OPTIMAL VELOCITY FEEDBACK CONTROL ON A BEAM

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1 INTRODUCTION

This paper considers active vibration reduction on a simply supported beam excited by a disturbance. Though MIMO controllers of various types can be used to control these vibrations, this research focuses on the use of SISO-systems to attenuate the vibrations. Several SISO strategies for the control of a beam already exist. For single frequency disturbances tuneable vibration neutralisers and wave absorption strategies can be particularly effective as demonstrated by Brennan and Dayou[1] and Brennan, Elliott and Pinnington[2]. For broadband disturbances the most widely used method of control is absolute velocity feedback, which was used by Rockwell and Lawther[3] in one of the first applications of active vibration control. The feedback gain can be tuned to an optimal value. Levine and Athans[4] described an algorithm that lets the velocity feedback gain converge to a point where the derivative of the cost function with respect to the gain is zero, a necessary condition for optimality. Geromel and Bernussou[5] modified this algorithm to calculate the separate optimal gains for decentralised controllers.

The purpose of this paper is to examine the relationship between local properties of the beam and the optimal velocity feedback gain that minimises the kinetic energy in the beam. These local properties are assumed to be the poles of the system and the way the actuator and sensor couple into the modes.

To achieve this several steps will be taken. First, the optimal velocity feedback gain is examined when the beam is excited by a single disturbance or primary force. It will be shown that, as the primary force location changes, the optimal velocity feedback gain also varies. This means some assumption has to be made with respect to the primary force, to be able to establish a relationship between the optimal value of the velocity feedback gain and the variables measured at that location. The assumption that is then made is that the primary excitation is a random distributed excitation. The Levine and Athans algorithm is used to find the optimal velocity feedback gain. The feedback gain is then examined as the secondary (control) force location is varied. The results for the optimal feedback gain are then compared to the optimal solution for a much simpler model. Finally the solutions are compared in a 2x2 MIMO setup.

2 POINT FORCE EXCITATION

The control of the beam is first examined when it is subject to a disturbance from a point force. The assumed locations of the primary and secondary force are depicted in figure 1. The velocity sensor is collocated with the secondary force. Both the sensor and the actuator are assumed to be ideal. A model of a simply supported Euler-Bernoulli beam is used, subject to a slight viscous modal damping. The number of modes that has been taken into account (N) is 30. Assumptions with respect to flexural rigidity (EI), density (m), length (L) and the amount of damping (ζ) are shown in table 1.

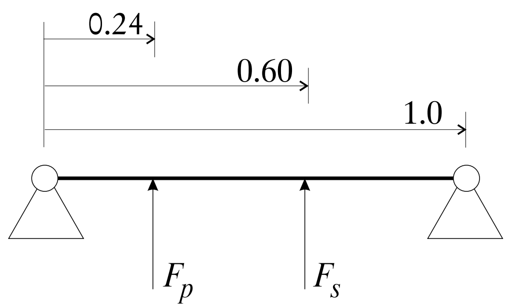


Figure 1: Location of primary and secondary force.

2.1 Frequency-domain method

The total kinetic energy of the beam is equal to the integral of the kinetic energy along the beam:

$$J_{ke\,t}(t) = \int_0^L \tfrac{1}{2} m v(x,t)^2 \, dx$$

The velocity at each point can be written as a sum of the modal velocities:

$$v(x,t) = \sum_{n=1}^{N} \Psi_n(x) v_n(t)$$

EI	1 [Nm²]
m	1 [kg/m]
L	1 [m]
ζ	0.01 [-]

Table 1: properties of the beam.

In this equation $\Psi_n(x)$ denotes the mode shape of mode n and $v_n(x)$ the velocity of that mode. For the simply supported beam the modes are sinusoidal and orthogonal. This means the kinetic energy of the beam is also equal to a sum of the kinetic energy in each of the modes[6]:

$$J_{ke\,t}(t) = \tfrac{1}{2} m \int_0^L \sum_{n=1}^{N} \Psi_n(x) v_n(t) \sum_{m=1}^{N} \Psi_m(x) v_m(t) dx = \tfrac{1}{2} m \sum_{n=1}^{N} \int_0^L \Psi_n(x)^2 \, dx v_n(t)^2 = \tfrac{1}{4} mL \sum_{n=1}^{N} v_n(t)^2$$

In the frequency domain, this results in:

$$J_{ke\,\omega}(\omega) = \tfrac{1}{4} mL \sum_{n=1}^{N} \left| V_n(\omega) \right|^2$$

where $V_n(\omega)$ is the modal velocity of mode n for excitation at that particular frequency. Parseval's theorem guarantees that the integral over the time domain, from minus infinity to infinity is equal to the integral over the frequency domain from minus infinity to infinity. This integral can be approximated with a Riemann sum of the values of $J_{ke\,\omega}(\omega)$ over a finite frequency range:

$$\int_{-\infty}^{\infty} J_{ke\,t}(t) dt = \int_{-\infty}^{\infty} J_{ke\,f}(f) df = \frac{1}{2\pi} \int_{-\infty}^{\infty} J_{ke\,\omega}(\omega) d\omega = 2\frac{1}{2\pi} \int_{0}^{\infty} J_{ke\,\omega}(\omega) d\omega \approx \frac{1}{\pi} \sum_{i=1}^{N_\omega} J_{ke\,\omega}(i\Delta\omega)\Delta\omega$$

It is assumed that the primary excitation force is unit variance white noise. The change in total kinetic energy of the beam is shown in figure 2 for varying velocity feedback gains. The optimal value for the feedback gain is clear. However, calculating the response at each frequency for each gain is a computational intensive method of finding the optimum.

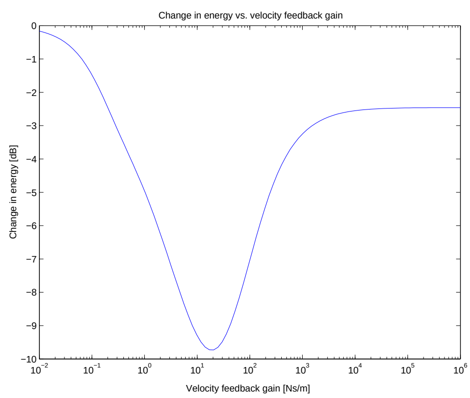


Figure 2: Variation of the total kinetic energy of the beam with velocity feedback gain

Figure 3 shows the kinetic energy at each frequency when this optimal value of velocity feedback gain is used. The velocity feedback mainly reduces the kinetic energy at the lower frequencies and at resonances. It does not reduce the kinetic energy at all resonances, because the feedback loop has no control over the modes that have a node at the location of the secondary force; when these modes are at resonance, no significant reduction is possible.

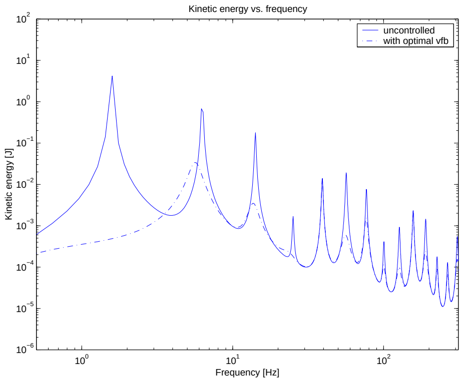


Figure 3: Kinetic energy with and without velocity feedback control.

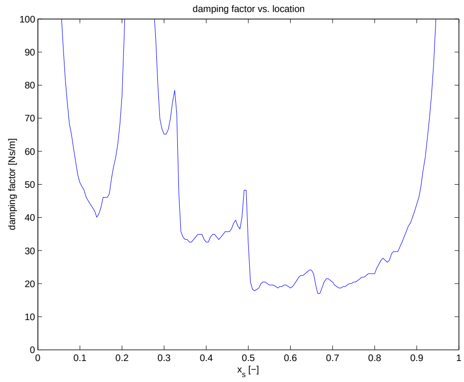


Figure 4: The optimal velocity feedback gain as a function of the location of the secondary force. The primary force location is 0.24.

2.2 Varying secondary force location

When the location of the secondary force and sensor is varied along the beam, the optimal feedback gain changes. This is depicted in figure 4. When primary and secondary force are collocated, the feedback gain tends to infinity, since the secondary force can suppress all vibration by pinning this point. It is clear that the location of the primary force matters for the selection of an optimal feedback gain. In other words, there is not a single optimal velocity feedback gain that can be derived from the variables of the beam at the control location. This means some assumption has to be made with respect to the excitation to obtain a unique optimal feedback gain.

3 DISTRIBUTED EXCITATION

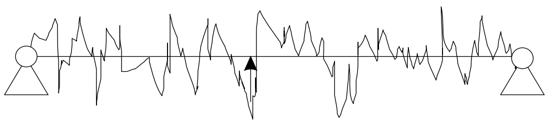


Figure 5: Spatially random distributed force.

The assumption that the primary force is located at a specific location, results in a situation where certain modes are excited more strongly than others. Also the phase of the movement of the separate modes is defined by the control location. One way to average out these effects is to assume that the beam is excited by a randomly distributed force, as depicted in figure 5. It also assumed that the distributed force also varies randomly in time. This type of excitation is also known as rain on the roof excitation. In this section the consequences of assuming this type of control are examined.

3.1 Spatially uncorrelated forces

Suppose a spatially random, distributed force excites the beam model. The distributed force has to be weighed and integrated to calculated the total force on the nth mode:

$$f_n(t) = \int_{x=0}^{L} p(x,t)\sin\left(\frac{n\pi x}{L}\right)dx$$

where $p(x,t)$ denotes a process that is random and uncorrelated in both space (x) and time (t). The correlation between the effective forces on different modes can be calculated with:

$$E[f_n(t)f_m(t)] = E\left[\int_{x_1=0}^{L} p(x_1,t)\sin\left(\frac{n\pi x_1}{L}\right)dx_1 \int_{x_2=0}^{L} p(x_2,t)\sin\left(\frac{n\pi x_1}{L}\right)dx_2\right]$$
$$= E\left[\int_{x_1=0}^{L}\int_{x_2=0}^{L} p(x_1,t)p(x_2,t)\sin\left(\frac{n\pi x_1}{L}\right)\sin\left(\frac{n\pi x_1}{L}\right)dx_2\,dx_1\right]$$
$$= \int_{x_1=0}^{L}\int_{x_2=0}^{L} E[p(x_1,t)p(x_2,t)]\sin\left(\frac{n\pi x_1}{L}\right)\sin\left(\frac{n\pi x_1}{L}\right)dx_2\,dx_1$$

Because $p(x,t)$ is uncorrelated in both x and t, $E[p(x,t)\ p(x,t)]$ is only non-zero when $x_1 = x_2$, where it is equal to its expected norm. As this norm will only scale the problem it can be chosen freely. Furthermore:

$$\int_{x=0}^{L}\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = \begin{cases} 0 \text{ if } n \neq m \\ \tfrac{1}{2}L \text{ if } n = m \end{cases}$$

Thus if the norm of the excitation is chosen equal to $2/L^2$, the expected norm of the total force on each mode ($E[f_n(t)\ f_n(t)]$) is equal to 1.

3.2 Optimisation using time-domain analysis

Levine and Athans[4] described the use of a time-domain analysis for the solution of a linear quadratic optimisation with a constant control matrix. The analysis is based on the response to an initial state of the system. In this analysis a state space description of the system is used:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}x(t)$$
$$u(t) = \mathbf{F}y(t)$$

Where $x(t)$ is the state vector of the system and $u(t)$ the control vector. $\mathbf{F}$ is the gain matrix that will be optimised. $x(t)$ is assumed to be ordered as:

$$x(t) = \begin{pmatrix} w(t) \\ v(t) \end{pmatrix}$$

where $w(t)$ are the modal displacements, and $v(t)$ the modal velocities. The cost function that was used is as follows:

$$J = \tfrac{1}{2}\int_{t=0}^{\infty}\left[x^T(t)\mathbf{Q}x(t) + u^T(t)\mathbf{R}u(t)\right]dt$$

Note that this cost function includes costs for both the state (through the matrix $\mathbf{Q}$) and the input (through $\mathbf{R}$). The matrix $\mathbf{Q}$ has to reflect the kinetic energy in modes. In this case:

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I} \end{bmatrix}$$

Levine and Athans use the fundamental transition matrix ($\mathbf{\Phi}(t,0)$) to write the cost function as a function of the initial state:

$$x(t) = \mathbf{\Phi}(t,0)x(0) = \exp([\mathbf{A} - \mathbf{B\Phi X}]t)x(0)$$

and thus:

$$J = \tfrac{1}{2}x^T(0)\int_{t=0}^{\infty}\left[\mathbf{\Phi}^T(t,0)\left[\mathbf{Q} + \mathbf{C}^T\mathbf{F}^T\mathbf{RFC}\right]\mathbf{\Phi}(t,0)\right]dt\,x(0)$$

This can also be written as:

$$J = \tfrac{1}{2}trace\left(\int_{t=0}^{\infty}\left[\mathbf{\Phi}^T(t,0)\left[\mathbf{Q} + \mathbf{C}^T\mathbf{F}^T\mathbf{RFC}\right]\mathbf{\Phi}(t,0)\right]dt\,x(0)x^T(0)\right)$$

The transient-matrix $[x^T(0)x(0)]$ of the system is chosen to match the model of random distributed excitation and will be noted as $\mathbf{P}$. The excitation is assumed to be steady state white noise. This results in a flat spectrum in the frequency domain. If a flat spectrum is transformed to the time-domain, it can also match an impulse function. It has already been established that the modes are excited independently. The response of each mode at time $t = 0$ to a unit impulse function is simply a velocity of $1/mL$. Because the modes are all excited in an uncorrelated fashion, the off-diagonal elements of $\mathbf{P}$ must be equal to 0. Combining these results, it can be shown that the matrix $\mathbf{P}$ must have the following form to match the assumed excitation:

$$\mathbf{P} = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I} \end{bmatrix}$$

Where $\mathbf{I}$ is an identity matrix of size N. This is slightly different from what was proposed by Levine and Athans, but does not change the algorithm in any major way. For the solution $\mathbf{F}$ to be optimal, it is necessary that:

$$\mathbf{F} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{KLC}^T\left[\mathbf{CLC}^T\right]^{-1}$$

where $\mathbf{K}$ is a positive semidefinite solution of:

$$\mathbf{KA}^* + \mathbf{A}^{*T}\mathbf{K} + \mathbf{Q} + \mathbf{C}^T\mathbf{F}^T\mathbf{RFC} = \mathbf{0}$$

and $\mathbf{L}$ a positive definite solution of:

$$\mathbf{A}^*\mathbf{L} + \mathbf{LA}^{*T} + \mathbf{P} = \mathbf{0}$$

Where $\mathbf{A}^* = \mathbf{A} - \mathbf{BFC}$. The iterative algorithm used by Levine and Athans to calculated $\mathbf{F}$ consists of three parts:

1) Choose a stabilising controller $\mathbf{F_0}$ and a sufficiently small convergence factor θ.
2) Set:

$$\mathbf{A}_n^* = \mathbf{A} - \mathbf{BF}_{n-1}\mathbf{C}$$

and solve:

$$\mathbf{A}_n^* \mathbf{L}_n + \mathbf{L}_n \mathbf{A}_n^{*T} + \mathbf{P} = \mathbf{0}$$
$$\mathbf{K}_n \mathbf{A}_n^* + \mathbf{A}_n^{*T} \mathbf{K}_n + \mathbf{Q} + \mathbf{C}^T \mathbf{F}_{n-1}^T \mathbf{R} \mathbf{F}_{n-1} \mathbf{C} = \mathbf{0}$$
$$\mathbf{F}_n = \mathbf{F}_{n-1} + \theta\left(\mathbf{R}^{-1} \mathbf{B} \mathbf{K}_n \mathbf{L}_n \mathbf{C} \left[\mathbf{C} \mathbf{L}_n \mathbf{C}^T\right] - \mathbf{F}_{n-1}\right)$$

3) Termination condition: $|\mathbf{F}_n\text{-}\mathbf{F}_{n-1}| < \varepsilon$, where ε is a minimum required change in $\mathbf{F}_n$. If the update is smaller than this factor, the algorithm stops.

Convergence of this algorithm (without the termination condition) is guaranteed, as longs as all solutions $\mathbf{F}_n$ stabilise the system and $0 < \theta \leq 1$, as was shown by Toivonen[7]. The disadvantage of using this system is that a weight, $\mathbf{R}$, has to be assigned to the control effort. This means that the algorithm will not minimise the kinetic energy as such, but that it can approach the optimal value if $\mathbf{R}$ is chosen very small. One advantage of this algorithm is that the actual cost and the kinetic energy can be calculated fairly easily:

$$J = trace(\mathbf{P}\mathbf{K}_n)$$

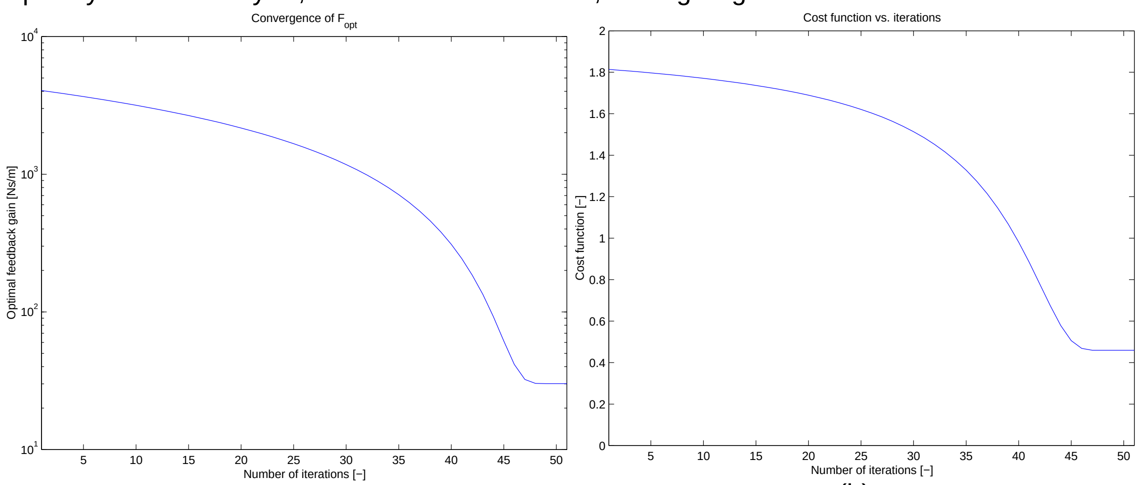
where $\mathbf{K}_n$ is solved from the aforementioned Lyapunov equation. J does include however, both costs, those associated with the kinetic energy and those with the input. If instead of $\mathbf{K}_n$ a matrix $\mathbf{M}$ is used, that is the positive semidefinite solution of:

$$\mathbf{M}\mathbf{A}_n^* + \mathbf{A}_n^{*T} \mathbf{M} + \mathbf{Q} = \mathbf{0}$$

then only the kinetic energy is calculated. This way of calculating the kinetic energy in a system is much less time consuming and more accurate than the frequency domain method. However, it is subject to the inaccuracies and limitations of the solution of the Lyapunov equation.

3.3 Results

When $\mathbf{R}$ is chosen to be small, optimality is approached by the iterative algorithm if the update parameter θ is also small enough. It has also been found that the calculation is quicker than the frequency domain analysis, described in section 2.1, while giving similar results.



(a) (b)

Figure 6: The convergence of the feedback gain to its optimal value (a) and the change in cost function (b) as the gain converges. Secondary force location $x_s = 0.6L$.

4 EFFECT OF CONTROLLER LOCATION

With the assumption of randomly distributed excitation, the effect of the controller location on the optimal velocity feedback gain can be examined. Figure 7 shows the optimal velocity feedback gains as calculated with the Levine-Athans algorithm, with an $\mathbf{R}$ of $1\cdot 10^{-5}$. For secondary force positions between 0.1 and 0.9 times the length of the beam, the optimal velocity feedback gain does not change very much. However as the controller position approaches the pinned ends of the beam, the gain varies a lot more. A direct relation between the local variables and the feedback gain is not clear from these results. To get more insight into this problem a simpler model is studied.

Two-mode model

The simplest useful model that can still be used for this problem is a beam of which only two modes have been taken into account. It is furthermore assumed that the modes are undamped. Using the theory from Levine and Athans, it can be shown that if the kinetic energy is the only relevant cost ($\mathbf{R}$ = 0):

$$J = trace\left(\mathbf{P} \int_{t=0}^{\infty} \mathbf{\Phi}^T(t,0)\mathbf{Q}\mathbf{\Phi}(t,0)dt \right) = trace(\mathbf{PK})$$

where and $\mathbf{K}$ is the 4x4 semi-positive definite solution of:

$$\mathbf{KA}^* + \mathbf{A}^{*T}\mathbf{K} + \mathbf{Q} = 0$$

and $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I} \end{bmatrix}$$

for this undamped case $\mathbf{A}$ is equal to:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 & 0 & 0 & 0 \\ 0 & -\omega_2^2 & 0 & 0 \end{bmatrix}$$

From the equation for J it can now be seen that large parts of $\mathbf{K}$ are not relevant for the calculation of the cost. In fact, only the 3rd and 4th on-diagonal elements, $\mathbf{K}_{33}$ and $\mathbf{K}_{44}$, are needed. These can be calculated analytically from the Lyapunov equation. J is equal to the sum of these two terms. Differentiating J with respect to the single channel feedback gain f, equating to 0 and solving for f, results in two solutions that are equal but opposite. The negative solution can be dismissed as it does not stabilise the system.
The equation for the optimal feedback gain, minimising kinetic energy for the undamped two-mode model, then is:

$$f_{opt} = \frac{mL}{2} \frac{\omega_2^2 - \omega_1^2}{\sqrt{\Psi_2^4\omega_1^2 + \Psi_1^2\Psi_2^2\omega_1^2 + \Psi_1^2\Psi_2^2\omega_2^2 + \Psi_1^4\omega_2^2}}$$

This equation will be referred to below as the 'two-mode formula'. The assumption has been that both Ψ_1 and Ψ_2 existed and are non-zero. If either is zero, this would result in an infinite cost for one mode, because the initial impulse would not die out. The other mode could be clamped with an infinite f to minimise the cost of that mode. It should also be noted that the derivative of the cost function J also does not actually have a root at that point. However, even though the optimal value should logically be infinite and the two mode formula was derived for non-zero Ψ_1 and Ψ_2, it still gives a finite value if either is zero. The optimal feedback gains calculated with the two-mode formula for the beam are shown in figure 7. It can be seen that these values are considerably lower in the area of 0.25 - 0.75 of the length of the beam, than for the Levine-Athans algorithm. When these gains are applied to the full beam model with 30 modes, the performance can be compared with the optimal gains calculated with the Levine-Athans algorithm. In figure 8 the change in kinetic energy of the beam relative to the uncontrolled case, has been depicted for both sets of control gains. The fact that the performance of the Levine-Athans algorithm drops off right at the edge in comparison to the two-mode formula shows that the cost associated with the input does indeed influence the result. Even though the two-mode formula is not optimal for the 30-mode model, the performance is close to what is optimally achievable in this single-channel case.

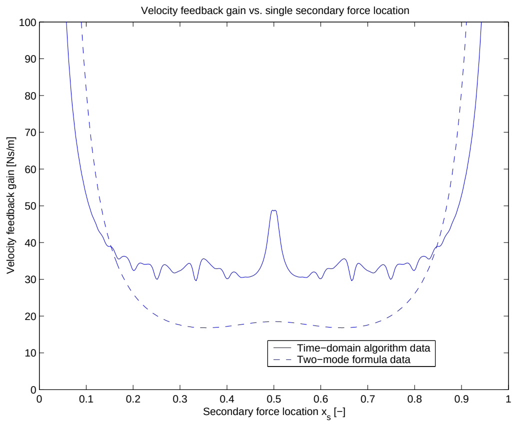


Figure 7: The optimal velocity feedback gain as a function of the secondary force location. Solid lines from iterative algorithm, dashed line from two mode theory.

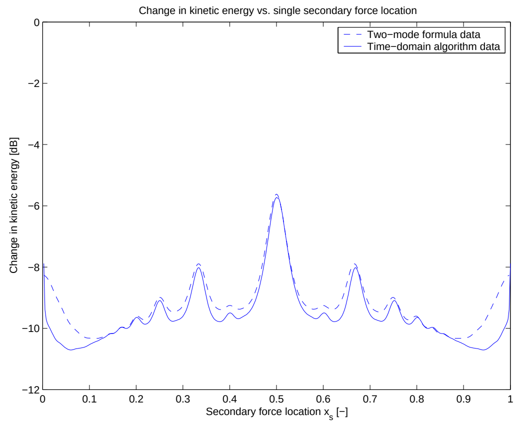


Figure 8: Change in kinetic energy as a function of secondary force location. Solid line from iterative algorithm, dashed form two mode theory.

5 MULTICHANNEL SYSTEMS

In the previous sections three different methods of calculating an optimal feedback gain were examined: by examining the kinetic energy upon a set of gains (gain-by-gain), the Levine-Athans algorithm and the two-mode formula. In this section these methods will be compared in terms of performance in a 2x2 multichannel system. The gain-by-gain method and the Levine-Athans algorithm can both be used to calculate both a centralised controller and a decentralised controller. The purpose of this analysis is two-fold, first, to establish if the two-mode formula still gives results that are close to the optimal performance and, second, to investigate whether if the performance of a centralised controller is significantly better than a decentralised controller.

5.1 Gain-by-gain method

The gain-by-gain method is a computationally intensive method for finding an optimal gain for a single control location. For the multichannel case the number of calculations needed increases dramatically, as it is linearly related with the number of possible combinations of feedback gains. If 100 values are allowed for the four elements of the feedback gain matrix $\mathbf{F}$, then the total number of combinations is 100^4.

For this reason, the decentralised controller is considered first. This requires the calculation of only 100^2 possible combinations. The excitation is assumed to be randomly distributed as described in section 3.1. Figure 9 shows the change in the cost calculated through solving the Lyapunov equations as described in section 5.2. This change has been calculated for different gains of the on-diagonal elements of $\mathbf{F}$. The control forces are assumed to be at 0.11 and 0.6 of the length of the beam. It can be seen that the performance is relatively insensitive to changes in the feedback gain: a large change in gain is required to give a 1 dB drop in performance. This confirms the result that was obtained for the single channel case.

The centralised controller is based on the decentralised controller. It is assumed that the on-diagonal elements of the controller will not differ much from the actual optimal gains of the centralised controller. Therefore only the off-diagonal elements are changed to see if better performance can be obtained. Figure 10 shows the result of changing the off-diagonal terms. The improvement in performance is minimal between the centralised case and the decentralised case and again the performance is not very sensitive to changes in the gain. The values of the optimum decentralised controller and the performance values are shown in table 2.

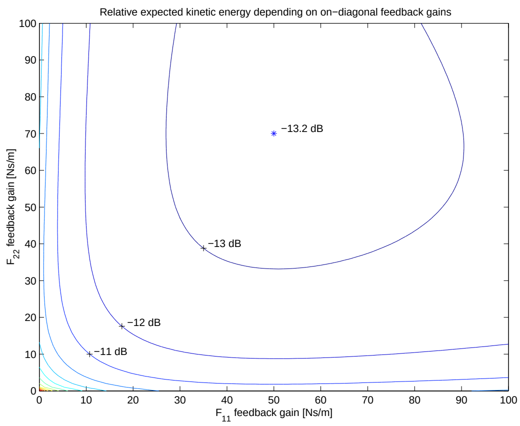


Figure 10: Contourplot of the performance as a function of the feedback gains, x_s=[0.11,0.6].

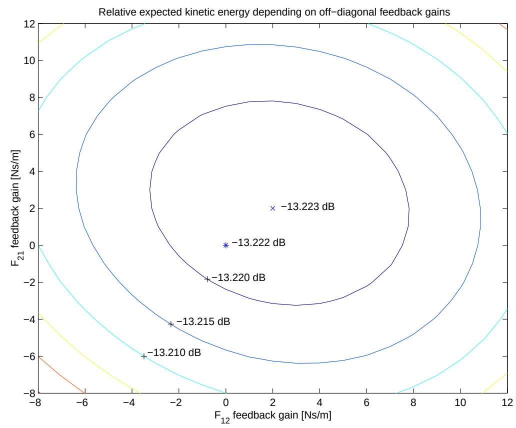


Figure 9: Contourplot of the performance as a function of the off-diagonal gains, x_s=[0.11,0.6].

5.2 Levine-Athans algorithm

The Levine-Athans algorithm converges to an optimal value, rather than having to go through a set of possible gains and selecting the optimum gain. Geromel and Bernussou[5] showed that by setting all off-diagonal elements in the update equation of $\mathbf{F}$ to zero, one obtains the optimal values for a decentralised constant gain feedback. The results for x_s = [0.11, 0.6] have been depicted in table 2.

5.3 Results based on two-mode formula

The two-mode formula can only give a value for a single feedback loop. Thus it can only give meaningful values for a decentralised controller. The results for these values have been depicted in table 2 for x_s = [0.11, 0.6].

Table 2 shows that for these locations of the control systems, there is no significant difference in performance between the centralised and decentralised controllers. The two-mode formula performs worse than the other optimisations. Whether the small performance difference between centralised and decentralised control holds in other cases or that it is location-specific is further examined in the next section, in which results obtained through the Levine-Athans algorithm will be examined.

	Gain by gain, decentralised	Gain by gain, centralised	Levine-Athans Algorithm, decentralised	Levine-Athans Algorithm, centralised	Two-mode formula
F	$\begin{bmatrix} 44 & 0 \\ 0 & 57 \end{bmatrix}$	$\begin{bmatrix} 44 & 4 \\ 4 & 57 \end{bmatrix}$	$\begin{bmatrix} 49 & 0 \\ 0 & 67 \end{bmatrix}$	$\begin{bmatrix} 49 & 2 \\ 2 & 66 \end{bmatrix}$	$\begin{bmatrix} 68 & 0 \\ 0 & 17 \end{bmatrix}$
Performance	-13.2 dB	-13.2 dB	-13.2 dB	-13.2 dB	-12.5 dB

Table 2: Feedback gains calculated in different ways for $x_s = [0.11, 0.6]$ and the corresponding change in kinetic energy with respect to the uncontrolled case.

5.4 Centralised and decentralised control

This section compares the results of the Levine-Athans algorithm for decentralised and centralised constant gain feedback. The results that are examined are the reduction in kinetic energy in comparison to the uncontrolled case, when randomly distributed forces excite the beam. The results are obtained for a grid of different control location for a 2x2 multichannel system. The locations of the control systems, x_{s1} and x_{s2}, are evenly distributed along the beam.

Figure 11 shows the change in kinetic energy obtained with the decentralised controller, calculated with the Levine-Athans algorithm. When the two controllers are located at the same position, then the problem reduces to the single channel case, because global knowledge of the system is available. Figure 12 shows the change in kinetic energy, when the controller is allowed to be a centralised, i.e. fully coupled controller. The differences between figures 11 and 12 appear small.

Figure 13 shows the difference between centralised and decentralised control, calculated with the Levine-Athans algorithm. The difference is small except when the two controllers are close together. This can be explained intuitively: as the controllers are further apart, the velocity at the two locations will lose correlation, thus if a constant feedback gain is used, the off diagonal elements may cause the controller to put energy into the system, rather than extract energy.

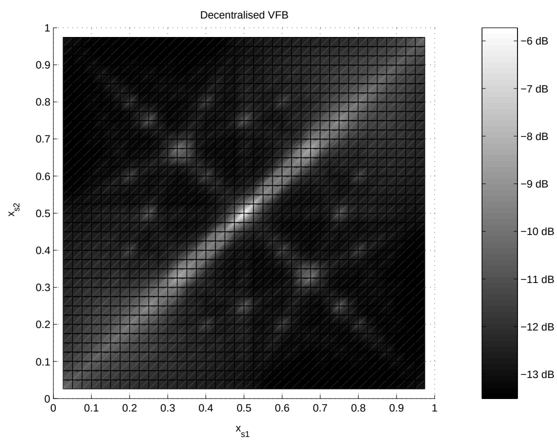


Figure 11: Performance for decentralised optimal feedback control for different control locations of a 2x2 multichannel system.

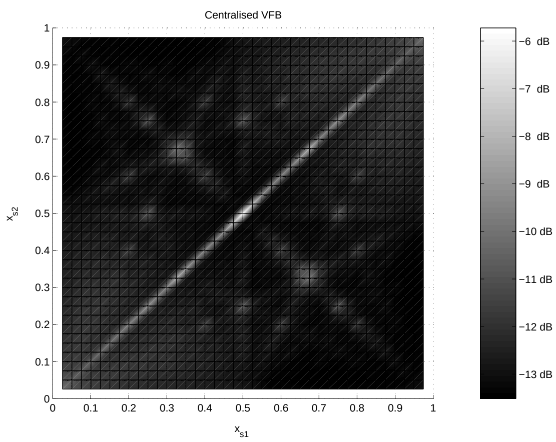


Figure 13: Performance for centralised optimal feedback control for different control locations of a 2x2 multichannel system.

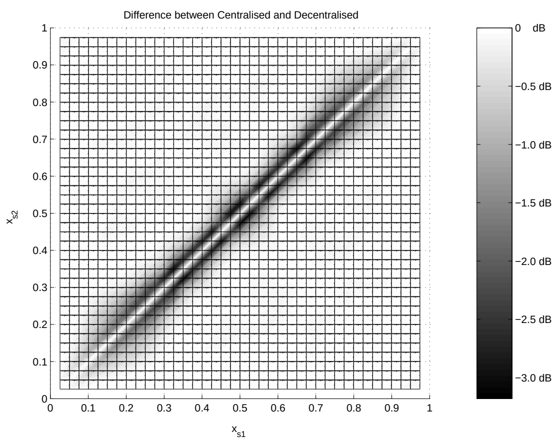


Figure 12: Difference in performance between the centralised and decentralised controllers for different control locations of a 2x2 system.

6 CONCLUSIONS

On the basis of the calculations presented here, the following conclusions can be drawn. First of all, it is necessary to make an assumption on how the system is excited, to be able to select a velocity feedback gain on the basis of the local properties of the structure. Assuming that randomly distributed forces excite the system is equivalent to assuming that the modes themselves are excited directly in an uncorrelated fashion. This assumption allows the selection of a single optimal velocity feedback gain minimising kinetic energy for a give secondary input position. For the single channel case the optimal performance can be approximated by a simple formula.

It was also shown that for random distributed excitation the performance is not very sensitive to changes in the value of the velocity feedback gain. The change in performance, when using a centralised velocity feedback controller instead of a decentralised velocity feedback is also minimal, unless the controllers are located close together.

Future work based on these results should investigate how these results compare to optimal power absorption strategies, as suggested by Elliott et al.[8] for sound fields and investigated by Hirami[9] and Sharp et al.[10] for structural vibrations. It is also interesting to examine how the performance difference of centralised and decentralised velocity feedback changes for larger numbers of sensors and actuators. Further examination of the two-mode formula may allow an improved selection of gains, such that the performance is closer to solution provided by the Levine-Athans algorithm.

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