

GROUND VIBRATIONS FROM RAIL AND ROAD TRAFFIC

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FOREWORD

The Rayleigh Medal is recognised in the international acoustics community as the highest honour that an acoustician can have. I am delighted that in the year 2000 the Institute of Acoustics have decided to confer this highly prestigious award to me. I consider this as recognition of some of my results, which were largely inspired by brilliant ideas of Rayleigh.

Lord Rayleigh is undoubtedly the greatest acoustician of all time. Such well-known terms, as 'Rayleigh criterion', 'Rayleigh equation', 'Rayleigh – Ritz method', etc., constitute only a small fraction of the scientific vocabulary that reflects his enormous impact on the development of Classical Physics in the 19th and early 20th centuries, especially Acoustics and Optics [1,2]. A special significance for me and for the topic of this Rayleigh Medal Lecture has the term which reflects one of Rayleigh's brightest discoveries that he has made in the year 1885, literally 'on the edge of his pen' [3]. This is '*Rayleigh waves*' - the first ever known surface waves of truly acoustical nature, in contrast to water waves caused by capillary forces and gravity. Rayleigh himself was well aware of the importance of his discovery. He believed that these surface waves might be important in seismology, for they propagate essentially in two dimensions and therefore experience less of geometric attenuation. All the history of seismology in the 20th century has confirmed this brilliant hypothesis (see, e.g. [4]). It is now well understood that it is indeed Rayleigh waves that carry most of the seismic energy from the epicentre to remote locations. The same property of Rayleigh waves defines their leading role also in generation and propagation of environmental ground vibrations, in particular from rail and road traffic, mining operations, piling, etc. However, these are not the only fields where Rayleigh waves are important. They play a fundamental role in fracture mechanics, where they limit the speed of crack propagation [5], and in solid state physics, where they define crystal lattice dynamics of surfaces and thin films [6].

Probably the most important technical applications of Rayleigh waves include ultrasonic non-destructive testing of materials and constructions, and miniature surface acoustic wave devices of signal processing (see, e.g. [7,8]). The latter field has undergone a tremendous development during the last three decades, having resulted in unique acousto-electronic devices using surface acoustic wave generation and propagation on piezoelectric crystal surfaces, which are now part of almost every television and mobile phone.

According to a very modest estimate, the total number of books, papers, patents and reports on Rayleigh waves and their applications now exceeds thirty thousand. In this lecture we will dwell only on one aspect among the above mentioned – ground vibrations generated by rail and road traffic.

1. INTRODUCTION

First investigations of ground vibrations generated by rail traffic have been initiated about 30 years ago. However, only in the last decade this topic has attracted the increased public attention, partly because of the increased network of underground trains in urban areas and fast development of high-speed surface trains as means of cost-effective and fast communication between cities. In

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both these cases ground vibrations often represent the main element of radiated acoustic energy as directly radiated air-borne noise almost does not exist in the first case and is relatively low in the second.

A number of experimental and theoretical investigations of railway-generated ground vibrations have been carried out for conventional passenger and heavy-freight trains travelling both above- and underground (see, e.g. [9-14] and references there). Later on, such investigations have been extended to high-speed trains, thus contributing to understanding the reasons why an increase in train speeds is normally accompanied by higher levels of generated ground vibrations [15-20]. Also, it has been theoretically demonstrated that especially large increase in vibration level should take place if train speeds v exceed the velocity of Rayleigh surface waves in the ground c_R [15,16]. If this happens, a *ground vibration boom* takes place, similar to a sonic boom studied by E. Mach more than a century ago.

In the fall of 1997 the above mentioned theoretical prediction of ground vibration boom has been experimentally confirmed by a research team working on behalf of the Swedish National Railway Administration (Banverket) on their newly opened West Coast Line - from Gothenburg to Malmö (see, e.g. [21,22]). The speeds achievable by the X2000 high-speed trains operating on this line (up to 200 km/h) can be larger than Rayleigh wave velocities in this part of south-western Sweden characterised by very soft ground. In particular, at the location near Ledsgård (25 km South of Gothenburg) the Rayleigh wave velocity in the ground was as low as 42 m/s, so that an increase in train speed from 140 to 180 km/h lead to about 10 times (20 dB) increase in generated ground vibrations. This indicated that '*supersonic*' or (more precisely) '*trans-Rayleigh*' trains have become today's reality.

As in the case of railway lines, ground-borne vibrations generated by road vehicles represent one of the major adverse environmental impacts of roads, especially in the light of increased numbers and speeds of heavy lorries travelling along town and country roads.

Theoretical and experimental investigations of ground vibrations generated by road traffic have been carried out mainly with regard to impact of heavy lorries. A number of works has been done for vehicles travelling on statistically rough surfaces of rather good quality [23-25], for roads with single obstacles, such as road humps and cushions used for traffic calming [26-28] and for accelerating and braking vehicles on ideally even roads [29,30]. A sufficient level of understanding the problem has been achieved and partly confirmed experimentally.

In what follows, we discuss the results of recent research into ground vibrations generated by rail and road traffic, with the emphasis on the author's original investigations.

2. RAILWAY-GENERATED GROUND VIBRATIONS

2.1 Interaction of Train with Track and Ground

Ground vibrations are generated by trains as a result of their interaction with a system comprising track and ground. There are several mechanisms of generating ground vibrations by trains: the quasi-static wheel-axle pressure onto the track, the roughness of rails and wheels, the effects of joints in unwelded rails and the dynamically induced forces of carriage- and wheel-axle vibrations excited mainly by roughness of wheels and rails. The most common generation mechanism is the above-mentioned quasi-static pressure of wheel axles onto the track. It is always present, even for ideally flat rails and wheels, whereas the contributions of all other mechanisms can be significantly reduced (at least in theory) if rails and wheels are ideally smooth and no carriage or wheel-axle bending vibrations are excited. For high quality tracks and wheels, the wheel-axle pressure mechanism is a major contributor to trainspeed-dependent components of the low-frequency vibration spectra, including the so-called main sleeper passage frequency $f_p = v/d$.

The wheel-axle pressure generation mechanism is characterised by downward deflections of the track beneath each wheel axle, which result in distribution of the wheel loads over the sleepers involved in the deflection area [13,16,31-33]. Thus, each sleeper acts as a vertical force $P(t)$ applied to the ground during the time necessary for a deflection curve to pass through the sleeper. In the framework of the wheel-axle pressure mechanism it is these forces that result in generating ground vibrations by passing trains.

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A railway track supported by the ground is a complex vibration system with distributed parameters which itself can sustain different wave modes propagating at different frequency-dependent phase velocities. In most cases one can treat a track (i.e., two parallel rails with periodically fastened sleepers) as two Euler - Bernoulli elastic beams of uniform mass m_0 lying on an elastic (Winkler) foundation. Such a system can support the so-called track bending waves in both rails. These waves are dispersive and have a velocity minimum at certain frequency. For high-speed trains it may happen that train speeds v become close to this minimal phase velocity of track waves $c_{min} = (4\alpha EI/m_0^2)^{1/4}$, where E and I are Young's modulus and the cross-sectional momentum of the rail, and α is the proportionality coefficient of the elastic foundation (the velocity c_{min} is often called 'track critical velocity'). In this case the track wave resonance may play a noticeable role in determining the train-related force distribution over the sleepers.

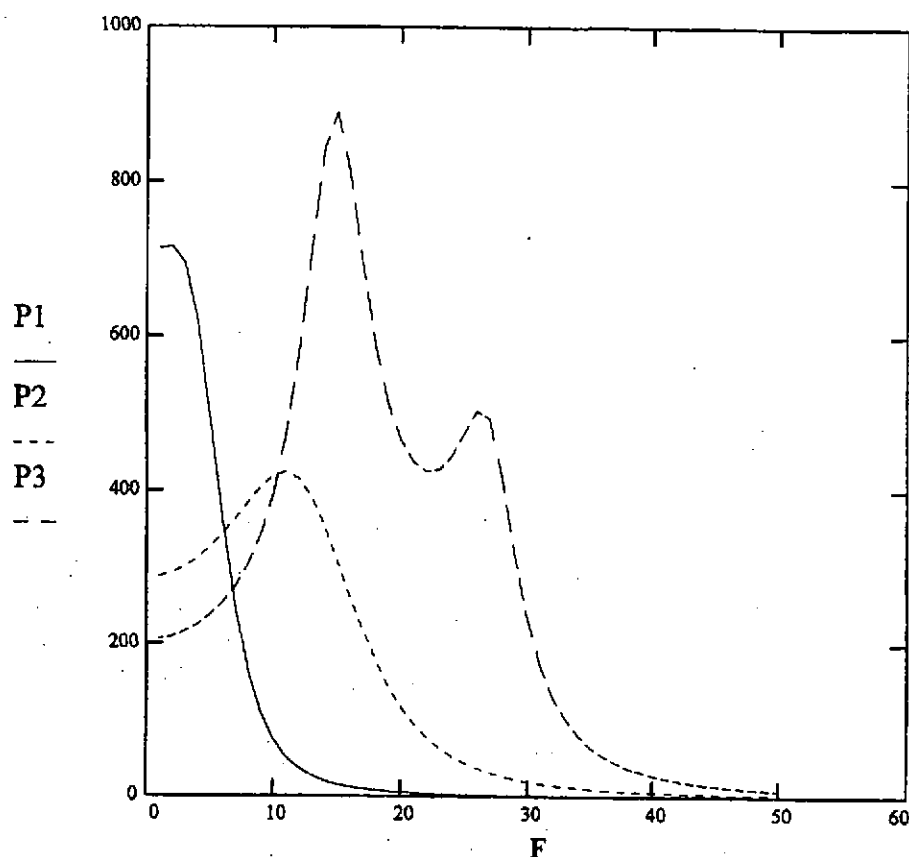


Fig. 1. Spectra of vertical forces applied from each sleeper to the ground (in Ns) for three values of train speed v corresponding to the cases $v < c_R$, $c_R < v < c_{min}$, and $v > c_{min}$ respectively: $v = 20$ m/s (curve P1), 50 m/s (curve P2), and 70 m/s (curve P3); frequency F is in Hz.

For typical parameters of track and ballast in the UK $c_{min} = 200\text{-}300$ m/s (720-1080 km/h) which is essentially larger than even the highest known train speed ($v = 515$ km/h for French TGV). However, for very soft soils c_{min} may be as low as 50-60 m/s, which is comparable with speeds of modern high-speed trains (typically, track critical velocity is by 10-30% higher than Rayleigh wave velocity c_R on the site concerned). If a train speed approaches the track critical velocity, then rail deflections and the associated dynamic forces applied from sleepers to the ground increase in a resonance way and are limited only by damping in the system track-ground that can be characterised by a normalised damping coefficient g .

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Typical forms of the vertical force frequency spectra $P(\omega)$ calculated for a train-wheel load T travelling on very soft soil at speeds $v = 20, 50$ and 70 m/s (corresponding to the cases $v < c_R$, $c_R < v < c_{min}$, and $v > c_{min}$ respectively) are shown on Fig. 1. Calculations have been performed using the following parameters of train and soil: $T = 50$ kN, $c_R = 45$ m/s, $c_{min} = 65$ m/s, and $g = 0.1$. As train speeds increase and approach or exceed the minimal track wave velocity, the spectra $P(\omega)$ become broader and larger in amplitudes, and a second peak appears at higher frequencies. The effect of track damping on the spectra $P(\omega)$ is more pronounced for $v > c_{min}$. For low train speeds, $v < c_R$, the effect of damping is negligibly small.

Note that possible large rail deflections at train speeds approaching track critical velocity may result even in train derailling, thus representing a serious problem also from the point of view of train and passenger safety. Different aspects of this problem are now widely investigated in the UK and abroad (see, e.g. [34, 35]).

2.2 Ground Response to an Elementary Force

The response of the ground to an elementary force applied either to the ground surface or to the bottom of a tunnel (in the case of underground trains) is a fundamental feature determining the whole ground vibration event on a given particular site. Mathematically such a response is called the Green's function of the problem, in honour of George Green, a self-taught 19th century mathematician from Nottingham who for the first time has introduced this fundamental solution as a sequence of what is now called the Green's theorem [36,37]. The physical meaning of such a Green's function for the problem under consideration is that it describes ground vibrations generated by an individual sleeper which in a good approximation can be regarded as a point source in the low-frequency band typical for recorded railway-generated ground vibration signals.

2.2.1 Case of Homogeneous Ground

For a homogeneous elastic half space the corresponding Green's function can be obtained using the results of the well-known axisymmetric problem solved in the year 1904 by another great British scientist – Horace Lamb [38]. The solution of this problem, which satisfy the dynamic equations of elasticity for a homogeneous medium subject to the stress-free boundary conditions on the surface, gives the corresponding components of the dynamic Green's function (Green's tensor) G_{zi} for an elastic half space. This Green's function describes Rayleigh waves propagating in radial directions along the ground surface and longitudinal and shear bulk waves (P- and S-waves respectively) propagating into the depth. In the case of surface trains, only Rayleigh wave contribution (the Rayleigh part of the Green's function) can be considered since Rayleigh waves transfer most of the vibration energy to remote locations along the ground surface.

There are two reasons why Rayleigh waves prevail in this particular case. The first is that Rayleigh wave propagate in two dimensions along the surface and, their geometrical amplitude attenuation as function of distance ρ to the observation point is proportional to $1/\sqrt{\rho}$, rather than to $1/\rho$ for bulk waves. This is exactly what Rayleigh foresaw more than a century ago. Another reason is simply that the total energy radiated into Rayleigh waves generated by point surface forces is noticeably larger than that radiated into bulk longitudinal and shear waves together. The corresponding energy calculations could have been made by Lamb himself. But historically they have been done only in the 50-ties [39]. According to [39], about 70 % of radiated acoustic energy go to Rayleigh waves. This enforces even more the possibility to take into account Rayleigh waves only.

For radiated Rayleigh waves the spectral density of the vertical velocity component of ground vibrations generated by one sleeper at the surface of homogeneous half space ($z=0$) is proportional to $P(\omega)(1/\sqrt{\rho}) \exp(ik_R\rho - \gamma k_R\rho)$, where $P(\omega)$ is a Fourier transform of the force $P(t)$ applied from a sleeper to the ground, ρ is the distance between the source (sleeper) and the point of observation on the ground surface, $\omega = 2\pi F$ is a circular frequency, $k_R = \omega/c_R$ is the wavenumber of a Rayleigh surface wave, c_R is the Rayleigh wave velocity, and $\gamma = 0.001 - 0.1$ is an empirical constant describing the "strength" of dissipation of Rayleigh waves in soil [40].

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2.2.2 Effect of Layered Ground

To consider the influence of layered geological structure of the ground on generating ground vibrations, one would have to use the Green's function for a layered elastic half space, instead of that for a homogeneous half space. As a rule, such a function, that contains information about the total complex elastic field generated in a layered half space considered (including different modes of surface waves and modes radiating energy into the bulk (leaky waves)), can not be obtained analytically. We recall that in layered media surface waves become dispersive, i.e., their phase velocities c_R depend on frequency: $c_R = c_R(\omega)$. For shear modulus of the ground μ normally having larger values at larger depths, there may be several surface modes characterised by different phase velocities and cut-off frequencies. As a rule, these velocities increase at lower frequencies associated with deeper penetration of surface wave energy into the ground (see, e.g., [41]). All these complicate the problem enormously and make its analytical solution almost impossible.

Several numerical approaches to this problem are now in use (see, e.g. [42-44]). In addition to these, simplified analytical or semi-analytical engineering approaches can be applied, in particular the approach that takes into account the effects of layered structure on the amplitude and phase velocity of only the lowest order surface mode only [33,45,46]. This lowest order surface mode goes over to a Rayleigh wave in the limit of a homogeneous elastic half space. The propagating modes of higher orders and leaky modes are generated less efficiently by surface forces associated with a train impact on the track and ground and can be neglected.

The dependence of Rayleigh wave velocity on frequency, $c_R(\omega)$, which is present directly or indirectly in all approaches, is determined by the particular profile of layered ground, characterised by the dependence of its elastic moduli λ , μ and mass density ρ_0 on vertical co-ordinate z . For all ground profiles, the determination of the velocity $c_R(\omega)$ is a complex boundary-value problem which generally requires numerical calculation.

2.2.3 Underground Tunnels

For underground trains the contribution of bulk shear and longitudinal elastic waves (S- and P-waves) is often more essential than that of Rayleigh waves, at least for relatively short distances from the 'epicentre' or, more precisely, from the tunnel projection on the ground surface (the problem of separation of Rayleigh waves from bulk waves generated by underground point sources and their relative contributions for different source depths and observation distances from the epicentre has been first considered by the Japanese seismologist Nakano (see, e.g. [47])). In comparison with surface trains, the case of underground trains is therefore more difficult for theoretical description. Another very significant complication comes due to the influence of the tunnel geometry and tunnel lining, making the problem of constructing the corresponding Green's function extremely complex. Therefore, most of the existing theoretical approaches to this problem use numerical methods and consider two-dimensional models that do not take into account the effects of train speed (see, e.g. [14,48]).

For simple engineering calculations which do take train speeds into account, the approximate analytical approach can be applied [33,49] that considers the problem in the low-frequency approximation, i.e., characteristic wave-lengths of generated bulk acoustic waves in the ground are assumed to be essentially larger than the tunnel diameter. According to the above mentioned approximate approach, the vertical force F applied from a train axle to the bottom of the tunnel can be represented as a superposition of two pairs of forces applied to the top and bottom of the tunnel and having the amplitudes $F/2$ - the pairs of forces acting respectively in downward direction and in opposite directions. The pair of forces acting in opposite directions give a dipole source which contribution is proportional to the product of characteristic wavenumber of bulk waves k_{li} and the tunnel diameter a . At low frequencies this product is small, and hence the dipole contribution can be neglected. On the other hand, the pair of forces acting in the same downward direction give the axle force F , thus indicating that in the low-frequency approximation the Green's function for an underground tunnel can be approximated by field radiated by a single force located underground, without referring to the tunnel properties. The expression for this Green's function, which describes contributions of radiated longitudinal and shear bulk waves, is well known and can be written out analytically. Note that the above mentioned low-frequency approach is inaccurate for frequencies higher than 10-15 Hz. To improve the situation, one can take into account the above mentioned

dipole term in the series expansion of the Green's function, which is proportional to the product of tunnel diameter and characteristic wavenumber of radiated ground vibrations [50].

2.3 Ground Vibrations from Moving Trains

2.3.1 General Remarks

To calculate ground vibrations generated by a train one needs superposition of waves generated by each sleeper activated by wheel axles of all carriages, with the time and space differences between sources (sleepers) being taken into account. Using the Green's function this may be written in the form of convolution of the Green's function with the space distribution of all forces acting to the ground along the track in the frequency domain. This distribution can be found by taking a Fourier transform of the time and space distribution of load forces applied from each sleeper of the track to the ground.

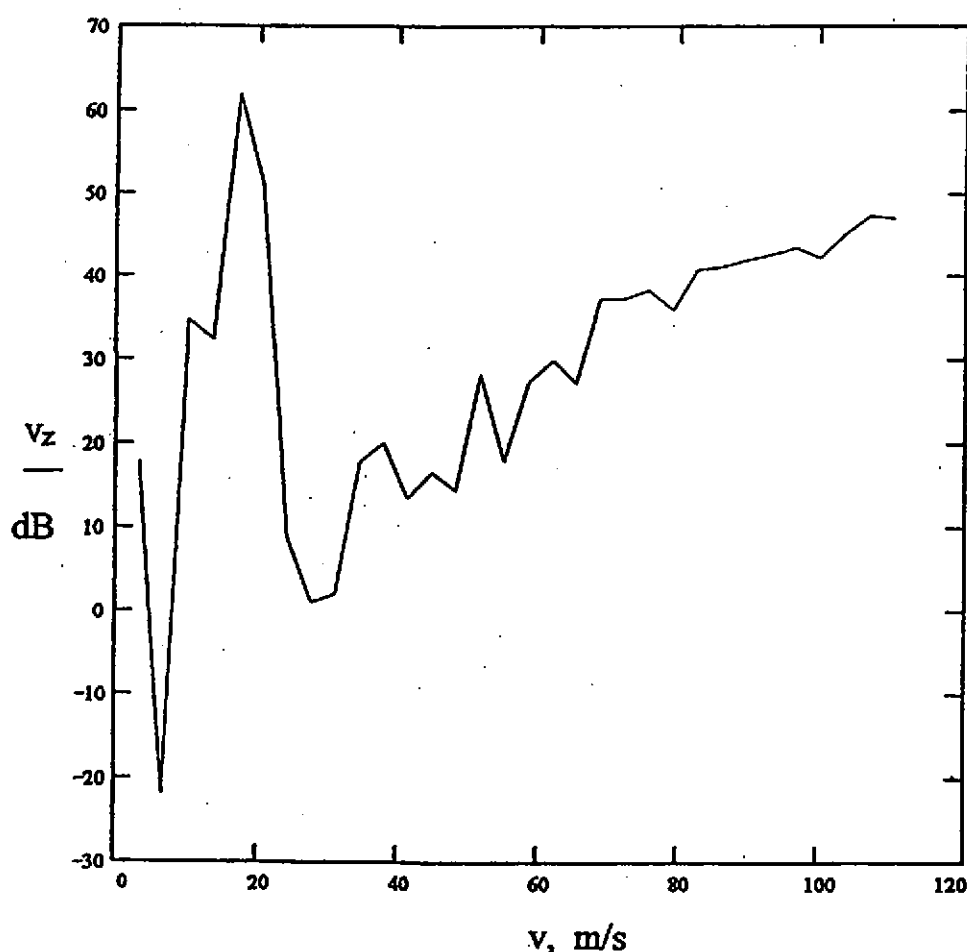


Fig. 2. Amplitude of 1/3-octave component of ground vibration spectra at the central frequency of 25 Hz (in dB, relative to the reference level of 10^{-9} m/s) generated by 'slowly moving' TGV or Eurostar trains as a function of train speed v .

For a single axle load moving at speed v along the track, this function takes into account the train speed, the axle load, the track dynamic properties, and sleeper periodicity. The resulting formula for vertical vibration velocity of ground vibrations generated by a single axle load moving along the track at speed v describes a quasi-discrete spectrum of generated vibrations with

frequency peaks close to the sleeper passage frequency $f_p = v/d$ and its higher harmonics. To take account of all axles and carriages one should use a more complex expression for the force distribution function which, apart from the train speed, the axle load, the track dynamic properties, and sleeper periodicity, also takes into account the number of carriages N , the total carriage length L , the distance between the centres of bogies in each carriage M , and the distance between wheel axles in a bogie a .

2.3.2 Conventional Surface Trains

For simplicity we assume that all carriages are identical and their masses are equal. The resulting vertical velocity component of ground vibrations generated by a complete train moving along the track at speed v has maximum values at frequencies determined by the condition $(\omega/v)(sd + qL) = 2\pi l$, where $s, q, l = 1, 2, 3, \dots$ [13]. This means that radiation of vibrations takes place not only at the earlier mentioned sleeper-passage frequencies and their harmonics, but also at frequencies proportional to v/L and at their combinations with sleeper-passage frequencies.

Effect of train speed v on the amplitude of 1/3-octave spectral component of ground vibrations generated by a 'slowly moving' TGV or Eurostar train at the central frequency of 25 Hz (in dB, relative to the reference level of 10^{-9} m/s) is shown on Fig. 2 for speeds not exceeding the Rayleigh wave velocity ($c_R = 125$ m/s in the case considered) [31, 32]. A train consists of $N=5$ equal carriages with the parameters $L = 18.9$ m, $M = 15.9$ m and $a = 0$. Note that the bogies of TGV and Eurostar trains have a wheel spacing of 3 m and are placed between carriage ends, i.e., they are shared between two neighbouring carriages, so that one should consider each carriage as having one-axle bogies separated by the distance $M = 15.9$ m. One can see that, in agreement with practical observations, the averaged level of vibrations increases with the increase of v . The maximum around $v = 17.5$ m/s relates to the sleeper passage condition $f_p = v/d$ which for $f_p = 25$ Hz and $d = 0.7$ m gives exactly $v = 17.5$ m/s. This type of speed-dependent behaviour of generated ground vibrations has been mentioned also by other investigators (see, e.g. [51]).

2.3.3 Trans-Rayleigh Surface Trains

For the specific case of "trans-Rayleigh trains", i.e., trains travelling at speeds higher than Rayleigh wave velocity in the ground, a particular analytical treatment can be done to elucidate the special features of the problem [15, 16, 31-33]. The analysis shows that maximum radiation of ground vibrations takes place if the train speed v and Rayleigh wave velocity $c_R(\omega)$ in the ground satisfy the following simple relation: $\cos \theta = c_R(\omega)/v$, where θ is the observation angle relative to the direction of train movement. Since the observation angle θ must be real ($\cos \theta \leq 1$), the value of $v/c_R(\omega)$ should be larger than 1, i.e., the train speed v should be larger than Rayleigh wave velocity $c_R(\omega)$. Under this condition ground vibrations are generated as quasi-plane Rayleigh surface waves symmetrically propagating at angles θ with respect to the track, and with amplitudes much larger than for 'sub-Rayleigh trains'. This phenomenon can be called 'ground vibration boom', similarly to sonic boom from supersonic aircraft.

Figure 3 illustrates the ground vibration spectra generated by complete TGV or Eurostar trains travelling on homogeneous ground for both sub-Rayleigh and trans-Rayleigh train speeds (respectively $v = 50$ km/h - curve Vz1 and $v = 500$ km/h - curve Vz2), and for monotonous layered ground (with a soft upper layer) at the same train speeds (curves Vz3 and Vz4 respectively). One can see that for homogeneous ground (curves Vz1 and Vz2) the averaged ground vibration level from a train moving at trans-Rayleigh speed 500 km/h (138.8 m/s) is approximately 70 dB higher than from a train travelling at speed 50 km/h (13.8 m/s). Including the effect of layered ground results in decrease of ground vibration level from a trans-Rayleigh train at low frequencies (curve Vz4). Note that for trains travelling at low speed the effect of layered structure is small (curves Vz1 and Vz3 are almost indistinguishable).

If a three- or four-layer ground model is used, with the softest layer being in the depth of the ground, the Rayleigh wave velocity may have a minimum at a certain frequency (for example, around 4-6 Hz on the location near Ledsgård, Sweden). In such cases a decrease in levels of ground vibrations generated by a trans-Rayleigh train may take place for both low and high frequencies, the high-frequency reduction being the most essential. However, for the range of

frequencies corresponding to the minimum of Rayleigh wave velocity the levels of generated ground vibrations remain very high.

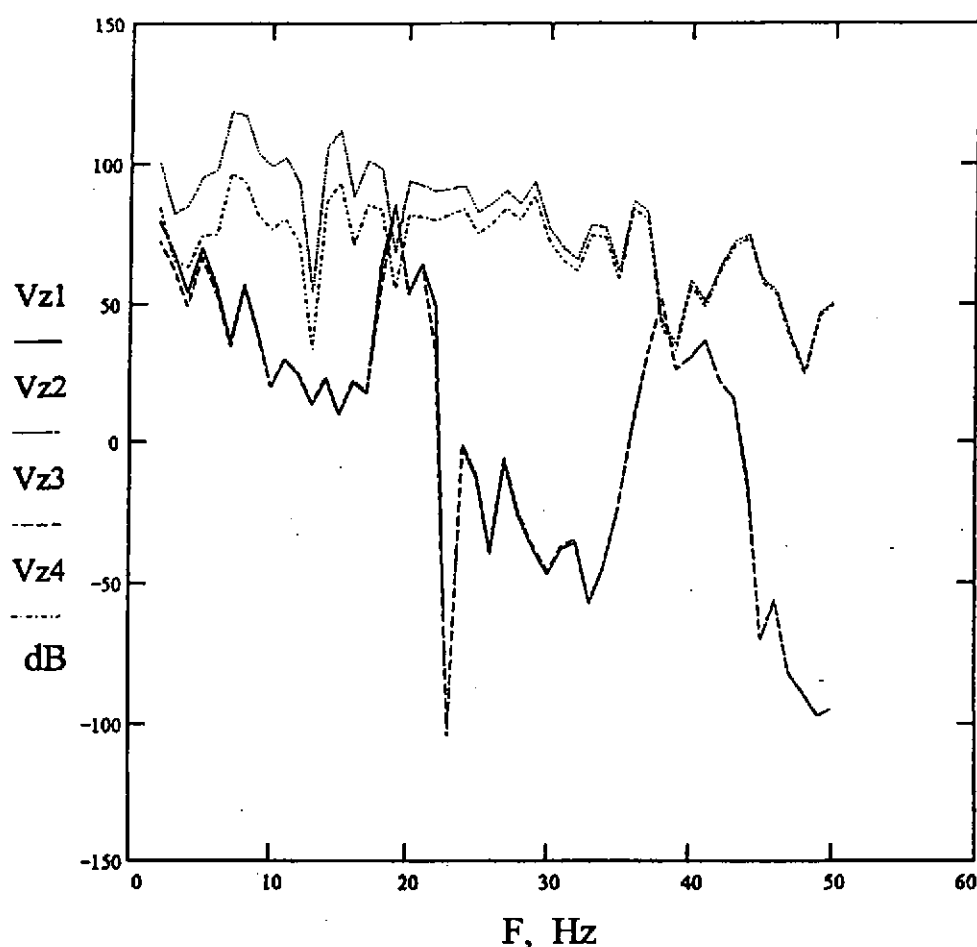


Fig. 3. Ground vibration spectra (in dB, relative to the reference level of 10^{-9} m/s) generated by TGV or Eurostar trains travelling on homogeneous ground at sub-Rayleigh and trans-Rayleigh speeds (respectively: $v = 50$ km/h – curve Vz1 and $v = 500$ km/h – curve Vz2) and on layered ground at the same speeds (curves Vz3 and Vz4 respectively).

2.3.4 Underground Trains

Calculations undertaken in the framework of low-frequency approach (see Section 2.2.3) show that, similarly to the case of surface trains, spectra of ground vibrations from conventional underground trains are quasi-discrete, with the maxima at frequencies determined by the condition $(\omega/v)(sd + qL) = 2\pi l$, where $s, q, l = 1, 2, 3, \dots$ [33, 49]. The main contribution to the vertical component v_z of the total ground vibration field at the ground surface is usually due to the radiated shear bulk waves rather than to the longitudinal bulk waves.

In the case of high-speed underground trains, the analysis shows that the resulting radiated field has two maxima for the values of observation angles satisfying the conditions: $\cos\theta_1 = c/v$ - for radiated shear waves, and $\cos\theta_2 = c/v$ - for radiated longitudinal waves. Since $\cos\theta_1$ and $\cos\theta_2$ must be less than 1 it follows from these conditions that the maxima can be achieved if the train speeds are high enough and the conditions $v > c_1$ or even $v > c_2$ hold. In such cases the

corresponding waves are radiated into the ground as conical waves propagating at the angles $\theta_1 = \arccos(c/v)$ and $\theta_2 = \arccos(c/v)$ relative to the track [33].

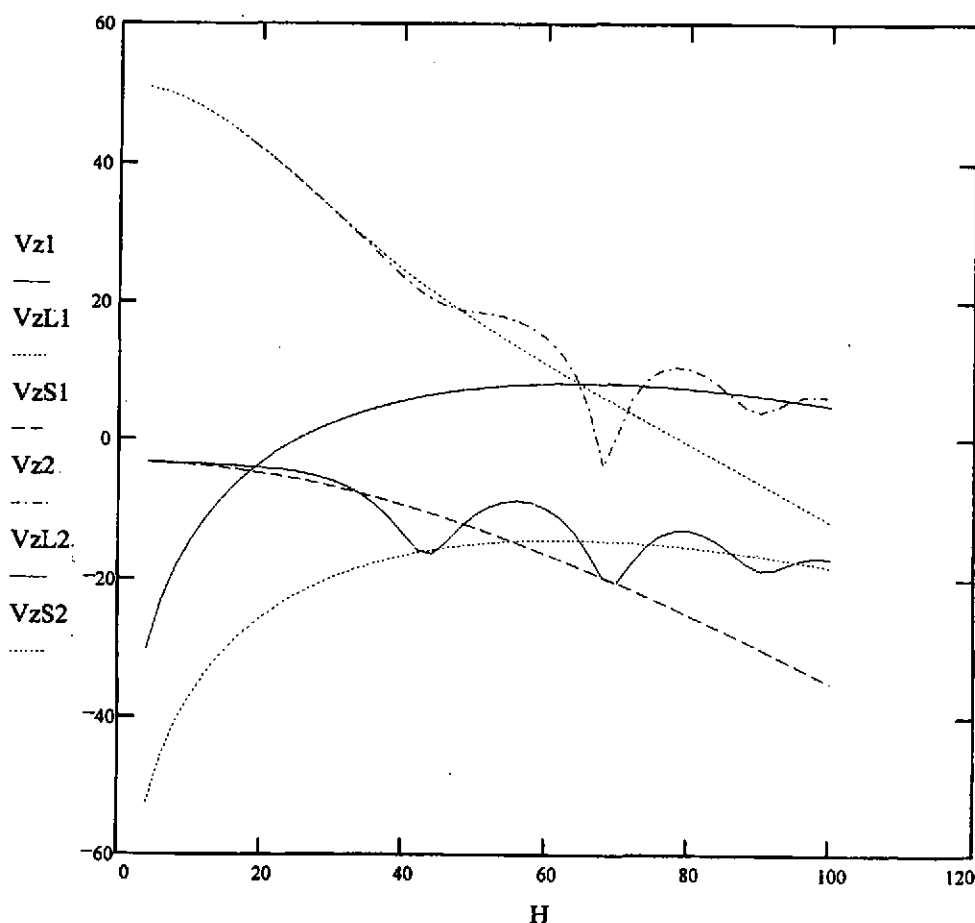


Fig. 4. Ground vibration amplitudes (in dB, relative to the reference level of 10^{-9} m/s) generated by underground TGV trains at frequency $f = 15$ Hz as functions of the tunnel depth H (in m); the results are shown for two values of train speed: $v = 13.8$ m/s (conventional speed) and $v = 80$ m/s (trans-shear speed for the ground considered); the vertical components of ground vibration velocity on the ground surface are indicated as $Vz1$ and $Vz2$ respectively for $v = 13.8$ m/s and $v = 80$ m/s. The corresponding separate contributions of longitudinal and shear elastic waves are shown as $VzL1$, $VzS1$ and $VzL2$, $VzS2$.

The results of calculations for the spectral amplitudes of generated ground vibrations at frequency 15 Hz as functions of the tunnel depth H for $Y_0 = 30$ m are shown on Fig. 4 for two values of train speed: $v = 13.8$ m/s (conventional speed) and $v = 80$ m/s (trans-shear speed for the ground considered). The parameters of a train correspond to a TGV high-speed train with 5 carriages. Other parameters are the following: the elastic parameters of the ground are $c_1 = 76$ m/s, $c_2 = 129$ m/s, and $c_R = 70$ m/s (corresponding to the Poisson's ratio $\sigma = 0.25$), the mass density of soil ρ_0 is 2000 kg/m³, and the wheel load T is 100 kN. The soil attenuation parameters χ_i were chosen as 0.05 , and the effect of track wave velocity was neglected.

It is seen from Fig. 4 that ground vibrations generated by an underground train travelling at speed v higher than shear wave velocity in the ground c_2 are essentially larger than those

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generated by the same train moving at conventional speed (the speed-related amplification of the total ground vibration field varies from about 50 dB to 20 dB for the tunnel depth H changing from 2 m to 100 m). For practical values of H (less than 60-70 m) the contributions of shear waves for the given Poisson's ratio is essentially higher than the contribution of longitudinal waves. For larger depths, the contributions of shear and longitudinal waves first become comparable with each other, causing an oscillatory behaviour of the resulting field versus H , and then longitudinal waves prevail.

Calculations of ground vibration spectra generated by underground trains show that shapes of spectra from underground trains are very similar to those generated by the surface trains travelling at the same speeds. This implies that the shapes of ground vibration spectra are determined mainly by track and train geometrical parameters rather than by the tunnel depth and consequently by types of predominantly generated elastic waves.

2.3.5 Effect of Sleepers on Generated Ground Vibrations

An important conclusion following from the above treatment is that, in the framework of the quasi-static generation mechanism, radiation of ground vibrations by trans-Rayleigh trains may take place also on tracks without sleepers. However, for conventional low-speed trains ($v < c_R$), ground vibrations in the form of waves are not generated on tracks without sleepers if only the quasi-static generation mechanism is considered. This agrees with the well known result of the elasticity theory that, for loads moving along a free surface of an elastic half space at speed $v < c_R$, radiated wave-fields do not exist (only localised quasi-static fields can accompany the moving load) [52]. Thus, the presence of sleepers is paramount for generating ground vibrations by conventional trains due to the mechanism of quasi-static wheel-axle pressure considered here. Changes of d to smaller values may result in noticeable reduction in high-frequency components of generated ground vibration spectra.

Note that the process of generating ground vibrations by quasi-static wheel forces transmitted to the ground via periodic sleepers is nothing more but another manifestation of the general wave phenomenon called 'transition radiation' [53] which was initially investigated in respect of electromagnetic waves radiated by charged particles, such as electrons, moving at constant speed through a transparent plate with the different refraction coefficient. As it is well known, an electron moving at constant speed in a homogeneous medium does not emit electromagnetic waves and is accompanied only by its electro-static field. The situation is absolutely similar in the case of train moving over sleepers, which represent inhomogeneities for applied train-induced quasi-static forces.

2.3.5 Effects of Rail Roughness

On tracks with rough surfaces or on unwelded tracks, the ground vibration spectra predicted according to the above theory may differ from experimentally observed ones because the wheel-axle pressure generation mechanism may not dominate at conventional train speeds. The corresponding mechanisms of generating ground vibrations have not been adequately described for real situations yet, although there exist some calculations of idealised problems for moving vertical forces simultaneously oscillating at given frequencies [19,54]. As expected, generation of ground vibrations by such model forces does not depend on the presence of sleepers and is present at any train speed.

2.4 Comparison with Some Experiments

2.4.1 Measurements in Sweden.

In autumn 1997 the research team working on behalf of the Swedish National Railway Administration (Banverket) have carried out measurements of ground vibrations generated by Swedish high-speed trains X2000 on their newly opened West Coast Line - from Gothenburg to Malmö (see, e.g. [21,22,55]). At the location near Ledsgård (25 km South of Gothenburg) the Rayleigh wave velocity in the ground was only 42 m/s, so that an increase in train speed from 140 to 180 km/h lead to about 10 times (20 dB) increase in generated ground vibrations.

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The actual ground stratification at Ledsgård can be approximated by a non-monotonous four-layered system, with the 'slowest' layer of organic clay being positioned beneath the top layer. Using the theoretical model described above, calculations of the vertical ground vibration velocity averaged over the frequency range 0-50 Hz have been carried out (this roughly corresponds to a peak level of vibration velocity used in the experimental observations). Also, the reported lowest value of Rayleigh wave velocity in the ground ($c_R = 42$ m/s at frequency 5 Hz) has been used. A typical result of the calculations for an X2000 train comprising $N = 5$ equal carriages and travelling on a layered ground is shown on Fig. 5. Geometrical parameters of the train were considered as follows: $L = 24.4$ m, $M = 14.8$ m and $a = 2.9$ m. For comparison, on the same figure the result for a homogeneous elastic ground with $c_R = 42$ m/s is shown as well.

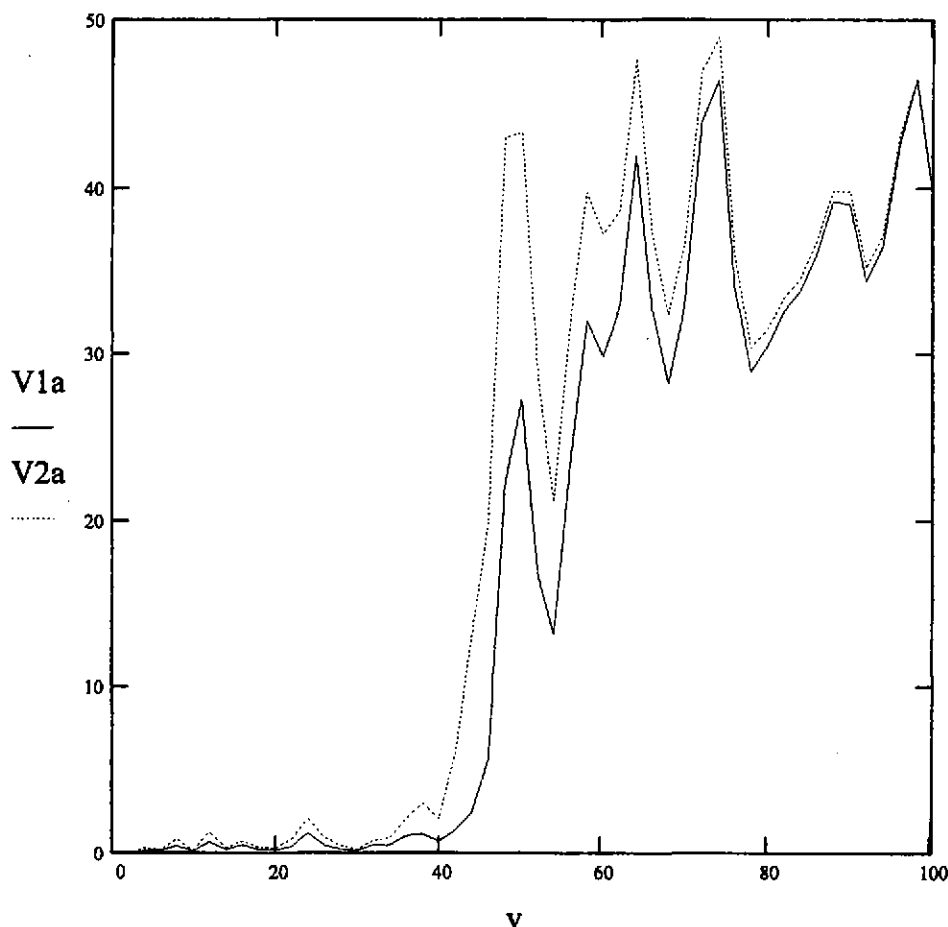


Fig. 5 Effect of train speed v (in m/s) on the averaged ground vibration velocity (in 0.01 mm/s) generated by X2000 high-speed trains in a layered ground with the parameters typical for Ledsgård (curve V1a) and in a homogeneous ground with the value of Rayleigh wave velocity $c_R = 42$ m/s (curve V2a).

As one can see from Fig. 5, the ground stratification reduces the growth of the vibration amplitudes with the increase of train speed. The predicted amplitudes of the peak vertical velocity change from about 2×10^{-5} m/s at $v = 140$ km/h (38.8 m/s) to 24×10^{-5} m/s at $v = 180$ km/h (50 m/s). Thus, the estimated 12 times increase in ground vibration level following from the above theory for the given train speeds and Rayleigh wave velocity is in reasonable agreement with the 10 times increase observed experimentally for typical runs of the X2000 train at Ledsgård. Calculations on Fig. 5 also took into account the effect of track critical velocity c_{min} for train speed approaching or exceeding its value ($c_{min} = 60$ m/s in this example). This effect implies that the level

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of generated ground vibrations becomes larger (by approximately 1.5 times, as compared to the case of absence of track dynamics effects). This increase is not as large as in the case of ground vibration boom. However, since it occurs in combination with the latter, this gives a noticeable additional amplification of the resulting ground vibration impact [56].

In March 2000, the Swedish National Railway Administration (Banverket) have organised the first international seminar on high-speed trains on soft ground which has attracted about 50 participants from 10 European countries, USA and Japan. The culmination of the seminar was the visit of all the participants to the experimental site at Ledsgård. A temporary speed limit of 140 km/h has been recently imposed on this part of the line to avoid the above-mentioned undesirable speed-related effects. However, in spite of this speed limitation, two test runs of the high-speed train X2000 at speed of 200 km/h have been specially arranged by the Banverket to demonstrate the phenomenon. During the high-speed train passages, very strong ground vibrations could be felt on the embankment through the legs. Also, a very strong feeling of ground vibrations was present on the open field at distances 15-20 m from the track.

In July 2000 the remediation measures on the site at Ledsgård have been taken by means of concrete piling of the embankment. This must have stiffened the track, thus reducing the dynamic forces applied to the ground and increasing the track critical velocity beyond the reach of high-speed trains. This would result in reduction of generated ground vibrations. Indeed, the preliminary measurements made in late autumn 2000 indicated some improvements in ground vibration behaviour. More detailed measurements on this site are planned for the near future. In addition to Ledsgård, there are about 20 other sites on the Swedish West Coast Line that have similar dynamic behaviour and need attention. The effects identical to those observed in Sweden were reported also from Holland, Switzerland and Japan. All these clearly indicate that, with the general trend of increase in operating train speeds, the problems associated with ground vibration boom will appear sooner or later in many countries in Europe, Asia and North America.

2.4.2 Measurements in Belgium

In December 1997 the research team from the KU Leuven had carried out detailed experimental measurements of ground vibrations generated by Thalys high-speed trains during the homologation tests organised by the Belgian railway company on the newly built high-speed line from Brussels to Paris before its inauguration [20,57]. Although the highest train speed used in this series of experiments, $v = 314 \text{ km/h}$ (87 m/s), was lower than Rayleigh wave velocity on the site and a ground vibration boom did not occur, the quality and accessibility of the experimental results allowed using them for a thorough validation of the above theoretical model for trains travelling at conventional (sub-Rayleigh speeds). This comparison has been made by both the Belgian team (who used our theoretical model) and by us. The results of such a comparison show that at conventional train speeds the model works well at low and relatively high frequencies, corresponding to sleeper passage frequencies. However, it underestimates the observed ground vibration level at medium frequencies, where a significant contribution is apparently due to the rail-roughness-induced wheel vibrations that were ignored in the above theoretical model. Note in this connection that in the case of ground vibration boom the rail-roughness contribution is negligible and the model of ideally flat welded rails is perfectly justified.

3. GENERATION OF GROUND VIBRATIONS BY ROAD VEHICLES

3.1 General Remarks

There are two main mechanisms of generating ground vibrations by road vehicles [58].

The first one is associated with vehicles travelling on rough or bumpy road surfaces. Note that, in addition to natural road defects, such as bumps and holes, these may be road humps and speed cushions installed by local authorities for the purpose of traffic calming. Installation of road humps and speed cushions results in significant reduction of the severity and number of road accidents. However, such a gain is achieved partly in expense of the increased traffic noise and ground-borne vibration [59].

The second mechanism of generation is associated with acceleration and braking of road vehicles. It is essential in the places where vehicles stop, start or change direction of their movement, e.g., before traffic lights or in supermarket yards.

3.2 Vibrations Generated on Bumpy Roads

3.2.1 Vehicle Interaction with a Bump

Typical mechanical model of a road vehicle travelling on uneven roads possesses four degrees of freedom corresponding to four main resonance frequencies of low-frequency vibrations related to body bounce and pitch, or to front- and rear-axle hops [25, 60]. Frequencies of body bounce and pitch resonances are normally very low (in the range of 1-3 Hz). Axle-hop resonance frequencies are essentially larger (from 8 Hz to 12 Hz) and are therefore more important from the point of view of generating ground vibrations (we remind the reader that ground wave generation efficiency is higher at higher frequencies).

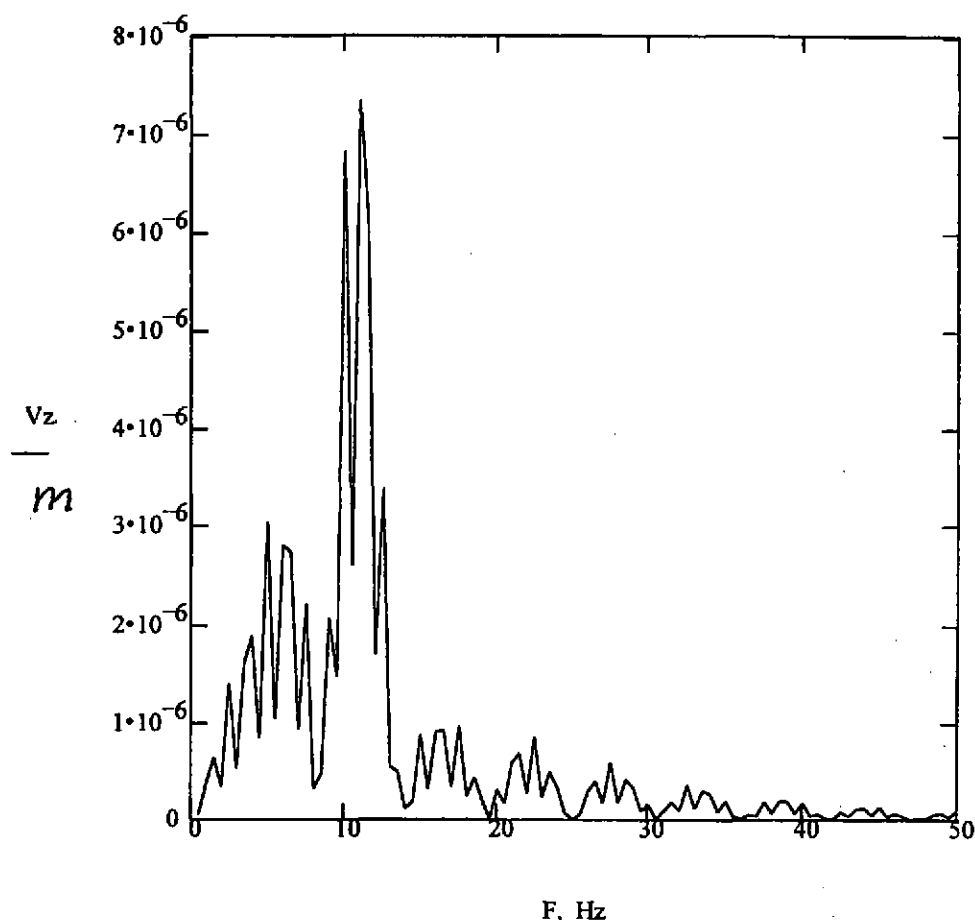


Fig. 6. Spectrum of ground vibrations generated by a road vehicle travelling at speed $v = 5 \text{ m/s}$ over the cosine-shaped hump with the parameters $l = 0.9 \text{ m}$ and $h = 0.074 \text{ m}$

Keeping this in mind, one can use the simplified model of a vehicle, considering its carriage as immobile in vertical direction and taking into account only axle vibrations. This model consists of two identical vibrating systems, each having one degree of freedom and comprising an axle mass

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m and two springs with constants K_1 and K_2 modelling respectively the elasticity of tyre and suspension. Axles are separated from each other by the distance L (wheel base) and the bump cross-section is described by the function $z_1 = f(x)$.

According to the model considered, the vertical displacements of each axle versus its static position z_2 are described by the dynamic equation of a typical vibrating system with one degree of freedom, with the right-hand side proportional to $K_1 z_1(vt)$, where v is a vehicle speed. Solving this equation and calculating z_2 as function of t , one can easily determine the related normal stress $T_{zz}(t)$ applied to the ground at the bump. The important parameters involved are the axle hop resonance frequency $\omega_b = (K/m)^{1/2}$ (where $K = K_1 + K_2$ is a combined elasticity of tyre and suspension), the tyre 'jumping' resonant frequency $\omega_t = (K_1/m)^{1/2}$, the normalised damping coefficient α , and the Fourier spectrum corresponding to the bump profile $Z_1(\omega)$.

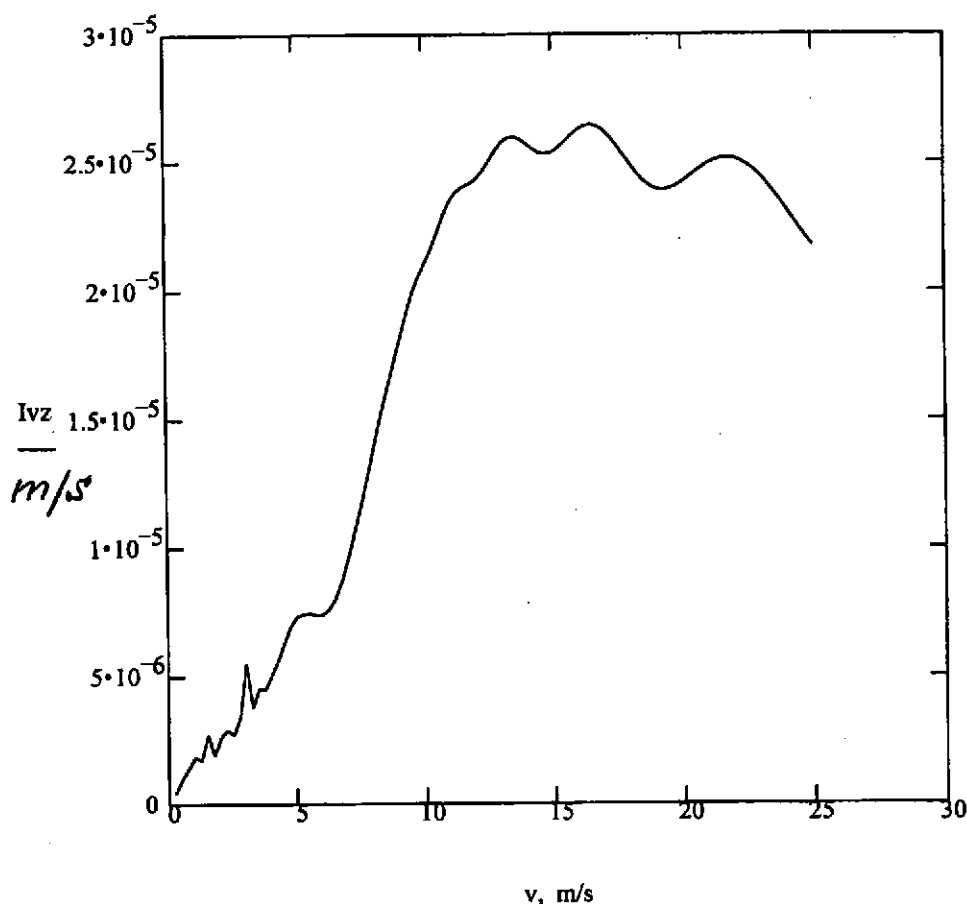


Fig. 7. Integral value of vehicle-generated ground vibrations as a function of vehicle speed. Parameters of the hump are the same as in Fig. 6.

3.2.2 Calculation of generated elastic waves

As in the case of railway-generated ground vibrations, the ground vibration field generated by vehicles in an elastic half space, which we assume to be homogeneous and isotropic, should satisfy the elastic Lamé' equation and the boundary conditions on the ground surface taking into account the vertical force $T_{zz}(t)$ resulting from the interaction of a vehicle with an uneven surface.

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Using again the Green's function method for solving the problem and taking into account only generated Rayleigh waves, one can derive the relatively simple expression that describes the spectrum of vertical component of the surface vibration velocity. Calculations of ground vibrations spectra generated by vehicles crossing road bumps at different speeds show that, if the bump spectrum $Z_1(\omega)$ has components around the axle hop resonance frequency ω_0 , then the axle vibrations are effectively excited and noticeable generation of ground vibrations takes place (see Fig. 6). Amplitudes and spectra of generated ground vibrations depend on hump or cushion profile, their characteristic gradient (ratio h/l), vehicle speed v and load, and elastic parameters of the ground.

In particular, the smoother the bump the lower the level of generated ground vibrations. For example, for typical vehicle speeds v , a bell-shaped hump, $z_1(x) = h \exp(-x^2/l^2)$, results in a significant reduction in the level of generated ground vibrations in comparison with a cosine-shaped hump, $z_1(x) = h \cos(\pi x/l)$, characterised by the same values of h and l .

The dependence of generated ground vibrations on vehicle speed v is quite complex for each particular spectral component. In particular, for small and medium speeds v ground vibration amplitudes grow with oscillations (depending on hump length l and wheel base L) and reach a maximum at certain v . Then, for larger values of v , the level of generated vibrations decays inversely proportionally to v . Calculation of the integral level of generated ground vibrations shows that it depends on v in a more simple way (Fig. 7). The dependence is roughly linear for typical values of v at which drivers chose to travel in traffic calming areas (from 0 to 7 m/s).

Calculations for cosine-shaped road humps of different dimensions demonstrate that the integral level of generated ground vibrations grows roughly in direct proportion to h/l , which is in good agreement with the experimental results [59]. The comparison of calculated peak vibration velocities as functions of vehicle speed with the corresponding measurements made for humps of different profiles also shows rather good agreement [28].

3.3 Ground Vibrations Caused by Acceleration and Braking

3.3.1 Traction Forces Applied to the Ground

The problem of generating ground vibrations during acceleration and braking can be considered for vehicles accelerating (decelerating) with a constant acceleration a from rest to a constant speed v , or braking to a stop from a constant speed v with constant acceleration $-a$. In the low-frequency approximation considered, an accelerating or braking vehicle can be modelled as a horizontal traction force proportional to its mass and acceleration and moving along with the vehicle. The assumption of constant acceleration (deceleration) is in agreement with general requirements for vehicle performance, e.g., in the process of braking. According to these requirements [61], deceleration produced during braking should ideally be uniform throughout an application.

It should be emphasised that, in contrast to the normal load forces considered in the previous section, the main mechanism of ground vibration generation by accelerating or decelerating vehicles is the action of horizontal traction forces which are applied from tyres to the ground only during the time of accelerating or braking (the contribution of normal load forces is relatively small for speeds and accelerations typical for road traffic).

Road braking statistics show that in the majority of cases decelerations range from 0.1 to $0.2g$, i.e. from about 1 to 2 m/s^2 . However, in emergency braking decelerations of 0.4 - $0.6g$ are typical, and greater than $1.0g$ are possible. We consider vehicle acceleration $\pm a$ as a given value resulting from joint application of engine or braking forces, drag forces caused by wind resistance, rolling resistance, friction in the wheel bearings etc [60,61]. It is also assumed that the most important internal degrees of freedom of the vehicle (corresponding to the pitch, bounce and wheel resonances) are not excited by horizontal inertial forces produced during acceleration or deceleration. Thus, in the low-frequency range of generated ground vibrations, an accelerating or braking vehicle of mass M can be considered as a point horizontal traction force $F_x = aM$ applied to the ground and moving along the road in the direction x with the vehicle. In fact, the resulting point force is the sum of partial forces applied to the ground from each wheel of the vehicle. For braking or accelerating respectively the direction of this force coincides with the direction of vehicle movement or is opposite to it. We also assume that adhesion between road and

tyres is perfect. Thus, the limitations on acceleration (deceleration) caused by sliding or skidding are not considered.

3.3.2 Calculation of generated Vibrations

The solution of the above problem, which now involves horizontal traction forces moving with acceleration along the ground surface, can be obtained using the Green's function for a horizontal unit force in a Fourier domain in respect to time and space. The analysis shows that the resulting very bulky expressions for ground vibrations generated by accelerated and braking vehicles characterised by the same absolute value of acceleration a differ from each other only by the phase factor. Thus, if one is interested only in the amplitudes of generated vibrations, it is sufficient to consider just one of these cases, e.g. the case of braking vehicles. Moreover, if to consider only 1/3 octave spectra of generated ground vibrations, the resulting integrals can be simplified without losing accuracy of calculations (for more details see [29,30]).

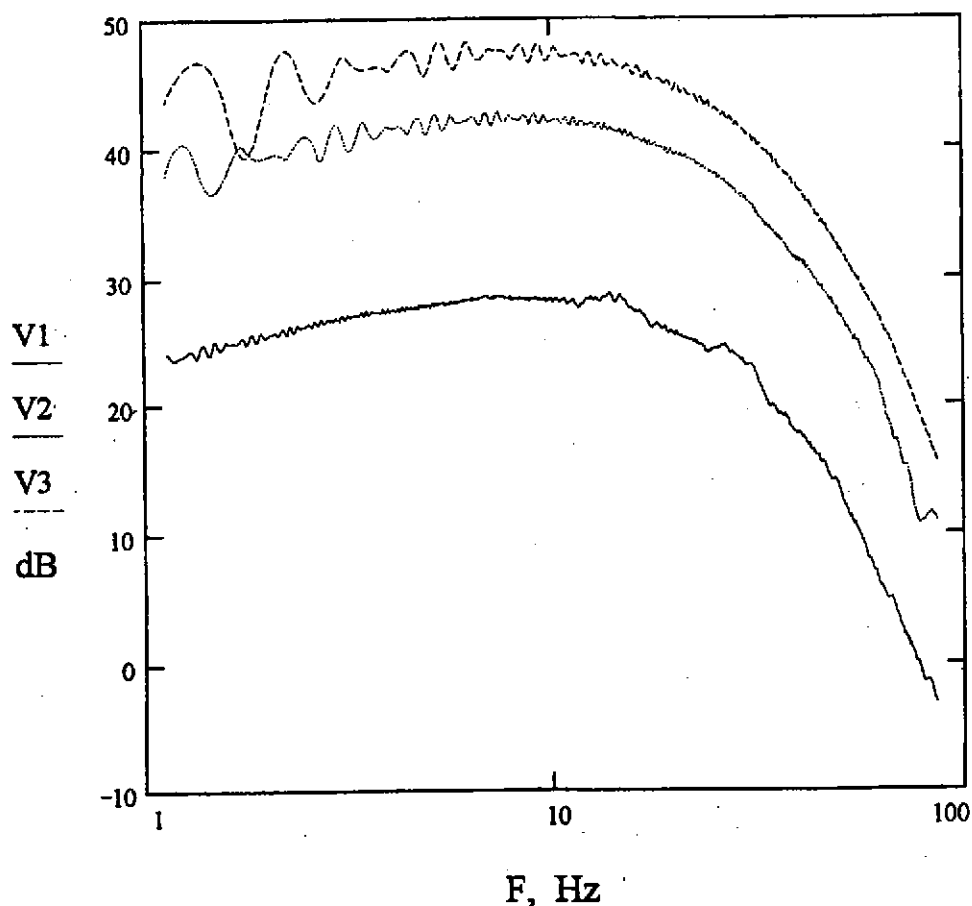


Fig. 8. 1/3-octave ground vibration spectra calculated for three values of vehicle acceleration (deceleration) a : $a = 1 \text{ m/s}^2$ (curve V1), $a = 5 \text{ m/s}^2$ (curve V2) and $a = 9 \text{ m/s}^2$ (curve V3)

The behaviour of 1/3-octave spectra of ground vibrations generated by a braking lorry with $M = 20000 \text{ kg}$ for three different values of deceleration: $a = 1, 5$ and 9 m/s^2 is illustrated on Fig. 8. The initial speed is $v = 10 \text{ m/s}$. Other parameters are the following: mass density of the ground ρ_0 is

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2000 kg/m³, velocity of longitudinal bulk waves - $c_l = 471$ m/s, shear waves - $c_t = 272$ m/s and Rayleigh surface waves - 250 m/s. It follows from Fig. 8 that amplitudes of generated ground vibrations at all frequencies increase with increase of a . It can be shown that this increase is roughly proportional to a .

Calculations of generated ground vibrations as functions of initial (final) vehicle speed v show that for small values of v and low frequencies significant oscillations of ground vibration velocity versus v take place, whereas for larger v and higher frequencies the ground vibration velocity almost does not depend on v . It is interesting that, for low-frequency components of ground vibration spectra (4 and 8 Hz) the level of generated vibrations may differ by about 12 dB if initial speed v changes from 1 to 2 m/s. Thus, even small changes of v in this range may result in large difference in ground vibrations generated in the infrasonic frequency range. This may cause very high statistical deviations in levels of ground vibration generated by slowly approaching breaking vehicles.

4. CONCLUSIONS

Problems of ground vibrations from rail and road traffic have received a lot of public attention during the last decade. This stimulated intensive theoretical research in this area, especially in respect of vibrations from high-speed trains and heavy lorries. The current level of understanding the main features of generating ground vibrations by rail and road traffic is quite satisfactory. However, further research is needed to take into account additional generation mechanisms, in particular rail-roughness-induced ground vibrations. Also, the effects of some infra-structure details, such as tunnel lining, embankments, road pavement, etc., require more substantial investigation.

Recent experimental observations of ground vibration boom made for high-speed trains in Sweden indicate that 'supersonic' or (more precisely) 'trans-Rayleigh' trains have become today's reality. The effects identical to those observed in Sweden were reported also from Holland, Switzerland and Japan. All these clearly indicate that, with the general trend of increase in operating train speeds, the problems of excessive ground vibrations associated with the boom will appear sooner or later in many countries in Europe, Asia and North America. The obvious possible solution to this problem is the reduction of train speeds. However, this is not always acceptable. Therefore, costly remediation methods, such as the ballast stiffening, have to be applied. It is now recognised that international co-ordination of efforts is needed to tackle the problem.

Theoretical analysis of ground vibrations from road vehicles travelling on bumpy roads demonstrates that amplitudes and spectra of vibrations depend strongly on the surface irregularity profile, relation between its height and length, and vehicle speed and load. If a vehicle accelerates or brakes, e.g. before traffic lights or turning, additional vibrations can be generated even on ideally flat road surfaces. The measures to reduce levels of generated vibrations are quite trivial: road surfaces should be maintained in very good condition. Also, as for many other good reasons, it is advisable to avoid using small town and country roads for passage of heavy lorries. This will reduce not only air pollution and traffic congestion, but traffic-induced ground vibrations as well.

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