A DESIGN PROCEDURE FOR TRANSDUCERS'ARRAYS BASED ON FINITE ELEMENT AND BOUNDARY ELEMENT METHODS

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1. INTRODUCTION

While designing an acoustic array, one would like to use numerical models to predict performances and optimize parameters. Several models (from simple plane wave models to sophisticated F.E. models) exist to modelize single transducers in air (i.e. before mounting into array), and very good accuracy is obtained. But, as soon as a transducer is mounted into an array, modelling becomes much more difficult. The major difficulty lies in the computation of mutual impedances between transducers (including self-impedances).
Numerical tools already exist for the computation of mutual interaction between transducers. For example, using the Boundary Element Method one can compute the mutual interaction between transducers in any bi- or tri-dimensional array provided there is no direct elastic coupling between transducers.

Now if one is interested in predicting the performances of an array where transducers are embeded in a massive polyurethane dome (acoustic window), additional difficulties arise:

- in addition to the radiative coupling (compressional waves in the fluid), there is an extra coupling of elastic nature(shear waves)
- radiative coupling becomes more difficult to compute in case of a thick acoustic window

Tentatives have been made to solve the complete problem using a 3-D elastic Finite Element Method. But, due to the size of any realistic array compared to the shear wavelength, the system of equations is far too large.

In this paper an hybrid numerical model is used, based on a two-scales approach :

- elastic coupling is assumed to be "local" and is computed using a 2-D F.E. Method.
- radiative coupling is computed on the whole system, considering the acoustic window as a fluid. Characteristics of the window are taken into account, as well as the external shape of the system. The method used is a combination of the 3-D fluid Finite Element Method (F.E.M.) and the Boundary Element Method (B.E.M.)
- total interaction is computed as the " sum " of the two partial interactions

2. GENERAL DESCRIPTION OF THE METHOD

The method is based on the assumption that the coupling between transducers can be decomposed into two parts :

- elastic coupling due to shear waves
- radiative coupling due to compressional waves

Moreover, it is assumed that the shear phenomena remains local. Shear waves are mainly generated at the border of transducers, and, due to the high losses of shear waves, they are attenuated very rapidly. In addition, for low shear wave velocities, the shear wavelength is small compared to the dimensions of the radiating face of one transducer. This allows to compute the elastic interaction in two-dimensions. After the elastic, short range interactions have been calculated, one still has to compute the long range, radiative interactions. A 3-D model has to be used. Because the elastic interactions have been calculated apart, this 3-D model can make the assumption of a fluid acoustic window. The total mutual interactions consist in a combination of the elastic coupling and radiative coupling. See figure 1.

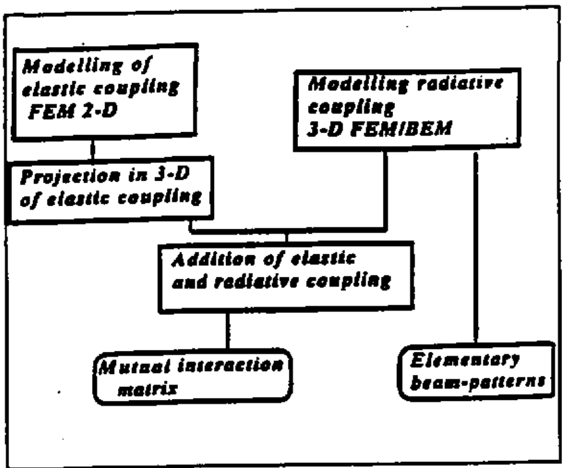

Figure 1: Flow-diagram of the method

3. MODELLING OF RADIATIVE COUPLING

3.1 Geometry of the problem (see figure 2):

A fluid region Ω_I, representing the acoustic window, is bounded on one side by the structure and, on the other side by an unbounded fluid region Ω_E. The two fluids have different characteristics and, in some cases, the region Ω_I can be composed of several sub-regions. Mutual interaction between transducers T_1 and T_2 is :

$$Z_{12}=F_2/v_1$$

where F_2 is the total force resulting on the face of transducer T_2 when the face of transducer T_1 is vibrating with a uniform velocity v_1 (harmonic time dependance is assumed all along) and the faces of all other transducers, including T_2 are blocked. A local mechanical admittance Y can be imposed on each of the boundaries Γ_2 and Γ_4.

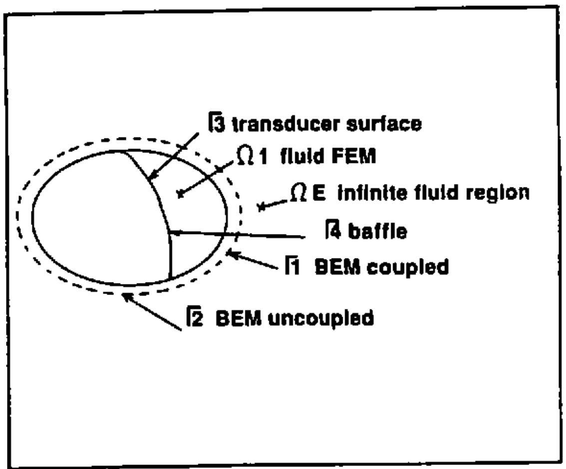

Figure 2: Geometry of the problem

3.2 Choice of a method:

To solve the equations for the unbounded region Ω_E, a Boundary Element Method is chosen. Due to the dimensions of a realistic array, the Finite Element Method generally leads to gigantic systems of equations.

To solve the equations for the bounded region Ω_I, both B.E.M. and F.E.M could be used. F.E.M. has been chosen for the following reasons:
- although the number of degrees of freedom is sligthly higher with F.E.M., the resultant linear system is much simpler (sparse matrices, simple dependance with frequency)
- in case of multi-domain acoustic window, F.E.M. is a more natural method
- in case of a window with elongated shape, B.E.M. leads to ill-conditioned linear systems

3.3 Description of the Acoustic FEM/BEM method used:

The theoretical description of the method is given in ref. 1 and 2. A special purpose software, called AMFIBIE, has been developped using SYSNOISE environment.

4. MODELLING OF ELASTIC COUPLING

4.1 Choice of a method:

As said previously, elastic coupling can be considered as local and can be computed in two-dimensions. Analytical/numerical methods could be used (such as decomposition in plane waves) but better versatility is obtained using the Finite Element Method. The F.E.M. code must take into account losses in a proper manner, i.e. at the material level, in order to discriminate between shear and compressional losses. In case of a thick acoustic window, the F.E. mesh must extend until a distance where shear waves are sufficiently attenuated.

4.2 Elimination of compressional waves:

Using any numerical model like F.E.M. one generates both shear and compressional waves. The latter must be eliminated from the 2-D results before it is added to the 3-D results which contain the actual compressional waves field. This elimination can be done by using an auxiliary computation, where shear is set to zero.

5. VALIDATION

The methodology used in this paper cannot be validated by comparison with analytical results because even the simplest application necessitates a numerical approach. Therefore, the validation has been made by comparison with another numerical approach which consists in modelling the complete phenomenon by elastic F.E.M. In order for the reference model to be of reasonable size, the configuration consists in a single transducer. This means that only the self-impedance is checked. Moreover, the selected transducer is axisymmetrical so that the reference numerical datas can be obtained using an axisymmetrical F.E.M. code. The validation procedure and results are described in ref.1.

6. APPLICATION TO AN ARRAY OF CERAMIC RINGS EMBEDED IN A MASSIVE POLYURETHANE DOME

The application presented here consists in a "linear array" of 6 ceramic rings embeded in a massive polyurethan dome. The dome has different material characteristics than the surrounding water (c_p=1500 m/s, ρ=1000 kg/m^3):

compressional waves velocity:c_p=1550 m/s
Shear waves velocity:c_s= 100 m/s
density:ρ =1100 kg/m^3

The system also includes a low impedance baffle layer of characteristics:

c_p=1300 m/s
ρ = 255 kg/m^3

A rigid support plane xOz lies above the baffle layer.
A symmetry plane yOz is introduced in the model.
In order to be able to compute all the mutual interactions, no symmetry plane xOy is introduced in the model, although the geometry of the system is symmetrical.

Both the low impedance baffle and the dome are modellized using fluid finite elements (see figure 3). Radiation into the infinite exterior water is modellized using coupled boundary elements (see figure 4).

The transducer's ceramic rings are not modellized by FEM. They are represented by a unit normal velocity boundary condition applied on the correspondant faces of the fluid mesh. This means a "single-mode" representation of each ceramic ring, which is a reasonable approximation in the present case.

6.1 Radiative coupling results
Radiation mutual impedances are computed by integration of the pressure on the surface of each ceramic ring:

$$z_{ij}=\frac{F_j}{v_i} \qquad \textit{where} \qquad F_j=\int_{s_j} p\,ds$$

Figures 5 to 16 show the variation of the most significative radiative mutual impedances versus frequency in the range 10-14 kHz. Impedances are normalized by $\rho_0 c_0 S$. To check the accuracy of the results, the symmetry of the mutual impedances matrix is verified (see figures 6,7,12):

$$Z_{ij}=Z_{ji}$$

In our case we also have (due to the symmetry plane xOy):

$$Z_{41}=Z_{3i} \qquad Z_{5i}=Z_{2i} \qquad Z_{6i}=Z_{1i}$$

Figure 17 shows the sum of Z_{3i} for i=1 to 6, which represents the force on ceramic 3 when all (6) ceramics are driven with unit velocity. It is quite different from Z_{33} which is the force on ceramic 3 when only ceramic 3 is driven with unit velocity and the other are blocked.

Overdetermination : due to the size of the model, a rather large number of internal resonances have to be taken into account. An overdetermination procedure is used in order to compensate for the non-uniqueness of the solution at those frequencies. Convergence is checked by comparing solutions for increasing number of overdetermination points.

6.2 Elastic coupling
In the present system, elastic coupling can be furthermore decomposed into two parts. First part consists in a "pure" shear stiffness between ceramic rings due to the presence of a 1 mm rubber join. This part can be computed "by hand", being a "pure" shear spring:

$$Z_{i,i+1}=1/jC\omega$$

where C is the compliance of the rubber join: $C=e/\mu s$
Using a complex value for the shear modulus μ leads to a complex impedance. Results are given at figure 18 for a shear velocity of 400 m/s and a loss tangent of 10%.

Second part of elastic coupling is due to the local generation of shear waves at the interface between two ceramic rings and the polyurethan dome. This part is computed numerically, using elastic FEM, including losses, as described in paragraph 4. Results are given at figure 19 for a shear velocity of 400 m/s and a loss tangent of 10%. Results are also normalized by $\rho_0 c_0 S$. Radiative part has been removed from the results. It is interesting to notice that, in the present problem, this part of the elastic coupling remains small.

6.3 Combination of elastic and radiative coupling

Total coupling is computed as the sum of radiative coupling and elastic coupling (parts 1 and 2 in the present case). Results are shown at figure 20 for Z_{33}.

6.4 Response of the array

Mutual impedances as computed above are used to evaluate the electrical response of the array. The general electromechanical scheme of figure 21 is used.

7. CONCLUSION

The two scales approach for computing mutual impedances can be advantageously applied for all kinds of arrays where both elastic and radiative coupling exist. This is generally the case when transducers are embeded in a massive elastomeric dome or acoustic window. This approach is often the only feasable one because a full elastic FE method is practically unapplicable due to the dimensions of the system compared with the shear wavelength.

8. REFERENCES

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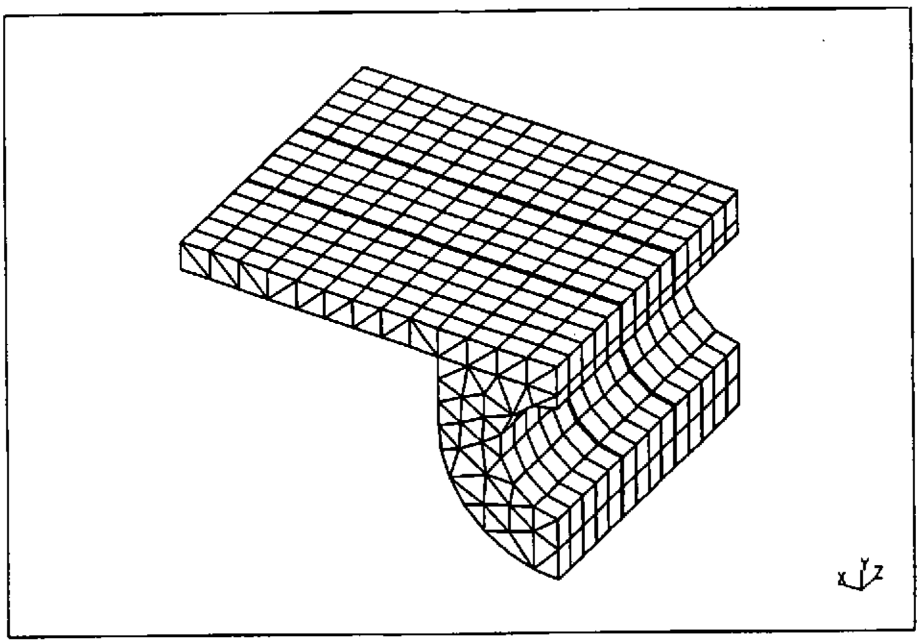

Figure 3: fluid FEM mesh

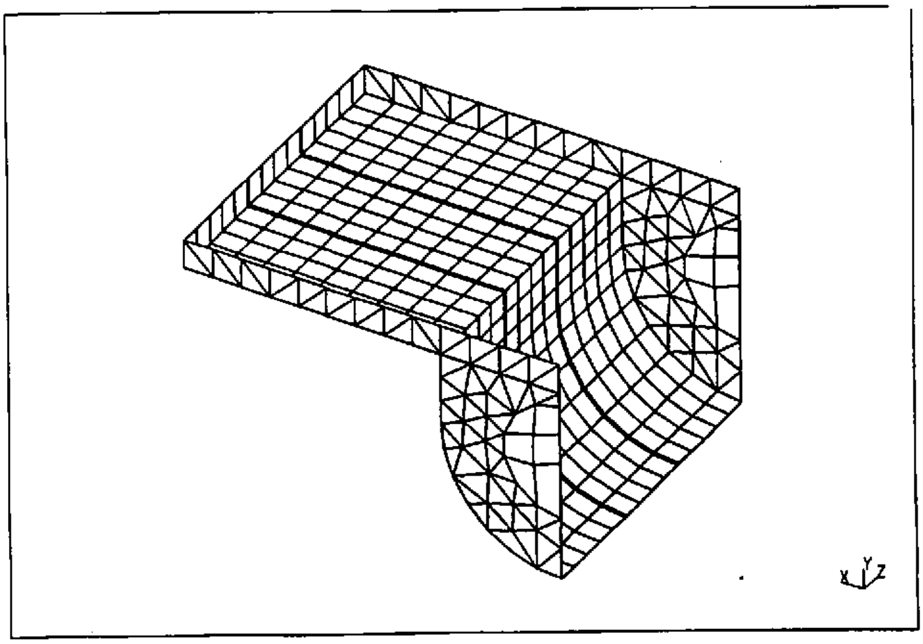

Figure 4: BEM mesh

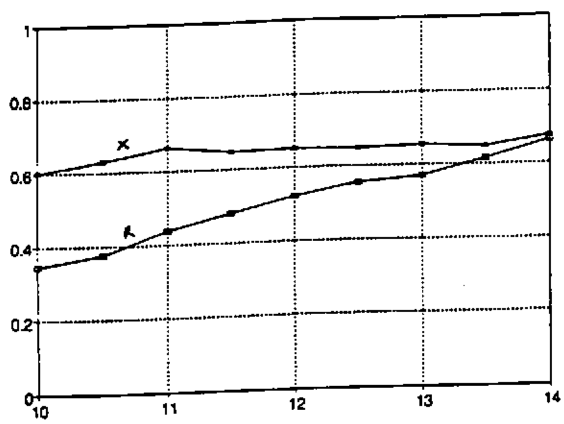

Figure 5: Z11

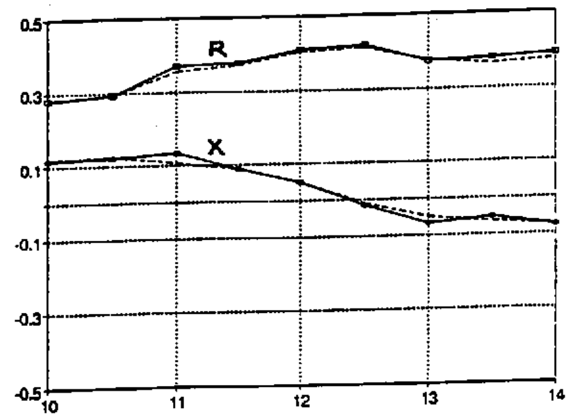

Figure 6: Z12

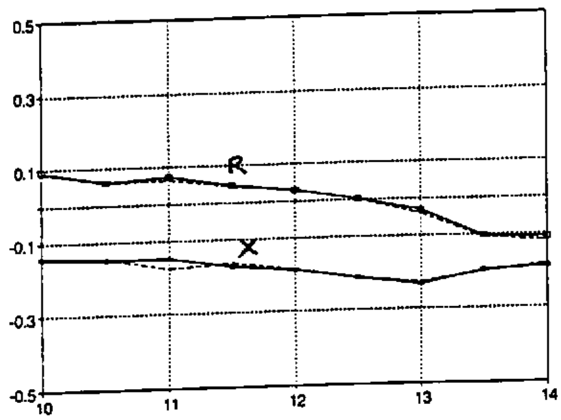

Figure 7: Z13

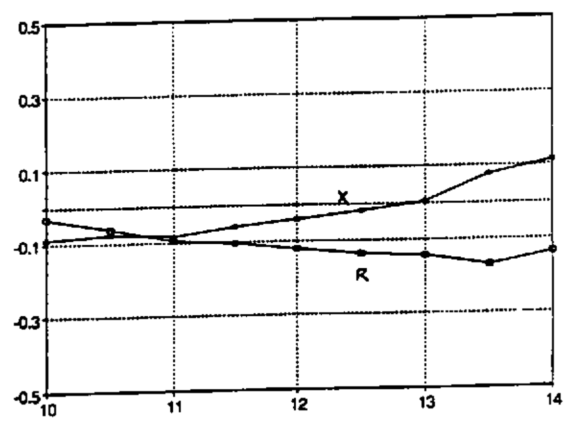

Figure 8: Z14

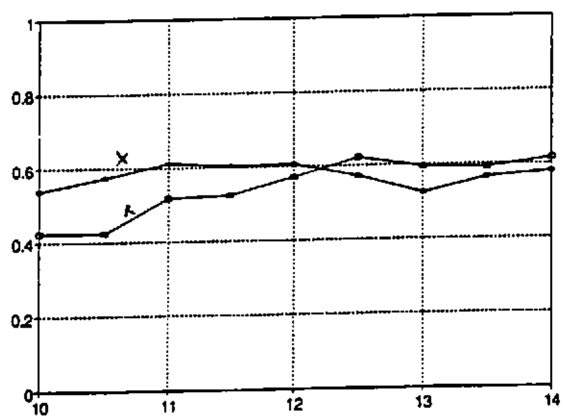

Figure 9: Z22

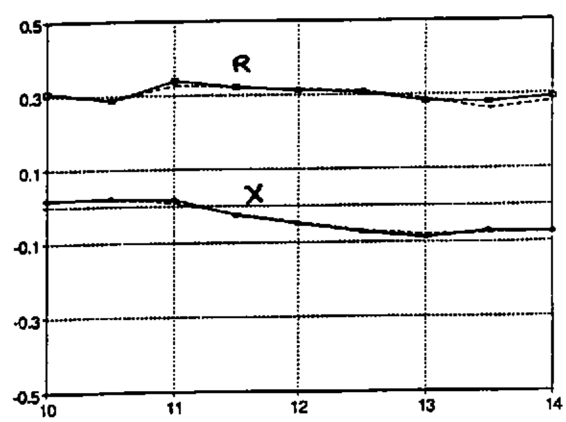

Figure 10: Z23

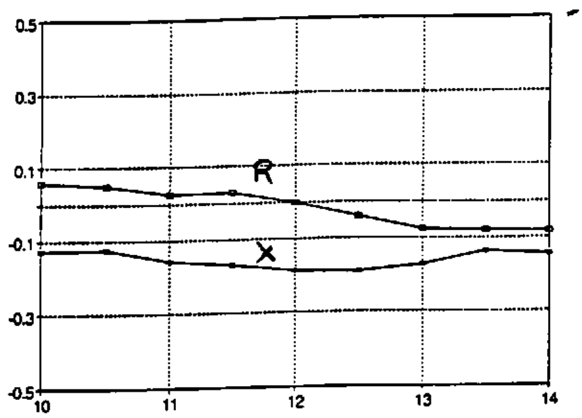

Figure 11: Z24

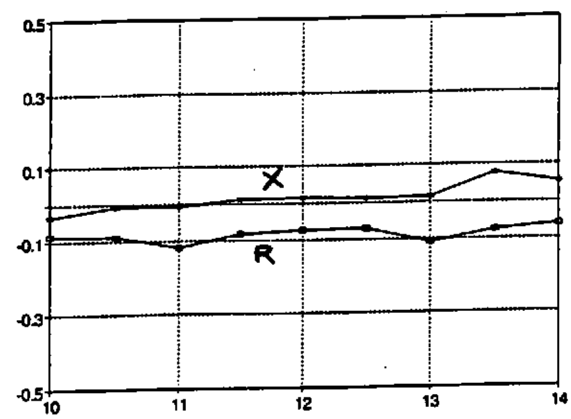

Figure 12: Z25

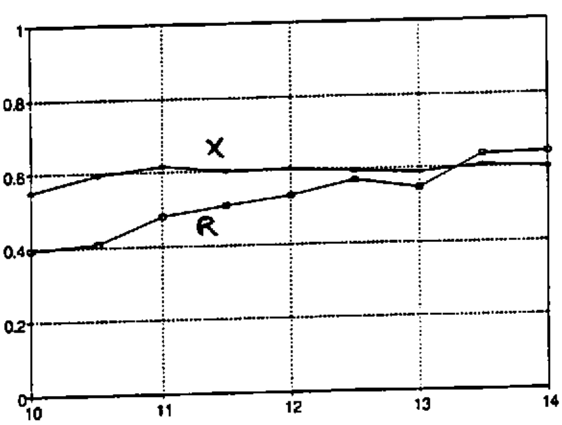

Figure 13: Z33

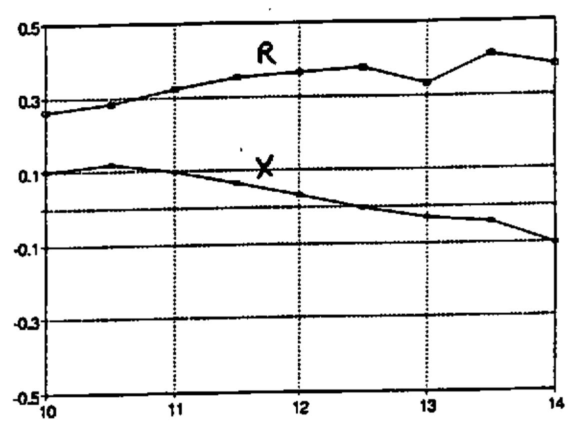

Figure 14: Z34

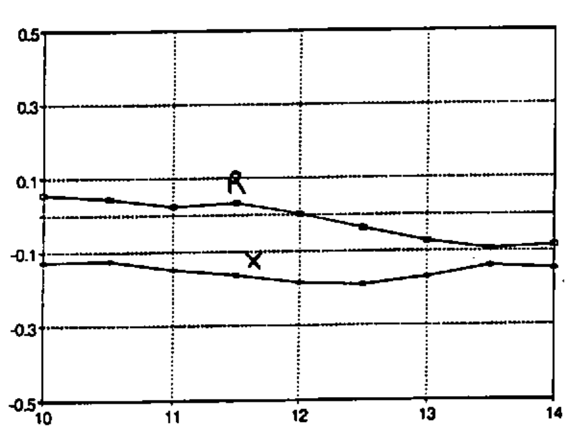

Figure 15: Z35

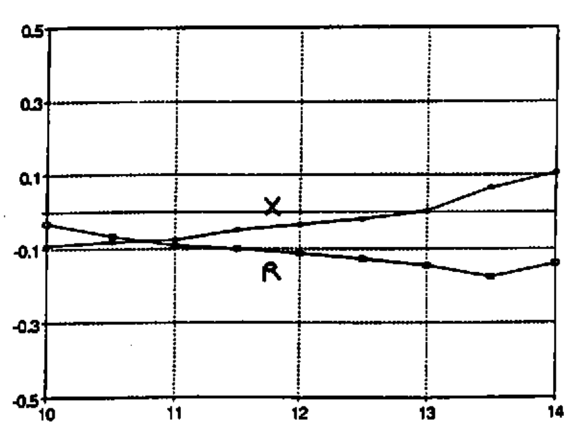

Figure 16: Z36

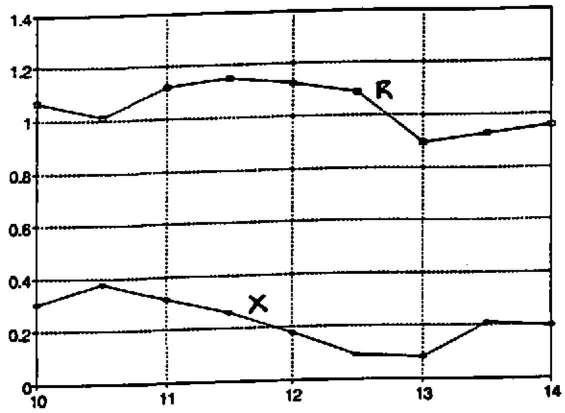

Figure 17:
sum of Z3i for i=1 to 6

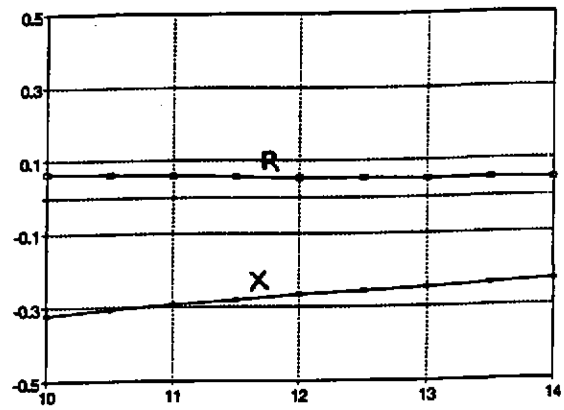

Figure 18:
elastic coupling Zi,i+1 (part 1)

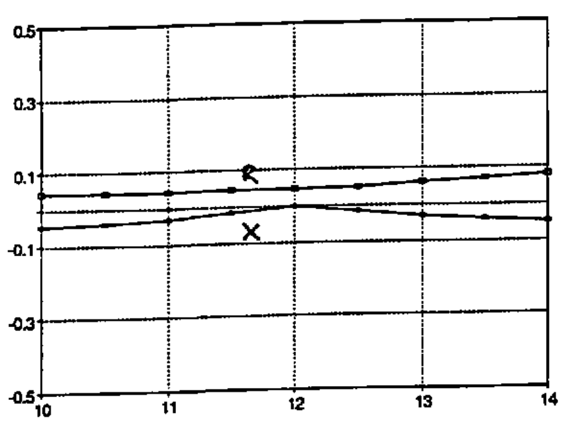

Figure 19:
elactic coupling Zi,i+1 (part 2)

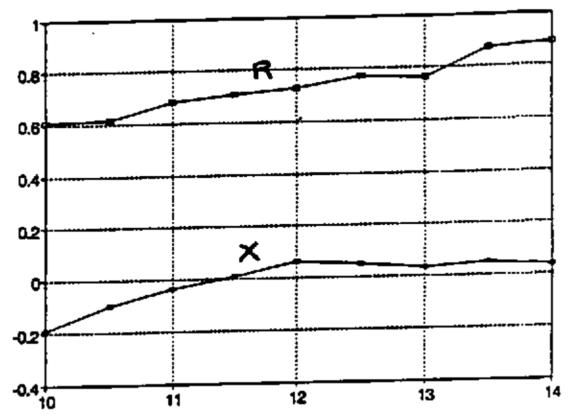

Figure 20:
Z33 total coupling

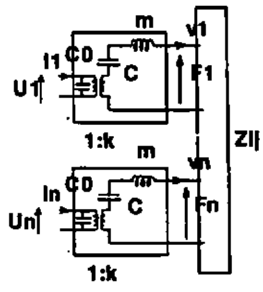

Figure 21: Electromechanical scheme