A NEW ENGINEERING MODEL FOR THE ACOUSTIC PROPERTIES OF UNCONSOLIDATED GRANULAR MIXES

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1 INTRODUCTION

Accurate modelling of the acoustic properties of loose granular media is of considerable interest in many areas of physics, engineering and industry. Hence, there are many theoretical and empirical models which have been developed to predict the acoustic characteristic impedance and propagation constant of porous granular materials[1-5]. These models depend largely on the measured values of the flow resistivity which, in the case of loose granular mixes, tends to vary significantly as a function of the compaction ratio. Recently, a new, simple empirical model has been proposed[6]. This model is based on the four directly measurable non-acoustic parameters: porosity, tortuosity, specific density and characteristic dimension of the loose granular mix. It does not require the flow resistivity data which proves to be an unreliable parameter in the case of the unconsolidated grains. The model accounts heuristically for the friction between the individual grains and grain micro-porosity.

The current paper is the continuation of the original work[6]. This paper presents the results of a practical application of the proposed model for the prediction of the acoustic characteristics of the hard-backed layers of loose granular mixes which can be used for the acoustic insulation. A comparison between the results of some existing models for the acoustic properties of porous media and the experimental data is made. It is shown that the theoretical models based on the theory by Biot can provide good agreement with the experimental data for granular materials in the medium frequency range. The limitations of the other models related to this class of porous media are discussed.

2 THEORETICAL BACKGROUND

On the basis of experimental data the new empirical relations for the normalised characteristic impedance, $W = W_a + iW_i$, and complex propagation constant, $\gamma = \alpha + i\beta$, have been proposed[6]

$$W_a^E = 1 + Q,\ f < f_{cr} \tag{1}$$

$$W_a^E \cong q / H,\ f \geq f_{cr}. \tag{2}$$

$$W_i^E = \frac{QH}{1+C}, \alpha^E = \frac{kQH}{1+A} \text{ and } \beta^E = k[1 + QH(1+B)]. \tag{3}$$

where the structural characteristic

$$Q = \frac{0.2(1-H)(1+H)^2}{H\sqrt{kD\chi}}. \tag{4}$$

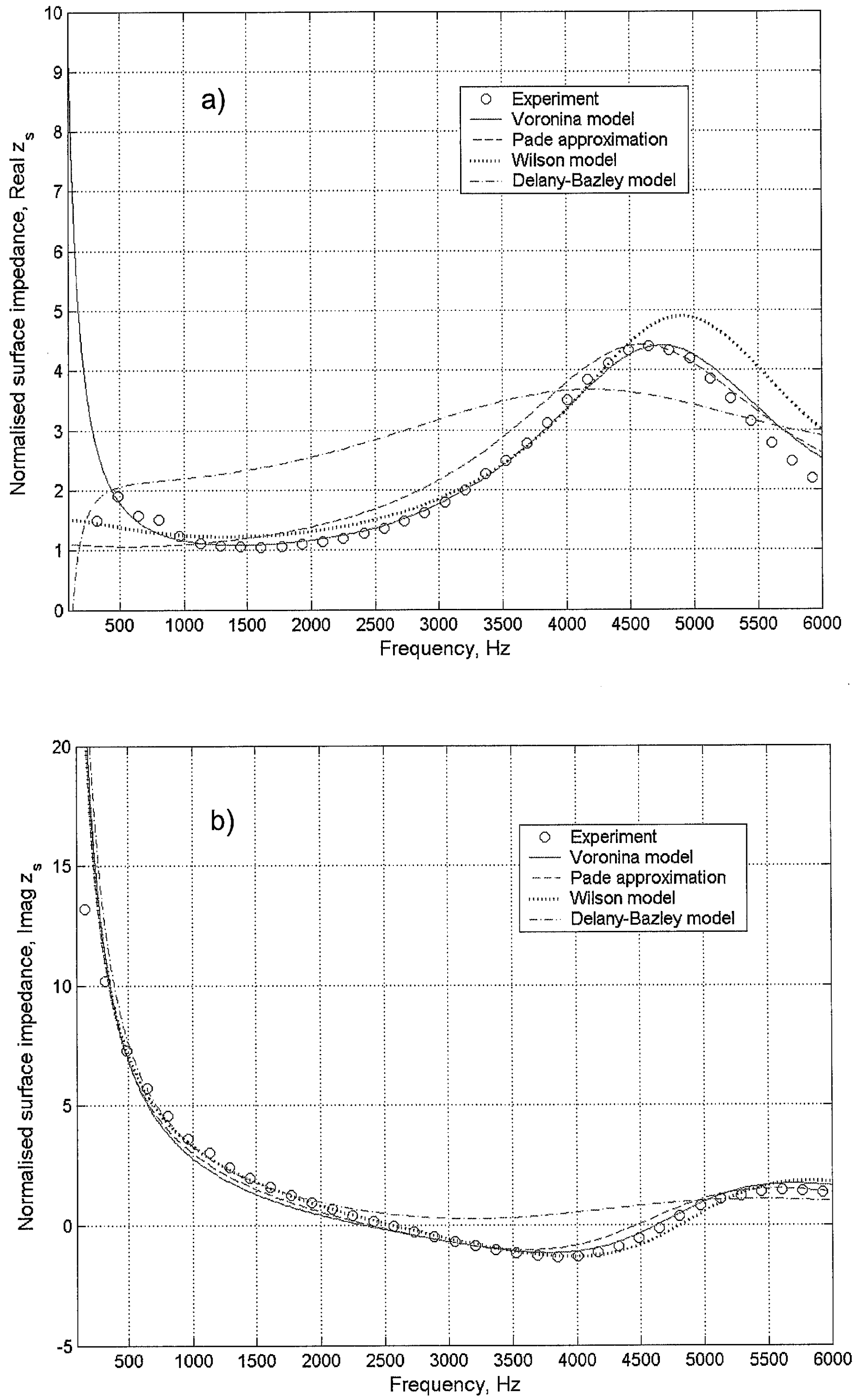


Figure 1. The real (a) and imaginary (b) parts of the normalised surface impedance of a 21mm hard-backed layer of 0.71-1.18mm clinkers of Alag.

In the above expressions H is the porosity, $q = \sqrt{\alpha_\infty}$ is the tortuosity, D is the characteristic

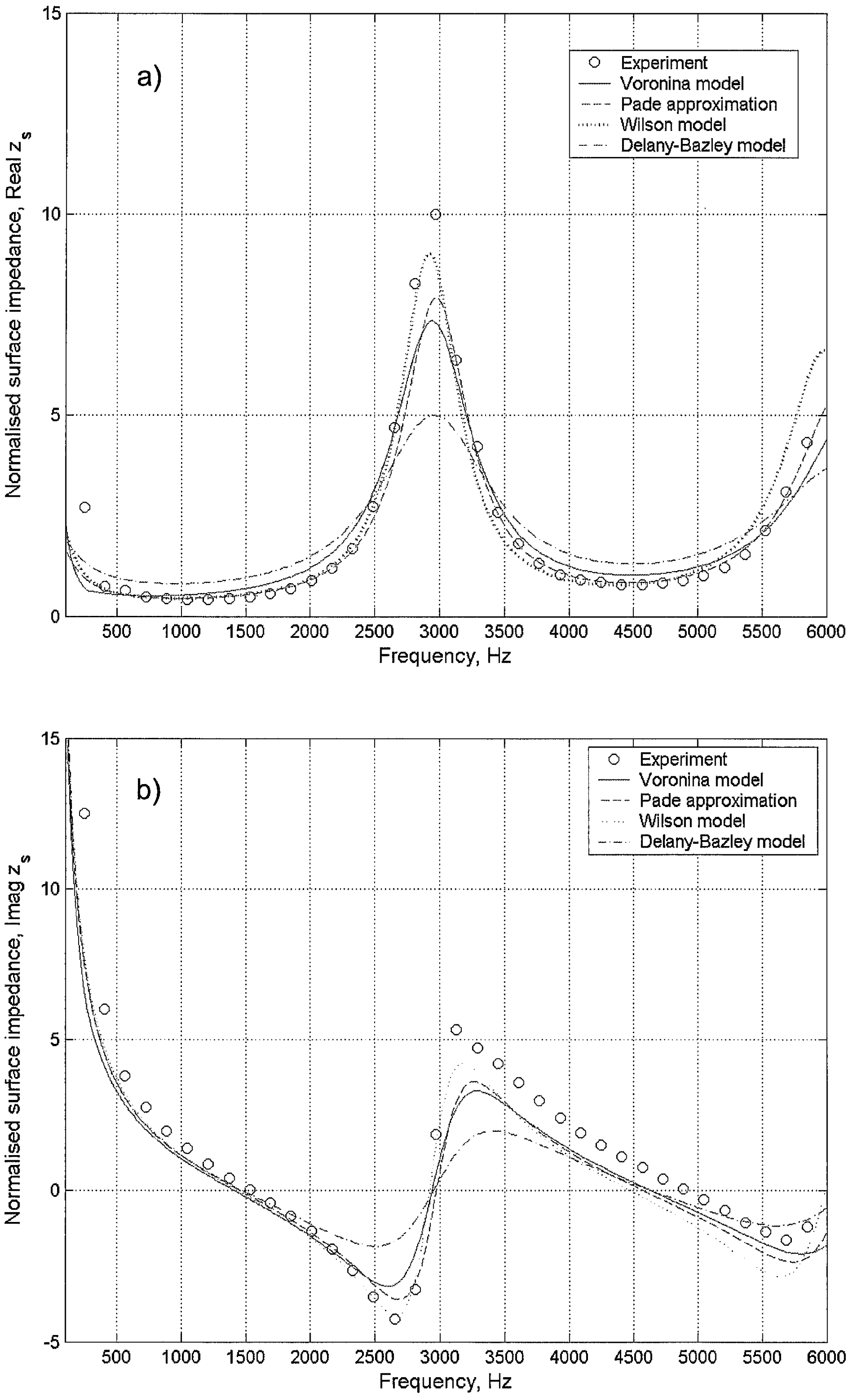
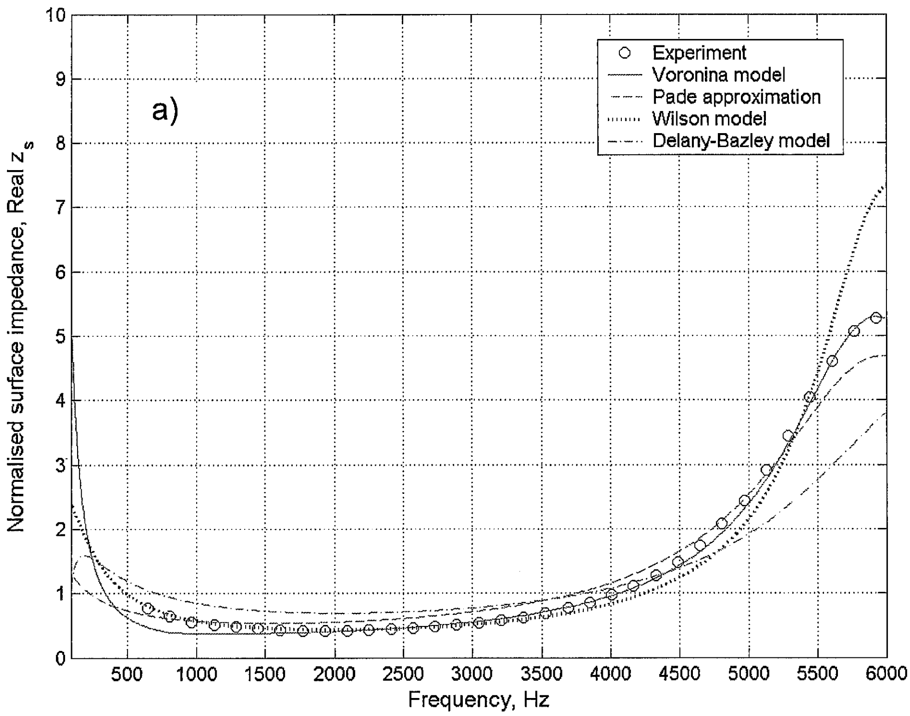


Figure 2. The real (a) and imaginary (b) parts of the normalised surface impedance of a 40mm hard-backed layer of 2-3mm expanded clay granulates.

dimension of the grain base, $k=\omega/c$ is the acoustic wavenumber in air, $\chi=\dfrac{D\rho_0 c}{\mu 10^4}$ is a



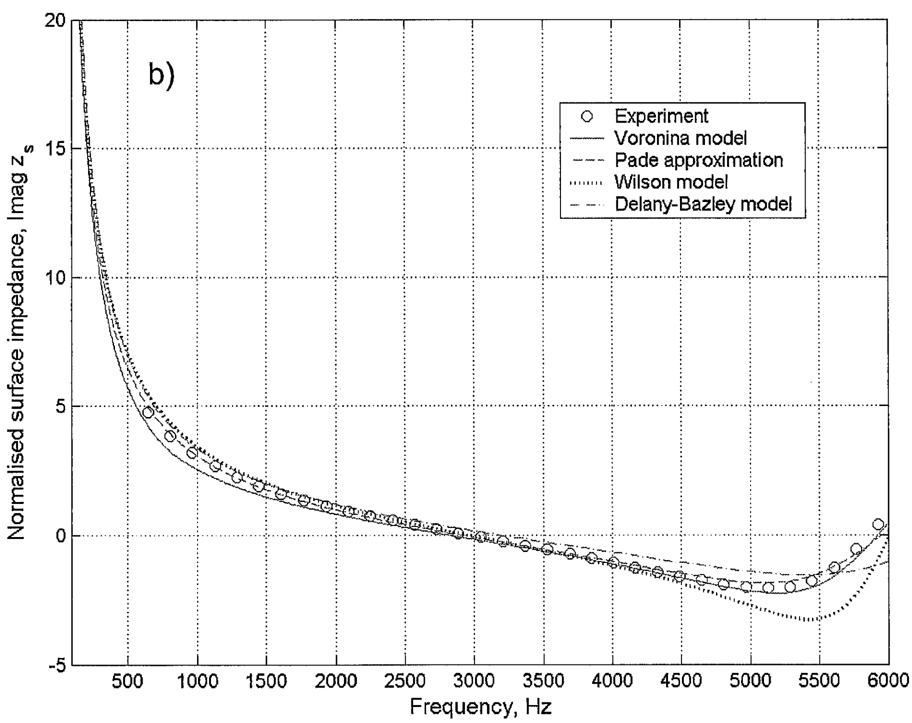


Figure 3. The real (a) and imaginary (b) parts of the normalised surface impedance of a 20mm hard-backed layer of 1-2mm recycled tyre granulates.

dimensionless parameter related to the grain size, ρ_0 is the density of air, c is the sound speed in air, μ is the dynamic viscosity of air. The superscript "E" denotes the "Empirically predicted" values of the acoustic characteristics. The critical frequency is determined from the equation $Q_{cr} + 1 = q / H$ and is given by

$$f_{cr} = \frac{0.02c(1-H)^2(1+H)^4}{\pi D\chi(q-H)^2}. \tag{5}$$

The coefficients C, B and A in eqs. (1) – (3) are predicted heuristically from

$$A = \frac{(1-H)M}{1+Q},\ B = \frac{1}{\sqrt{Q}(1+H)(1+Q^2M)} \ \text{and}\ C = \frac{1-H}{\sqrt{Q}}, \tag{6}$$

where the dimensionless parameter $M = \rho_g / \rho_0 10^3$ and ρ_g is the specific density of the grain material in the mix. It is easy to demonstrate that the coefficients C, B, $A \to 0$ as $f \to 0$. This asymptotic behaviour allows to obtain the useful expression for the flow resistivity of a loose granular mix as a function of the porosity and the structural characteristic[6]

$$R = \rho_0 ckQ^2 H(1+H),\ f \to 0 \tag{7}$$

3 COMPARISON WITH OTHER RESULTS

The model proposed in Ref. [6] was used to calculate the characteristic impedance and propagation constant of porous samples of loose granular media with different values of the flow resistivity, grain size and density so that the normalised surface impedance can be predicted from

$$z_z = W \coth(\gamma d), \tag{8}$$

where d is the thickness of the hard-backed sample. The selected materials were 0.71-1.18mm clinkers of Alag[7], 2-3mm expanded clay granular mix[8] and 1-2mm recycled tyre crumb[9]. The predicted impedance was compared against the experimental data and with the results predicted by the Pade approximation model[3], Wilson model[4] and the Delany-Bazley model[5]. The procedure for the acoustic and non-acoustic measurements is detailed in Refs. [6-8]. The summary of the non-acoustic properties used in this work is provided in Table 1.

Figures 1 (a) and (b) show the comparison of the measured real and imaginary parts of the normalised characteristic impedance of a 21mm hard-backed sample of Alag aggregates and those predicted using the models presented in Refs. [3-5]. An excellent agreement between the experimental data and the result from Voronina model[6] is observed across the considered frequency range. A good agreement is also observed for the results predicted by the Pade approximation model[5] and the Wilson's model[6]. The Pade approximation tends to underestimate the real part of the surface impedance in the lower frequency range below 1000 Hz, whereas the Wilson's model tends to overestimate this parameter in the higher frequency range above 4500 Hz. The model by Delany and Bazley fails completely to provide the accurate prediction. It is he expected result, because this model has been originally designed to predict the acoustic properties of fibrous media with high proportion of the open pores and it should not be used for granular materials.

Material	*Flow resistivity, Pa s m-2*	*Porosity*	*Tortuosity*	*Characteristic particle dimension, m, x10-3*	*Specific density of grains, kg/m3*	*Standard deviation, ϕ-units*	*Shape factor*
Alag, 21mm	31075	0.52	1.30	0.85	3200	0.77*	1.4*
Expanded clay, 40mm	3230	0.57	1.31	4.2	2400	0.52*	1.35*
Rubber crumb, 20mm	7060	0.62	1.20	1.85	1050	0.90*	1.3*

Table 1. The values of the non-acoustic parameters required for modelling of the acoustic properties of the investigated granular mixes. (*) indicates the fitted values.

The two theoretical models[3,4] exhibit the asymptotic behaviour similar to that found in the Biot-Allard[2] and Attenborough models[1]. These models assume that the porous material is formed by a stack of parallel capillary tubes so that it is possible to illustrate that in the case of $M \approx 1$ the behaviour of the characteristic impedance and propagation constant in the low- and high-frequency regimes is determined by the following asymptotic expressions

$$W^T \approx Q(1-i)\sqrt{(1+H)/2} \text{ and } \gamma^T \approx kQH(1+i)\sqrt{(1+H)/2}\,,\ f \to 0 \qquad (9)$$

$$W^T \approx q/H - i\frac{Q\sqrt{1+H}}{4(1-\tau)} \text{ and } \gamma^T \approx \frac{kQH\sqrt{(1+H)}}{4(1-\tau)} + ikq\,,\ f \to \infty \qquad (10)$$

where $\tau = 0.5Q(H/q)\sqrt{1+H}$. The superscript in the above quantities "T" denotes the "Theoretical" approach.

In the case of the proposed empirical model[6] the asymptotic expressions for the low- and high-frequency behaviour of the above characteristics are rather different and given by

$$W^E \approx 1+Q-iQH \text{ and } \gamma^E \approx kQH(1+i)\,,\ f \to 0 \qquad (11)$$

$$W^E \approx q/H - iQH/(2-H) \text{ and } \gamma^E \approx 0.5kQH(1+Q) + ik\left(1+QH+\frac{0.5H}{1+H}\right),\ f \to \infty \qquad (12)$$

A careful investigation of the above expressions reveals that in the low frequency regime a noticeable difference between the behaviour of the "theoretical" and "empirical" models is likely to occur in the real part of the characteristic impedance so that

$$W^T < W^E \text{ as } f \to 0. \qquad (13)$$

This mismatch explains some discrepancy between the considered theoretical models[4,5] and the experimental data which is pronounced below 1000 Hz.

Similarly, a more close examination of expressions (10) and (12) shows that the empirically and theoretically predicted values of the characteristic impedance are likely to be close. The major difference should occur between the attenuation coefficient, $\alpha = \mathrm{Re}\,\gamma$, which can be significantly underestimated by the theory in the case of a smaller grain mix ($\chi \le 1$). This discrepancy explains the particular increase in the real part of the normalised surface impedance which can be observed in Figure 1a.

For materials with higher values of porosity and larger grain size the differences between expressions (9)-(11) and (10)-(12) are reduced so that the results of the theoretical models should appear to be more accurate across the low and medium frequency range.

The real and the imaginary parts of the normalised surface impedance of a 40mm hard-backed layer of the loose expanded clay granulates are shown in Figures 2(a) and 2(b), respectively. In general, a good agreement between the predicted values and the measured data is observed for all the models considered here. However, neither of the considered models is capable of predicting the exact value of the real part of the surface impedance spectrum around the 2900 Hz resonance peak. The models also tend to underestimate the imaginary part of the surface impedance at frequencies above the first resonance (> 2900 Hz). This phenomenon is likely to be attributed to rather high values of micro-porosity in the expanded clay mix. It would be of interest to carry out a separate investigation of this phenomenon using the method proposed recently by Olny[10].

The last set of data relates to the acoustic properties of a 20mm hard-backed layer of 1-2mm recycled tyre granulates. The real and imaginary parts of the normalised surface impedance are shown in Figures 3(a) and 3(b), respectively. A good agreement between the experimental data and the predicted values the normalised surface impedance results by the Voronina's and Pade approximation models is observed across the considered frequency range. There is a noticeable difference between the measured values of the real and imaginary parts of the impedance and those predicted by the Wilson's model which is obvious at frequencies above 5000Hz. The Wilson's model tends to overestimate the real part of the impedance in the vicinity of the resonance, which can be explained by the rather low value of the attenuation coefficient α predicted by the model. A reasonable agreement between the experimental data and the result calculated by the Delany-Bazley model can be observed in the medium frequency range.

4 CONCLUSIONS

A new engineering model has been used to predict the acoustic properties of a representative set of loose granular mixes. Unlike many other models, the proposed set of expressions does not require the flow resistivity data which is extremely sensitive to the degree of compaction of a loose granular mix. The model is based on the four easy-to-measure non-acoustic parameters: the characteristic particle dimension, porosity, tortuosity and the specific density of the grain base. In addition, the comparison has been made between this model and the models by Wilson and Delany and Bazley which can predict the acoustic properties of porous media. An excellent agreement has been observed between the experimental data and the proposed model. It has been shown that the models based on the Biot formulation for a stack of parallel identical capillary tubes can underestimate the characteristic acoustic impedance of loose granular materials in the low frequency range and overestimate it in the high frequency regime. In the latter regime, the predicted values of the attenuation coefficient in the propagation constant are lower than expected. In this respect, the proposed model offers an obvious improvement. It has also been demonstrated that the popular model by Delany and Bazley fails to provide the accurate fit in the case of a small grain mix. The predictions by this model disagree with the experimental data in the case of mixes with a larger grain base. This is the expected result, because the Delany-Bazley model has been originally designed to predict the acoustic properties of fibrous media with high proportion of the open pores and, therefore, should not be used with this class of porous media.

5 REFERENCES

1. K. Attenborough, "Acoustical characteristics of rigid fibrous absorbents and granular materials, J. Acoust. Soc. Am. 73, 785-799 (1983).
2. J.-F. Allard, Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials Elsevier, Amsterdam, 1993.
3. K. V. Horoshenkov, Keith Attenborough and S. N. Chandler-Wilde, "Padé approximants for the acoustical properties of rigid frame porous media with pore size distribution", J. Acoust. Soc. Am., 104, 1198-1209 (1998).
4. K. Wilson, "Simple, relaxational models for the acoustic properties of porous media," Appl. Acoust. 50(3), 171-188 (1997).
5. M. A. Delany and E. N. Bazley, Acoustic properties of fibrous absorbing materials," Appl. Acoust. 3, 105-116 (1970).
6. N. N. Voronina, K. V. Horoshenkov, "A new empirical model for the acoustic properties of loose granular media," Appl. Acoust., 64 (4): 415-432 (2003).
7. K. V. Horoshenkov, D. C. Hughes and A. Cwirzen, "The acoustic attenuation in porous cementitious materials," Appl. Acoust. 64 (2): 197-212 (2003).
8. F. Asdrubali , K. Horoshenkov, "On the acoustic properties of expanded clay granulates," J. Build. Acoust., 9(2), (2002).
9. K. V. Horoshenkov, M. J. Swift, "Acoustic properties of consolidated granular mixes," Appl. Acoust., 62(6), 665-690 (2001).
10. X. Olny and C. Boutin, "Acoustic wave propagation in double porosity media," J. Acoust. Soc. Am. 114(1), 73-89 (2003).