to four main resonant frequencies of low-frequency vibrations associated with body bounce and pitch, and with front- and rear-axle hops [2, 12]. Frequencies of body bounce and pitch resonances are normally very low (in the range of 1-3 Hz). Axle-hop resonance frequencies are essentially larger (from 8 Hz to 12 Hz). Therefore, they are more important for generation of ground vibrations because the efficiency of elastic wave generation is higher at higher frequencies.

Keeping this in mind, we will use a reduced quarter-car model of a vehicle (for each wheel), considering the vehicle body as immobile (frozen) in the vertical direction and taking into account only axle vibrations [4, 6-8]. This model consists of two identical vibrating systems associated with the two vehicle axles, each having one degree of freedom and comprising the axle mass m (the unsprung mass) and the three springs with constants K_2 , K_T and K_L modelling respectively the stiffness of suspension, the tyre compliance, and the equivalent stiffness of the elastic layer of a flexible speed bump (Fig. 3). Note that the vibrating system associated with the second axle is not shown in Fig. 3 for brevity. The two axles are separated from each other by the distance L (wheel base). We also assume that a bump's exterior cross-section in the plane $y = 0$ (including the thickness h of the elastic layer) is described by the function $z_1(x)$ (see Fig. 2).

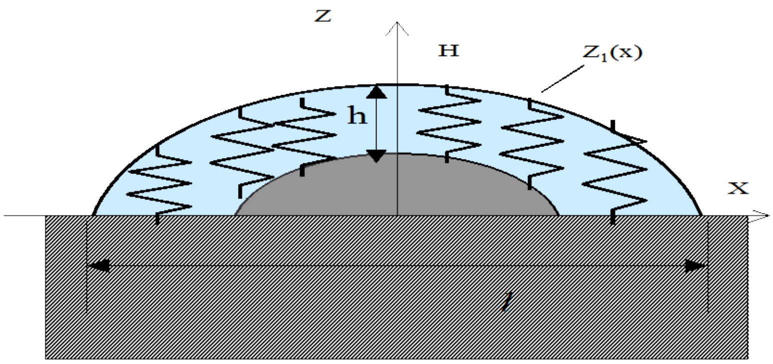

Fig. 2. Vertical cross-section of a flexible bump modelled as a rigid base covered by a rubber or plastic layer of constant thickness h.

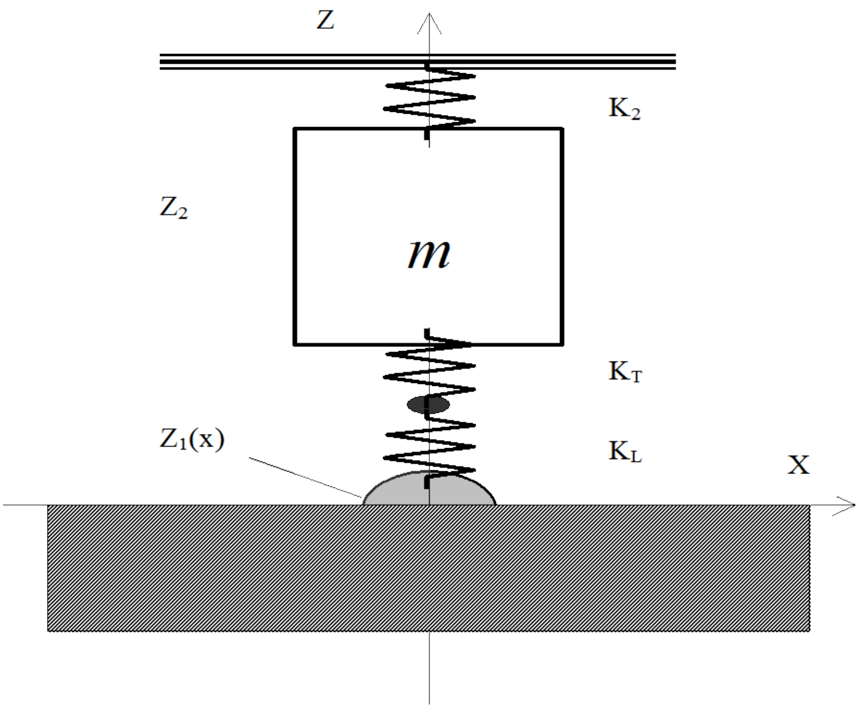

Fig. 3. A reduced quarter-car model of a vehicle travelling over a flexible speed bump: the unsprung mass m is affected by the suspension stiffness K_2, the tyre stiffness K_T, and the effective stiffness of a speed bump K_L. The second axle of a vehicle is not shown for brevity.

Applying Newton's second law to the model under consideration and assuming that the displacement δ of the elastic layer is small in comparison with its thickness h, one can write down the following equation describing vertical displacements z_2 of each vehicle axle versus its quasi-static vertical position that can be approximated as $z_1(vt)$:

$$m\frac{d^2 z_2}{dt^2} = -K_2 z_2 - Q\frac{dz_2}{dt} - K_1(z_2 - z_1(vt)). \tag{3}$$

Here $K_1 = (K_L K_T)/(K_L + K_T)$ is a combined stiffness of the elastic layer of the speed bump and of a vehicle tyre resulting from their series connection, and Q is a total damping coefficient. Introducing the notation $K = K_1 + K_2$ describing the combined action of the tyre along with the elastic layer and of the vehicle suspension on the axle, one can rewrite Eq (3) in the form:

$$m\frac{d^2 z_2}{dt^2} + Q\frac{dz_2}{dt} + Kz_2 = K_1 z_1(vt). \tag{4}$$

Assuming that the length l of a speed bump in x-direction is small in comparison with wavelengths of generated ground vibrations (elastic waves) and its centre is located at $x = 0$ and $y = 0$, the associated normal force $F_z(t)$ applied to the ground from both vehicle half-axles can be written in the form

$$F_z(t) = K_1[(z_2(t) - Z_1(t) + z_2(t - L/v) - Z_1(t - L/v)], \tag{5}$$

where $Z_1(t) = z_1(vt)$ and $Z_1(t\text{-}L/v) = z_1(vt\text{-}L)$ are the input functions for the front and rear half-axles respectively.

Solving Eqn (4) by Fourier method, one can obtain the following expression for the Fourier transform $z_2(\omega)$ of a front wheel axle vertical displacement $z_2(t)$:

$$z_2(\omega) = \frac{\omega_1{}^2 Z_1(\omega)}{\sqrt{(\omega_0{}^2 - \omega^2)^2 + (2\omega\alpha)^2}} \exp[-\mathrm{i}\tan^{-1}(\frac{2\omega\alpha}{\omega_0{}^2 - \omega^2})], \tag{6}$$

where $\omega_0 = (K/m)^{1/2}$ is the wheel hop resonance frequency, $\omega_1 = (K_1/m)^{1/2}$ is the tyre "jumping" resonant frequency (note that in the case of flexible speed bumps both these frequencies depend on the equivalent stiffness of the bump's elastic layer K_L, via K_1), $\alpha = Q/2m$ is a normalised damping coefficient, and $Z_1(\omega)$ is the Fourier spectrum corresponding to the bump profile. The Fourier transform for a rear wheel axle vertical displacement $z_2(t\text{-}L/v)$ differs from (6) only in the phase shift $\omega L/v$, and it is not shown here for shortness. The Fourier transform of the vertical force applied from the vehicle to the ground, $F_z(\omega)$, can be easily obtained from Eqn (5) via replacing $z_2(t)$, $Z_1(t)$, $z_2(t\text{-}L/v)$, and $Z_1(t\text{-}L/v)]$ with their Fourier spectra:

$$F_z(\omega) = K_1[(z_2(\omega) - Z_1(\omega) + z_2(\omega)e^{i\omega L/v} - Z_1(\omega)e^{i\omega L/v}]. \tag{7}$$

2.2 Calculation of Generated Ground Vibrations

The ground vibration field generated by vehicles in an elastic half space, which is assumed to be homogeneous and isotropic, can be calculated using Green's function method (see papers [4-8] and monographs [13,14] for more detail). Since the maximum height H of speed bumps is very small in comparison with characteristic wavelengths of generated ground vibrations (elastic waves), the influence of bump's elevation can be neglected, and it can be assumed that the vertical dynamic force $F_z(\omega)$ resulting from a vehicle interaction with a bump is applied to the free surface $z = 0$.

Taking into account only generated Rayleigh waves, one can write down the following expression for the vertical component of the surface vibration velocity spectrum [8]:

$$v_z(\rho,\omega) = \left(\frac{2\pi}{k_R\rho}\right)^{1/2} \frac{(-i\omega)k_R k_t^{\,2}\nu_l}{2\pi\mu F'(k_R)} \tilde{F}_z(\omega) e^{-k_R\gamma\rho} e^{ik_R\rho - i3\pi/4}. \qquad (8)$$

Here $\tilde{F}_z(\omega) = 2F_z(\omega)$ is the total dynamic force function taking into account (via the factor 2) that there are two wheels in each axle; $\rho = \rho(x,y)$ is the distance from the bump to the observation point; $k_R = \omega/c_R$ is the Rayleigh wavenumber, where c_R is Rayleigh wave velocity in the ground; $\nu_{l,t} = (k_R^2 - k_{l,t}^2)^{1/2}$ are nonspecified expressions, where $k_{l,t} = \omega/c_{l,t}$ are the wavenumbers of bulk longitudinal and shear elastic waves, c_l and c_t are their phase velocities; $F'(k_R)$ is the derivative $dF(k)/dk$ of the Rayleigh determinant $F(k) = (2k^2 - k_t^2)^2 - 4k^2 \nu_l \nu_t$ taken at $k = k_R$.

In writing Eqn (8) we have also taken account of attenuation of generated ground vibrations in the process of their propagation in the ground by replacing the real wavenumber of a Rayleigh wave in an ideal elastic medium $k_R = \omega/c_R$ with the complex wavenumber $k_R' = k_R(1 + i\gamma) = (\omega/c_R)(1+i\gamma)$. Here $\gamma << 1$ is the ground loss factor that describes the linear dependence of Rayleigh wave attenuation coefficient on frequency ω. For different types of ground, the values of γ are in the range from 0.01 to 0.2 (see e.g. [15]). In what follows we will be interested only in amplitudes of ground vibrations $V_z(\rho,\omega) = |v_z(\rho,\omega)|$, ignoring the phase information.

3 NUMERICAL CALCULATIONS AND DISCUSSION

Numerical calculations of ground vibrations generated by vehicles crossing flexible speed bumps have been carried out according to Eqn (8), with account of Eqns (1)-(7), for vehicles travelling at different speeds over road bumps covered by elastic layers of different stiffness. Calculations have been carried our for a typical passenger car with the following parameters: $m = 50$ kg, L = 2.8 m, $K_2 = 35\cdot10^3$ N/m, $K_T = 210\cdot10^3$ N/m, $\alpha = 15$ s^{-1}, $S = 40\cdot10^{-4}$.m^2. The bump was assumed to be of a cosine-shaped profile, and the values of the geometrical parameters used in the calculations were: H = 0.08 m, h = 0.04 m, and l = 0.9 m. It was assumed that the material of the elastic layer covering the bump was rubber. The Young's modulus of rubber E is in the range of (0.01 - 0.1)$\cdot10^9$ N/m^2. Initially a soft rubber was considered with $E = 0.01\cdot10^9$ N/m^2, which for the above values of S and h gave $K_L = 1\cdot10^6$ N/m. This was followed by a stiff rubber with $E = 0.1\cdot10^9$ N/m^2, which resulted in the value of $K_L = 10\cdot10^6$ N/m. The ground parameters were: Rayleigh wave velocity c_R = 125 m/s, Poison's ratio $\sigma = 0.25$, ground mass density $\rho_0 = 2000$ kg/m^3, ground attenuation constant $\gamma = 0.05$. Observation distance from the bump was $\rho = 30$ m. Comparisons have been made with the case of absolutely rigid bumps of the same profile and dimensions.

Figures 4 and 5 show the calculated ground vibration velocity spectra generated by a car travelling at speed v = 5 m/s (18 km/h) over the flexible bump described above in the cases of soft and stiff rubber layers respectively. The thickness of both layers is the same (h = 0.04 m), but the values of their Young's module E are different: $0.01\cdot10^9$ N/m^2 and $0.1\cdot10^9$ N/m^2 respectively. As one can see from Fig. 4, a soft rubber layer does indeed result in noticeable suppression of generated ground vibrations at some frequencies. However, a stiff rubber layer has very little influence on generated ground vibrations that show almost no difference from the case of a rigid speed bump of the same shape (see Fig. 5).

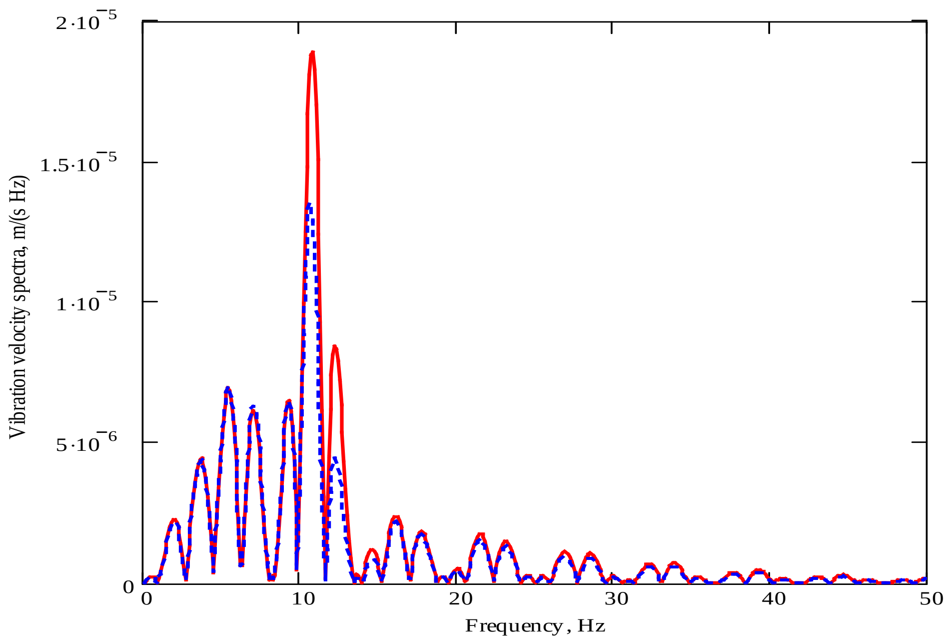

Fig. 4. Spectra of ground vibrations generated by a car travelling at speed *v = 5 m/s* over a rigid cosine-shaped bump - solid curve, and over a flexible bump containing a soft rubber layer (*h* = 0.04 m, *E* = 0.01 10^9 N/m^2) - dashed curve.

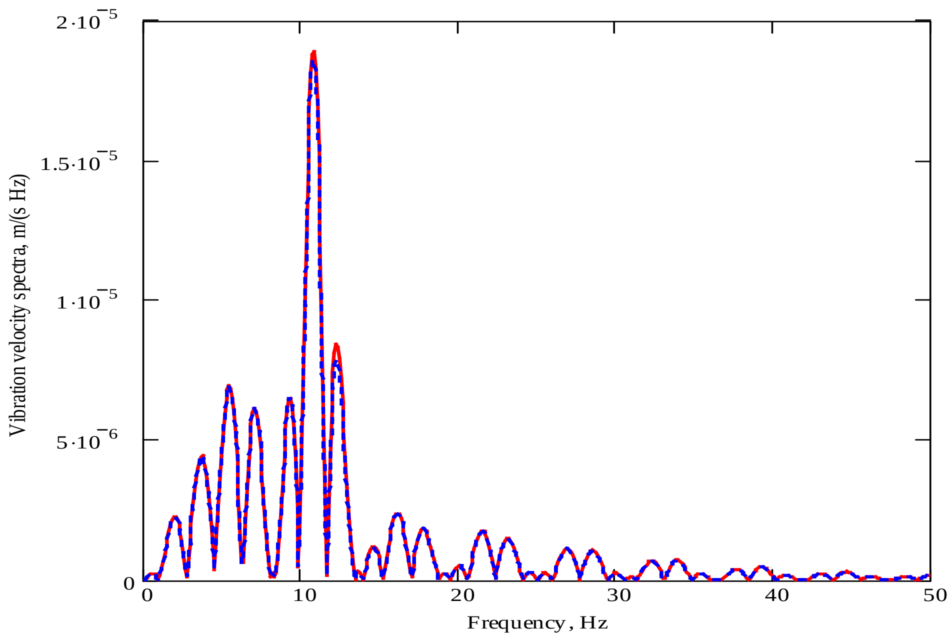

Fig. 5. Spectra of ground vibrations generated by a car travelling at speed *v = 5 m/s* over a rigid cosine-shaped bump - solid curve, and over a flexible bump containing a stiff rubber layer (*h* = 0.04 m, *E* = 0.1 10^9 N/m^2) - dashed curve.

Dependence of generated ground vibrations on vehicle speed v is quite complex for each particular spectral component. Calculated results for a cosine-shaped flexible bump (not shown here for brevity), in comparison with a rigid bump, demonstrate that with the increase of speed v ground vibration amplitudes grow with oscillations (depending on the bump length l and the vehicle wheel base L).

Calculations of the mean-square integral levels (over the whole frequency range) of generated ground vibrations carried out for the same flexible (containing a soft rubber layer) and rigid bumps shows that they depend on v in a more simple way (see Fig. 6). The dependence is roughly linear for typical values of v at which drivers chose to travel in traffic calming areas (from 0 to 7 m/s). Note that in this range of vehicle speeds the difference between integral levels of ground vibrations generated for the cases of rigid and flexible bumps of the same shape is rather small, although a flexible bump still provides some reduction. The difference becomes more noticeable at vehicle speeds between about 10 and 20 m/s.

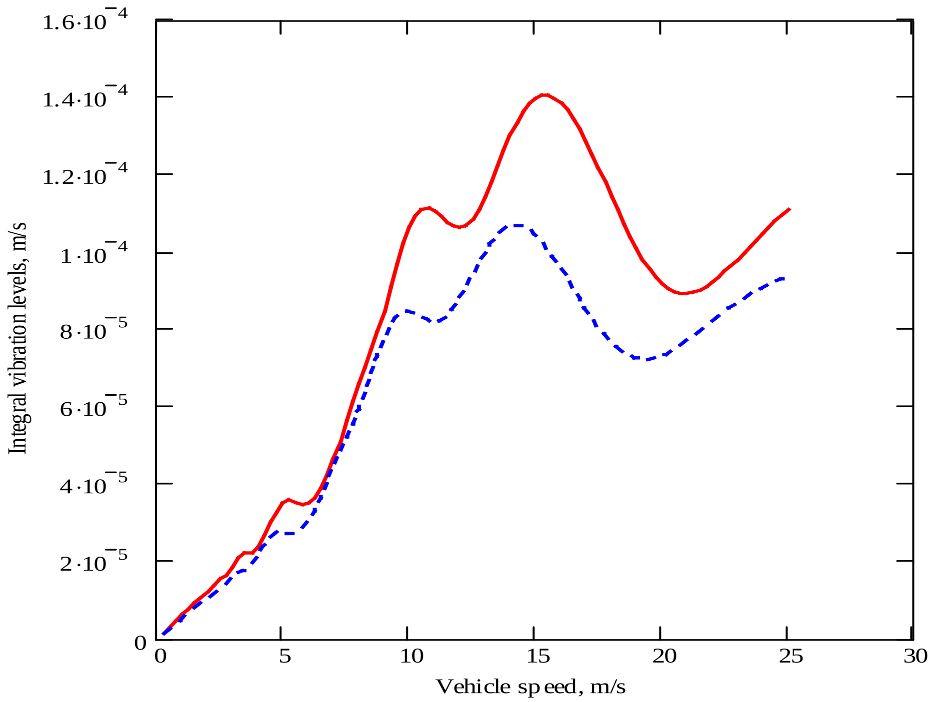

Fig. 6. Integral levels of generated ground vibrations as functions of vehicle speed for the cases of crossing a rigid bump (solid curve) and a flexible bump containing a soft rubber layer (dashed curve). Parameters of the flexible bump are the same as in Fig. 4.

In the case of a flexible speed bump containing a stiff rubber layer (h = 0.04 m, E = 0.1 10^9 N/m^2), the integral levels of generated ground vibrations (not shown here for brevity) are almost indistinguishable from the case of a rigid speed bump at all vehicle speeds.

4 CONCLUSIONS

Theoretical analysis of ground vibrations generated by vehicles travelling over flexible speed bumps shows that, as expected, the effect of flexibility may cause some reduction in generated ground vibrations (in comparison with rigid bumps of the same shape and dimensions). This is clearly visible in the case of bumps containing soft rubber layers (E = 0.01·10^9 N/m^2). However, even in the case of soft rubber layers this reduction is not significant enough to be of practical importance. The reason for this is that even for the above-mentioned case of very soft rubber layer the value of the resulting equivalent stiffness of a rubber layer K_L = 1·10^6 N/m is still about five times larger than a typical tyre compliance K_T = 210·10^3 N/m. This means that the combined stiffness K_1 of a tyre and a rubber layer is dominated by the tyre compliance due to the in-series connection of these two stiffnesses. A possible way to reduce values of K_L to the values comparable with a compliance of pneumatic tyres K_T could be making flexible speed bumps pneumatic as well. However, the practicality of such a solution is yet unclear.

In the case of stiff rubber layers (E = 0.1·10^9 N/m^2) having the same thickness h as above, there are almost no visible reductions in generated ground vibrations because the combined stiffness K_1 is now totally dominated by the tyre compliance. The same is true also for speed bumps made of plastics (E = (0.1 - 4.0)·10^9 N/m^2), which are even stiffer than the above-mentioned stiff rubber. In all these cases the levels of generated ground vibrations are roughly the same as in the case of rigid speed bumps made of asphalt or stones. Thus, in the majority of practical situations, flexible speed bumps are not expected to cause noticeable reductions in vehicle-generated ground vibrations.

REFERENCES

1. G.R. Watts, Vehicle generated ground-borne vibration alongside speed control cushions and road humps, In: Ground Dynamics and Man-made Processes: Prediction, Design, Measurement, Ed. B.O. Skipp, Thomas Telford, London, 87-101, (1998)
2. H.E.M. Hunt, Stochastic modelling of traffic-induced ground vibrations, Journal of Sound and Vibration, 144, 53-70, (1991).
3. T. Hanazato, K. Ugai, M. Mori and R. Sakaguchi, Three-dimensional analysis of traffic-induced ground vibrations. Journal of Geotechnical Engineering, 117, 1133-1151, (1991).
4. V.V. Krylov, Generation of low-frequency Rayleigh waves by heavy lorries, Journal of Low Frequency Noise and Vibration, 14(4), 165-172, (1995).
5. V.V. Krylov, Generation of ground vibrations by accelerating and braking road vehicles, Acustica - acta acustica, **82**(4), 642-649, (1996).
6. V.V. Krylov, Traffic calming and associated ground vibrations, Proceedings of the Institute of Acoustics, 20(1), 41-48, (1998).
7. G.R. Watts and V.V. Krylov, Ground-borne vibration generated by vehicles crossing road humps and speed control cushions, Applied Acoustics, 59(3), 221-236, (2000).
8. V.V. Krylov, Generation of ground elastic waves by road vehicles, Journal of Computational Acoustics, 9(3), 919-933, (2001).
9. G. Lombaert, G. Degrande and D. Clouteau, Numerical modelling of free field traffic-induced vibrations, Soil Dynamics and Earthquake Engineering, 19(7), 473–488, (2000).
10. G. Lombaert and G. Degrande, The experimental validation of a numerical model for the prediction of the vibrations in the free field produced by road traffic, Journal of Sound and Vibration, 262(2), 309–331, (2003).
11. M.A. Lak, G. Degrande and G. Lombaert, The effect of road unevenness on the dynamic vehicle response and ground-borne vibrations due to road traffic, Soil Dynamics and Earthquake Engineering, 31(10), 1357–1377, (2011).
12. J.Y. Wong, Theory of ground vehicles, John Wiley & Sons, New York, (1993).
13. W.M. Ewing, W.S. Jardetzky and F. Press, Elastic waves in layered media, McGraw-Hill, New York, (1957).
14. K.F. Graff, Wave motion in elastic solids. Clarendon Press, Oxford, (1975).
15. T.G. Gutowski and C.L. Dym, Propagation of ground vibration: a review, Journal of Sound and Vibration, 49, 179-193, (1976).