

## ON ONE POSSIBLE MECHANISM OF THE LOW-FREQUENCY HUM

V V Krylov

Centre for Research into the Built Environment, Nottingham Trent University,  
Burton Street, Nottingham NG1 4BU, UK

### 1. INTRODUCTION

The phenomenon of disturbing low-frequency noise and vibration, also called low-frequency hum, has been known in the UK for at least two decades [1,2]. However, in many aspects the nature of low-frequency hum still remains a mystery (see, e.g., "The Independent" of 22 June 1994). Although a current number of complaints about low-frequency noise (around 500 a year) represents just a small fraction of a total number of noise complaints, it is increasing at an alarming rate. This is why further theoretical and experimental research into the low-frequency hum is necessary, with emphasis on fuller understanding its physical nature.

The latest practical research into this problem just has been carried out by the Building Research Establishment (BRE) and the Sound Research Laboratories (SRL) on behalf of Department of the Environment [2]. In particular, the results of their survey show that in a relatively small number of cases "there was some evidence to suggest that a low level low frequency noise may occasionally be present that could be related to the noise complained of". However, neither noise itself, nor its sources have been identified. It is also mentioned in [2] that "if noise measurement had been possible over a long period of time it may have been possible in some of these cases to identify the noise that related to the source of the problem". In our opinion, despite a relatively small number of these "difficult cases", they deserve careful consideration, also from the point of view of providing opportunity of a deeper insight into the problem.

Obviously, there might be different physical mechanisms of the hum revealing in a variety of situations. In this paper the hypothesis is examined that one of such mechanisms may be related to the structure-borne sound caused by ground vibrations propagating to buildings as surface Rayleigh waves [3-5] (this might be one of the reasons why the sources of the low-frequency hum are so difficult to identify by traditional acoustic measurements carried out so far). In particular, we analyse the possibility of surface wave sources being buried underground gas or petrol pipes in which turbulent flows of gas or liquid generate sound waves of high amplitude propagating in a pipe-line as in a waveguide. The velocities of sound  $c_0$  inside the pipes (450 m/s for methane) may be often higher than the velocities of Rayleigh surface waves  $c_R$  in the ground at the frequencies of interest (5-50 Hz). Typical values of  $c_R$  are 300 - 600 m/s. If  $c_0 > c_R$ , then ground Rayleigh waves are expected to be effectively generated by sound waves propagating inside the pipes. The physical nature of this phenomenon is similar to that of sound boom from supersonic jets or to that of recently predicted Rayleigh ground wave boom from superfast trains [4,5].

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In what follows we demonstrate that ground Rayleigh waves can be effectively generated by sound propagating in buried pipes. The central frequencies of generated Rayleigh wave vibration spectra depend on depth of the pipe and are in the low-frequency range (5-20 Hz). The amplitudes of generated ground vibration velocity due to sound waves propagating in gas pipes buried at a depth from one to two meters may achieve about 70 dB (relative to  $10^{-9}$  m/s). This is quite enough to annoy some people both because of direct impact of vibrations and due to generated structure-borne noise. The theoretical results obtained may contribute to a fuller understanding the problem that will assist in working out practical recommendations on decreasing or eliminating the impact of low-frequency hum on people.

### 2. THEORETICAL BACKGROUND

We assume that a gas or petrol pipe of radius  $a$  and wall thickness  $d$  is buried at a depth  $h$  (Fig.1). Let us consider propagation of a time-harmonic sound wave inside this pipe (the lowest waveguide mode):

$$p(x,t) = p_0 \exp[i(k_0 x - \omega t)] \quad (1)$$

Here  $p(x,t)$  is time- and space-dependent sound pressure,  $p_0$  is the sound pressure amplitude,  $k_0 = \omega/c$  is the sound wavenumber,  $\omega = 2\pi f$  is circular frequency, and  $c_0$  is the velocity of sound in pipe gas (for certainty we limit our discussion by gas pipes only). We recall that total pressure inside the pipe is  $P(x,t) = P_{st} + p(x,t)$ , where  $P_{st}$  is a static pressure. The sources of sound wave excitation inside pipes may be different. It may be, for example, powerful compressors in gas compressor stations or instabilities of a gas flow in a pipe itself. We will not discuss these mechanisms here, considering the sound amplitude  $p_0$  as a given value.

Propagation of a sound wave in the pipe causes displacements of the pipe walls which in turn can generate elastic waves in the adjacent ground. Ignoring reaction of the ground on pipe deformations and using quasi-static solution of thin shell equations [6,7], one can obtain the expressions for sound-induced radial and horizontal displacements of the pipe walls respectively

$$w = (a^2/Ed) p(x,t), \quad (2)$$

$$v = i(a\sigma/Edk_0) p(x,t), \quad (3)$$

where  $E$  and  $\sigma$  are Young's modulus and Poisson's ratio of the pipe material.

The next step in the solution is to determine the amplitudes of elastic fields generated in the bulk of the ground due to the sound-induced displacements of pipe walls (2), (3). Introducing cylindrical coordinates  $r, x$  associated with the pipe axle, we can express radial and horizontal displacements in the ground in terms of the elastic potentials  $\varphi$  and  $\psi$ :

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$$u_r = \partial \varphi / \partial r + \partial^2 \psi / \partial r \partial x, \quad (4)$$

$$u_x = \partial \varphi / \partial x - \partial^2 \psi / \partial r^2 - (1/r) \partial \psi / \partial r. \quad (5)$$

Quantities  $\varphi$  and  $\psi$  describe potential and vortex parts of the elastic field respectively and satisfy the wave equations:

$$\Delta \varphi - (1/c_l) \partial^2 \varphi / \partial t^2 = 0, \quad (6)$$

$$\Delta \psi - (1/c_t) \partial^2 \psi / \partial t^2 = 0, \quad (7)$$

where  $\Delta = \partial^2 / \partial r^2 + (1/r) \partial / \partial r + \partial^2 / \partial x^2$  is the Laplace differential operator written in cylindrical coordinates,  $c_l$  and  $c_t$  are velocities of longitudinal and shear bulk waves in the ground. On the pipe walls, i.e., at  $r = a$ , the displacements of the ground,  $u_r$  and  $u_x$ , should satisfy the boundary conditions

$$u_r = w \quad \text{and} \quad u_x = v, \quad (8)$$

where  $w$  and  $v$  are determined by (2), (3).

The solution of the boundary value problem (2) - (8) is sought in the form

$$\varphi = A H_0^{(m)}(v_l r) \exp[i(k_0 x - i\omega t)], \quad (9)$$

$$\psi = B H_0^{(m)}(v_t r) \exp[i(k_0 x - i\omega t)], \quad (10)$$

where  $H_0^{(m)}(v_{l,t} r)$  is the Hankel function of the first kind and zero order,  $v_{l,t} = (k_{l,t}^2 - k_0^2)^{1/2}$ ,  $k_l = \omega/c_l$  and  $k_t = \omega/c_t$  are the wavenumbers of longitudinal and shear bulk waves,  $A$  and  $B$  are yet unknown coefficients. Note that (9), (10) satisfy the equations (6), (7) respectively. Obviously, if  $k_{l,t}^2 > k_0^2$ , then  $\varphi$  and  $\psi$  in (9), (10) describe conical longitudinal and shear elastic waves propagating away from the pipe. In the opposite case,  $k_{l,t}^2 < k_0^2$ , expressions (9), (10) describe localised quasi-static elastic fields accompanying the sound wave and travelling along the pipe at speed  $c_0$ .

Coefficients  $A$  and  $B$  are determined from the boundary conditions (8). Using the low-frequency approximation for the Hankel function  $H_0^{(m)}(v_{l,t} r) = (2i/\pi) \ln(v_{l,t} r)$ , valid for  $v_{l,t} r \ll 1$ , and substituting (9), (10) into (4), (8), (2), (3), one can obtain the following expressions for  $A$  and  $B$ :

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$$A = -[i\pi\sigma/2Edk_0^2 \ln(\nu\alpha)]p_0 \quad (11)$$

$$B = [(\pi\sigma/2Edk_0^3 \ln(\nu\alpha) - (\pi\alpha^3/2Edk_0)]p_0. \quad (12)$$

It is easy to see from (11), (12) and (4), (5) that contributions of  $\varphi$  and  $\psi$  to the ground displacement field are of the same order. Both these potentials are equally important and

contribute additively to generating Rayleigh surface waves at the ground surface. However, to demonstrate the effect in principle, we consider here only contribution of the potential  $\varphi$ . This essentially simplifies the problem yet allowing to achieve satisfactory estimation for the order of amplitudes of generated Rayleigh waves.

To calculate the amplitudes of elastic fields generated at the ground surface by the waves (9)-(12) one should solve the corresponding boundary-value problem for the elastic half-space. Excitation of longitudinal and shear elastic waves in the elastic half space by longitudinal conical waves in the form (9) has been considered in the paper [8]. According to this paper, the general integral representation of the ground surface vertical displacement associated with the excited elastic field has the form

$$u_z = D \int_{-\infty}^{\infty} \frac{2k_i^2 [2(k^2 + k_0^2) - k_i^2] \exp[i(ky + k_0x + ihs_i - \omega t)]}{F(\sqrt{k^2 + k_0^2})} dk, \quad (13)$$

where  $k$  is a current wavenumber,  $D = -2iA/\pi$  is the amplitude coefficient,  $F(\sqrt{k^2 + k_0^2}) = [2(k^2 + k_0^2) - k_i^2]^2 - 4(k^2 + k_0^2)s_i s_t$  is the Rayleigh determinant as a function of  $\sqrt{k^2 + k_0^2}$  (instead of  $k$  in usual notation), and  $s_{i,t} = (k^2 + k_0^2 - k_{i,t}^2)^{1/2}$ . Evaluation of the integral (13) with regard to calculation of generated bulk elastic waves has been carried out in the complex  $k$ -plane by the method of steepest descents [8]. Note that calculations in [8] were performed for high-frequency bulk longitudinal and shear elastic waves, with applications to active acoustic detection of leaks in underground gas distribution lines.

From the point of view of examining possible mechanisms of low-frequency hum, we need to investigate generation of Rayleigh surface waves which carry most of the energy of generated

low-frequency ground vibrations. To calculate radiated Rayleigh waves we have to take contribution of a residue of the integrand in (13) at  $k$  corresponding to  $F(k^2 + k_0^2) = 0$ , i.e., at  $k = \sqrt{k_R^2 - k_0^2}$ , where  $k_R = \omega/c_R$  is the Rayleigh wavenumber and  $c_R$  is Rayleigh wave velocity. After simple manipulations, this results in the following expression for the vertical component of the surface ground vibration velocity,  $v_z = \partial u_z / \partial t$ , associated with generated Rayleigh waves:

$$v_z = -\frac{2p_0\pi\alpha\sigma}{Edk_0^2 \ln(v_1\alpha)} \omega \frac{2k_i^2(2k_R^2 - k_i^2)}{[F'(k_R)/k_R] \sqrt{k_R^2(1+2i\gamma) - k_0^2}} \exp[-h(k_R^2 - k_i^2)^{1/2} + ik_0x \pm i[k_R^2(1+2i\gamma) - k_0^2]^{1/2}y]. \quad (14)$$

Here  $F'(k_R)$  is the derivative  $dF/dk$  of the "usual" Rayleigh determinant  $F(k) = (2k^2 - k_i^2)^2 - 4k^2(k^2 - k_i^2)^{1/2}(k^2 - k_0^2)^{1/2}$  taken at  $k = k_R$ , signs "+" and "-" in the exponential of (14)

correspond to the positive and negative values of  $y$  respectively, the factor  $\exp(-i\omega t)$  is omitted. In (14) we also have taken into account ground attenuation of Rayleigh waves where it is essential: in the exponential and in the denominator (otherwise a singularity could be expected when  $k_R = k_0$ ). We have accounted for attenuation in a traditional way: via replacing real wavenumber  $k_R$  by its complex value  $k_R(1+i\gamma)$ , where  $\gamma$  is the attenuation constant of the ground. It is assumed that sound waves inside the pipe propagate without attenuation.

### 3. DISCUSSION

According to the formula (14), radiation of Rayleigh waves in the ground takes place only for  $k_R > k_0$ , i.e., for  $c_R < c_0$ , i.e., similarly to a sonic boom from supersonic jets or to a ground Rayleigh wave boom from superfast trains [4,5]. This may happen quite often since, e.g., the velocity of sound inside the pipe filled with methane is 450 m/s and the velocities of Rayleigh surface waves in the ground are typically 300 - 600 m/s. It is seen that generated Rayleigh waves are quasi-plane waves propagating symmetrically with respect to the  $x$ -axis at the angles  $\Theta = \cos^{-1}(c_R/c_0)$  (Fig.1). If  $c_0 < c_R$  then Rayleigh waves are not generated and formula (14) describes the quasi-static elastic field accompanying the sound wave and exponentially decaying with distance  $y$  from the pipe.

The presence of the "resonance" expression  $\sqrt{k_R^2(1+2i\gamma) - k_0^2}$  in the denominator of (14) implies that for  $k_R > k_0$  the most efficient generation of Rayleigh waves is expected at  $k_R \approx k_0$ , i.e., at  $c_R \approx c_0$ . Maximum achievable Rayleigh wave amplitudes in this case are determined by the ground attenuation factor  $\gamma$ .

We recall that formula (14) describes radiation caused by time-harmonic sound wave (1) propagating in a pipe. If the sound wave in the pipe is not time-harmonic, then multiplication of (14) by the frequency spectrum of sound gives the spectral density of generated Rayleigh waves. One can assume that in the frequency range of interest (5 - 50 Hz) the spectrum of sound in the

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pipe is approximately uniform with a spectral density  $p_0$ . Then formula (14) represents straightway the spectrum of Rayleigh waves generated by the pipe.

Numerical calculations of the ground vibration amplitudes  $V = |v_z|$  have been carried out according to the formula (14) for the following parameters of the pipe and ground:  $a = 0.5$  m,  $d = 0.005$  m,  $E = 20 \cdot 10^{10}$  N/m,  $\sigma = 0.31$  (tempered steel),  $c_0 = 450$  m/s,  $p_0 = 100$  dB (relative to  $2\sqrt{2} \cdot 10^{-5}$  N/m<sup>2</sup>; Poisson ratio of the ground has been taken as 0.25; pipe depth  $h$  varied from 0.5 m to 1.5, Rayleigh wave velocity of the ground varied from 400 m/s to 500 m/s, and ground attenuation  $\gamma$  varied from 0.005 to 0.015; observation distance from the pipe  $y$  varied from 25 m to 100 m.

Fig. 2 shows Rayleigh wave ground vibration spectra for three values of pipe-depth:  $h = 0.5$ , 1.0 and 1.5 m (curves V1, V2, and V3 respectively). Here  $c_R = 440$  m/s,  $y = 50$  m and  $\gamma = 0.005$ . One can see that generated spectra have maxima with the magnitudes and locations dependent on  $h$ . The lower the  $h$  values, the higher the central frequencies and the larger the amplitudes of generated waves.

Fig. 3 represents ground vibration amplitudes as functions of Rayleigh wave velocity in the ground  $c_R$  for three values of attenuation:  $\gamma = 0.005$ , 0.010, and 0.015 (curves V1, V2, and V3 respectively). Distance  $y$  has been chosen as 50 m, pipe depth  $h$  was 1 m, and frequency  $f$  was equal to 20 Hz. One can see that at  $c_R$  approaching  $c_0$  from the left a resonance increase of generated ground vibrations occurs. For  $c_R > c_0$  a quick drop in amplitudes takes place characterising exponential decay of the accompanying quasi-static field with distance  $y$ .

### 4. CONCLUSIONS

In this paper, it has been demonstrated theoretically that one of the possible mechanisms of disturbing low-frequency hum may be related to ground vibrations propagating to buildings as surface Rayleigh waves. These waves can be effectively generated by buried underground gas pipes in which turbulent flows of gas or liquid excite sound waves of high amplitude propagating in a pipe-line as in a waveguide. Such a generation takes place if the velocities of sound  $c_0$  inside the pipes (450 m/s for methane) are higher than the velocities of Rayleigh surface waves  $c_R$  in the ground. Especially large resonance increase occurs for  $c_0$  slightly higher than  $c_R$ . The physical nature of this phenomenon is similar to that of sound boom from supersonic jets or to that of recently predicted Rayleigh ground wave boom from superfast trains.

The results of this investigation are based on the simplified model which does not take into account radiation of shear waves or the influence of layered structure of the ground. Therefore, they should be considered as preliminary ones giving merely qualitative picture of the phenomenon. Further theoretical investigation is required to provide more detailed quantitative

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description taking account of layered structure of the ground and particular mechanisms of sound excitation in pipes.

### REFERENCES

1. Tempest, W., A survey of low frequency noise complaints received by local authorities in the United Kingdom. *Journ. Low Frequency Noise and Vibration*, 8 (1989) 45.
2. Sargent, J.W., Low frequency noise. Report, Building Research Establishment, UK, Garston, October 1994.
3. Krylov, V. and Ferguson, C., Calculation of low-frequency ground vibrations from railway trains. *Applied Acoustics*, 42 (1994) 199-213.
4. Krylov, V.V., On the theory of railway-induced ground vibrations. *Journ. de Phys. IV*, 4 (1994) C5-769-772.
5. Krylov, V.V., Generation of ground vibrations by superfast trains. *Applied Acoustics*, 44 (1995) 149-164.
6. Junger, M.C. and Feit, D. *Sound, Structures, and Their Interaction*, Acoust. Soc. Amer., New York, 1993.
7. Jette, A.N., Morris, M.S., Murphy, J.C. and Parker, J.G., Active acoustic detection of leaks in underground natural gas distribution lines. *Materials Evaluation* 35 (1977) 90-99.
8. Jette, A.N. and Parker, J.G., Excitation of an elastic half-space by a buried line source of conical waves. *Journ. Sound and Vibration*, 67 (1979) 523-531.

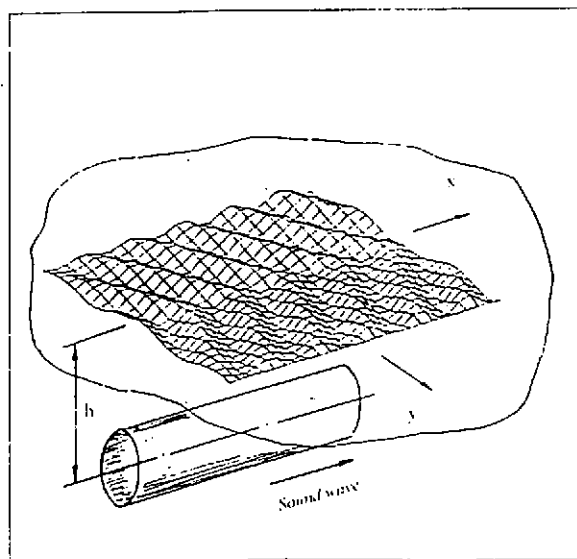


Fig 1. Geometry of the problem. Generated Rayleigh ground waves are quasi-plane waves radiated symmetrically from the pipe at the angles  $\Theta = \cos^{-1}(c_R/c_0)$  with respect to the direction of sound propagation

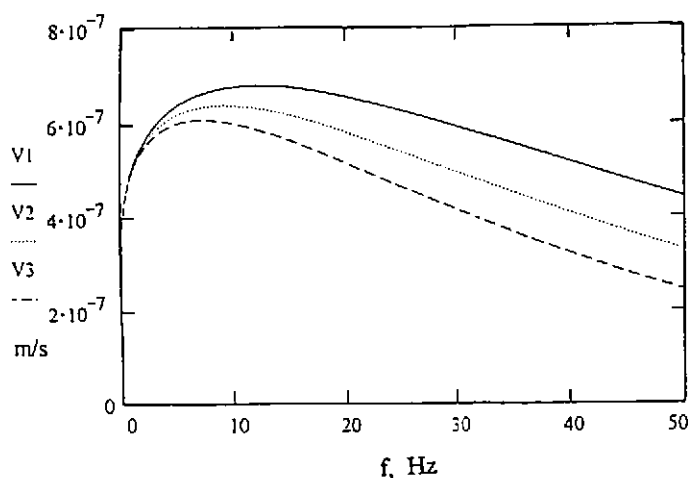


Fig.2. Ground vibration spectra for different pipe-depths

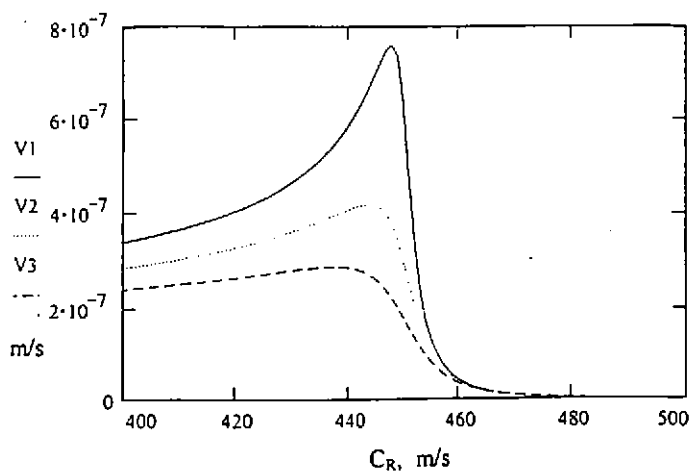


Fig.3. Ground vibration amplitudes as functions of Rayleigh wave velocity in the ground