LEAST SQUARES ESTIMATION OF ACOUSTIC REFLECTI ON COEFFICIENT

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1. INTRODUCTION

The broadband measurement of reflection coefficient in a standing wave tube undertaken with the two-microphone transfer function method is generally more convenient than the one-microphone S WR method even though the accuracy is less reliable.[1] The frequency range of the two-microphon e method is limited by the choice of microphone separation distance. To extend the effective band width into higher or lower frequencies, the microphone separation needs to be shorter or longer re spectively. However an increase in the spacing between the microphones improves low frequency performance but results in a deterioration of high frequency performance and vice versa. This pap er presents a 'least squares' solution such that the useful frequency range can be extended into m uch lower frequencies without sacrificing higher frequency results.

2. A REVIEW OF THE TWO-MICROPHONE TRANSFER FUNCTI ON METHOD

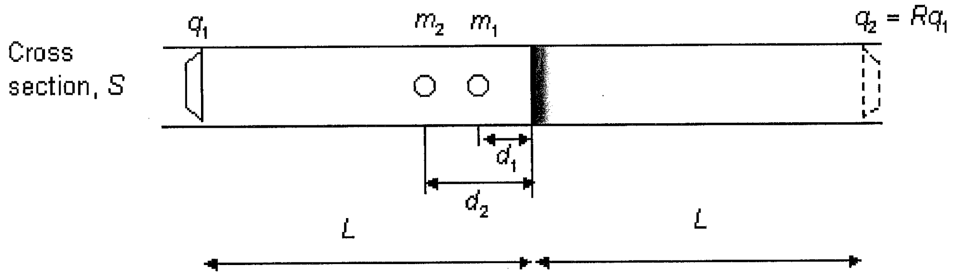

FIG. 1. A model of acoustic propagation and reflection model in an impedance tube. L is the leng th of the tube, d_1 and d_2 are distances from the reflecting surface to the microphone 1 an d 2 respectively. R is complex the reflection coefficient of the test sample, and q_2 is the s trength of the image source while q_1 is the strength of the original sound source.

With reference to Figure 1 the sound pressure detected at microphone 1 can be expressed as

$$p_1 = g_{11} q_1 + g_{12} q_2 = (g_{11} + g_{12} R) q_1 \quad (1)$$

where the Green functions g_{11} and g_{12} are defined by

$$g_{11} = \frac{\rho_0 c_0}{2S} e^{-jk(L-d_1)},\ g_{12} = \frac{\rho_0 c_0}{2S} e^{-jk(L+d_1)} \quad (2)$$

S is the cross sectional area of the tube, k is the complex wavenumber, and c_0 and ρ_0 are speed of sound and density of air in the tube respectively. Similarly p_2 can be expressed as

$$p_2 = g_{21}q_1 + g_{22}q_2 = (g_{21} + g_{22}R)q_1 \quad (3)$$

where

$$g_{21} = \frac{\rho_0 c_0}{2S} e^{-jk(L-d_2)},\ g_{22} = \frac{\rho_0 c_0}{2S} e^{-jk(L+d_2)} \quad (4)$$

The acoustical model suggests that the transfer function, H_{12}, is defined by

$$H_{12} = \frac{p_2}{p_1} = \frac{g_{21} + g_{22}R}{g_{11} + g_{12}R} \quad (5)$$

From Eq. (5), the complex reflection coefficient is found to be

$$R = \frac{g_{21} - g_{11}H_{12}}{g_{12}H_{12} - g_{22}} \quad (6)$$

Substituting Eqs. (2) and (4) into (6) gives

$$R = -e^{2jkd_1} \frac{H_{12} - e^{jk(d_2-d_1)}}{H_{12} - e^{-jk(d_2-d_1)}} \quad (7)$$

This is the usual form of solution for the reflection coefficient when deduced from a measurement of the transfer function H_{12}.[4] Note that in Figure 1 the usual microphone labelling of 1 and 2 is exchanged for the convenience of extending the theory into a multi-microphone method.

3. DEVELOPMENT OF THE LEAST SQUARES METHOD

3.1 Applying the least squares solution

The least square approach is to "best fit" the measured acoustic pressures to a model of the standing wave field. Define an error function, J, by

$$J = \left| \frac{\hat{p}_2 - p_2}{q_1} \right|^2 \quad (8)$$

where $\hat{p}_2$ is the measured pressure at microphone 2 and p_2 is the modelled pressure at microphone 2. This expression can be rearranged as

$$J = \left|\frac{\hat{p}_1}{q_1}\right|^2 \left|\frac{\hat{p}_2}{\hat{p}_1} - \frac{p_2}{\hat{p}_1}\right|^2 \quad (9)$$

Now the reference pressure in the model, p_1, can be chosen to be fixed such that it is equal to the measured pressure, $\hat{p}_1$. That is

$$p_1 = \hat{p}_1 \quad (10)$$

then Eq. (9) can be written as

$$J = \left|\frac{\hat{p}_1}{q_1}\right|^2 \left|\hat{H}_{12} - H_{12}\right|^2 \quad (11)$$

The optimum complex reflection coefficient, R_{opt}, is obtained from the process of minimising the er ror function, J. From Eq. (5),

$$J = \left|\frac{\hat{p}_1}{q_1}\right|^2 \left|\hat{H}_{12} - \frac{g_{21} + g_{22}R}{g_{11} + g_{12}R}\right|^2$$
$$= \frac{1}{\left|g_{11} + g_{12}R\right|^2} \left|\frac{\hat{p}_1}{q_1}\right|^2 \left|\left(g_{11}\hat{H}_{12} - g_{21}\right) + \left(g_{12}\hat{H}_{12} - g_{22}\right)R\right|^2$$

(12)

From Eqs. (1) and (10), this expression reduces to

$$J = \left|\left(g_{11}\hat{H}_{12} - g_{21}\right) + \left(g_{12}\hat{H}_{12} - g_{22}\right)R\right|^2 \quad (13)$$

For simplicity of further expansion of Eq. (13), the following terms are defined.

$$A \equiv g_{11}\hat{H}_{12} - g_{21}, \quad B \equiv g_{12}\hat{H}_{12} - g_{22} \quad (14)$$

Then the error function can be written as

$$J = \left|A + BR\right|^2 \quad (15)$$

which when expanded reduces to

$$J = |A|^2 + AB^*R^* + BA^*R + |B|^2\,|R|^2 \quad (16)$$

The error is at its minimum when

$$R_{opt} = -\frac{AB^*}{|B|^2} = -\frac{A}{B} = -\frac{g_{11}\hat{H}_{12} - g_{21}}{g_{12}\hat{H}_{12} - g_{22}} \quad (17)$$

Rearranging Eq. (17) gives

$$R_{opt} = \frac{g_{21} - g_{11}\hat{H}_{12}}{g_{12}\hat{H}_{12} - g_{22}} \tag{18}$$

Note that Eq. (18) is identical to Eq. (6) when $\hat{H}_{12}$ is substituted by H_{12}. This confirms that the resu lt obtained by using a least squares solution reduces to that used conventionally.

3.2 Extension to a multi-microphone method

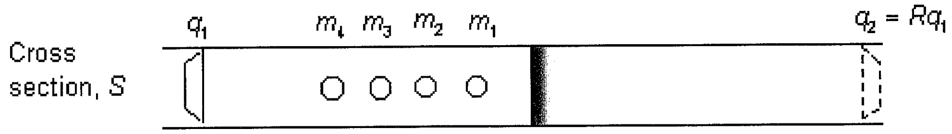

FIG. 2. Multi-microphone array in a standing wave tube.

With additional microphones placed in the tube, one may attempt to find R_{opt} that minimises the er ror between the measured and modelled transfer functions. The error function is defined by

$$J = \sum_{m=2}^{M} \left| \frac{\hat{p}_m - p_m}{q_1} \right|^2 \tag{19}$$

where M is the total number of microphones. With the same analysis shown in the previous sectio n referring to Eq. (10), the error function can be written as

$$J = \left| \frac{\hat{p}_1}{q_1} \right|^2 \sum_{m=2}^{M} \left| \hat{H}_{1m} - H_{1m} \right|^2 \tag{20}$$

where H_{1m} is analytically expressed from a model of the sound propagation process and $\hat{H}_{1m}$ is a measured transfer function between microphone 1 and microphone m. In general, H_{1m} is express ed as

$$H_{1m} = \frac{p_m}{p_1} = \frac{g_{m1} + g_{m2}R}{g_{11} + g_{12}R} \tag{21}$$

Substituting Eq. (21) into Eq. (20) gives

$$J = \left| \frac{\hat{p}_1}{q_1} \right|^2 \sum_{m=2}^{M} \left| \hat{H}_{1m} - \frac{g_{m1} + g_{m2}R}{g_{11} + g_{12}R} \right|^2 \tag{22}$$

which can be expanded to give

$$J = \frac{1}{|g_{11} + g_{12}R|^2}\left|\frac{\hat{p}_1}{q_1}\right|^2 \sum_{m=2}^{M}\left|\left(g_{11}\hat{H}_{1m} - g_{m1}\right) + \left(g_{12}\hat{H}_{1m} - g_{m2}\right)R\right|^2$$
$$= \sum_{m=2}^{M}\left|\left(g_{11}\hat{H}_{1m} - g_{m1}\right) + \left(g_{12}\hat{H}_{1m} - g_{m2}\right)R\right|^2 \qquad (23)$$

Now defining

$$A_m \equiv g_{11}\hat{H}_{1m} - g_{m1}, \quad B_m \equiv g_{12}\hat{H}_{1m} - g_{m2} \qquad (24)$$

shows that J can be expressed in the form

$$J = \sum_{m=2}^{M}|A_m + B_m R|^2 \qquad (25)$$

which then can be reduced to

$$J = \sum_{m=2}^{M}|A_m|^2 + \left(\sum_{m=2}^{M}A_m B_m^*\right)R^* + \left(\sum_{m=2}^{M}B_m A_m^*\right)R + \left(\sum_{m=2}^{M}|B_m|^2\right)|R|^2 \qquad (26)$$

The error function is minimised by

$$R_{opt} = -\frac{\sum_{m=2}^{M}AB_m^*}{\sum_{m=2}^{M}|B_m|^2} = -\frac{\sum_{m=2}^{M}\left(g_{11}\hat{H}_{1m} - g_{m1}\right)\left(g_{12}\hat{H}_{1m} - g_{m2}\right)^*}{\sum_{m=2}^{M}\left|g_{12}\hat{H}_{1m} - g_{m2}\right|^2} \qquad (27)$$

Note that this result was obtained by minimising the sum of errors between each measured and an alytically expressed pressures except the reference pressure which is armmed to be the same as t he modelled pressure. Eq. (27) is a general result for any number of microphones in an array. Whe n the total number of microphones, M, is two, the equation reduces exactly to Eq. (18) that is identi cal result obtained from two microphone transfer function method.

3.3 Microphone separation distances

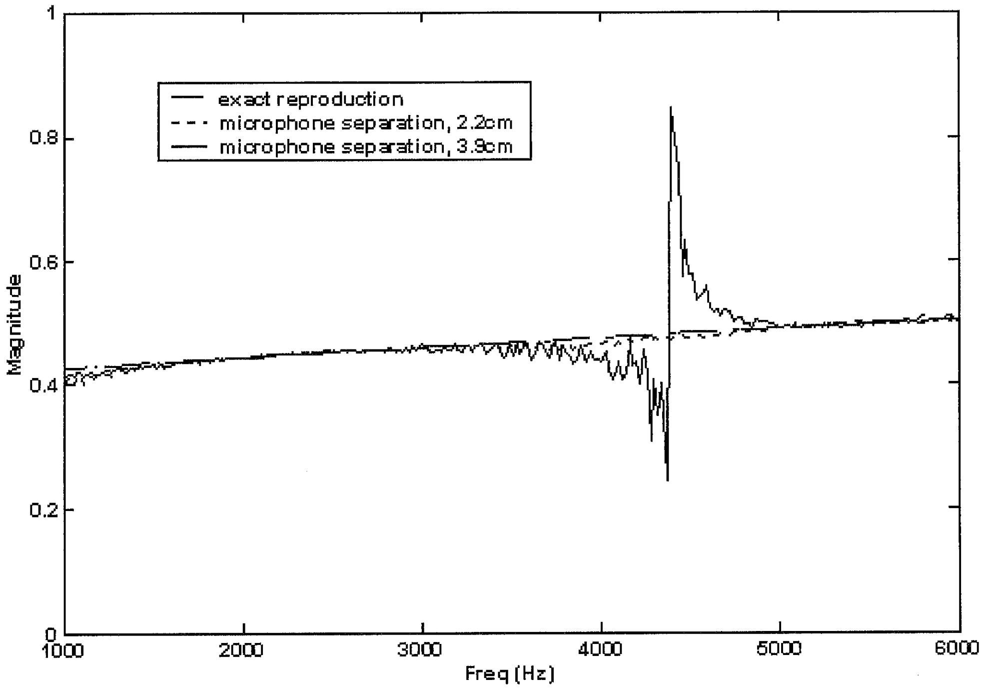

FIG. 3. Simulation results with two different analytical transfer functions that are determined by t he microphone separation distances, 2.2cm and 3.9cm.

With 1 inch-diameter circular tubes, the effective bandwidth for two-microphone transfer function m ethod is 1~6kHz.[4] The cross mode in a tube restricts the upper frequency for plane wave propagat ion and it cannot be extended unless smaller diameter tubes are used. Microphone separations ar e chosen no longer than the half wavelength of the maximum frequency because unwanted peaks occur in the measurement results at the frequency of which the half wavelength is equal to the mic rophone separation distance. Computer simulation shows that these peaks are caused by the pres ence of noise. 2% random noise is added to the transfer functions for this simulation. Figure 3 sho ws computer simulation result. The dashed line represents the modelled reflection coefficient. Soli d lines are the results obtained with the microphone separations of 2.2cm and 3.9cm, which gener ate peaks at 7.8kHz and 4.4kHz respectively

The longer microphone separation gives a far more limited upper frequency but slightly better resu lt at low frequency even if it may be marginal depending on the bandwidth of interest. Sometimes higher frequency results obtained from shorter microphone separations and lower from longer sep arations are combined by the centre frequency for better acquisition of lower frequency characteris tics, but as seen in Figure 3 the two lines do not exactly match at the dividing frequency.

The method based on the theory presented in the previous section makes it possible to extend the effective low frequency range. Equally separated microphone positioning must be avoided becaus e the half wavelength peaks are harmonic. For example, with a microphone separation distance of 4.4cm, the fundamental peak will be overlapped by the second harmonic peak of the result obtaine d with a microphone separation distance of 2.2cm. For the simulation with multi-microphone array, the microphone separation distances are chosen to be 2.2cm, 3.9cm, 6.4cm and 17.2cm. Each re sults in harmonic peaks of which the fundamental occurs at 7.8kHz, 4.4kHz, 2.7kHz and 1kHz res pectively.

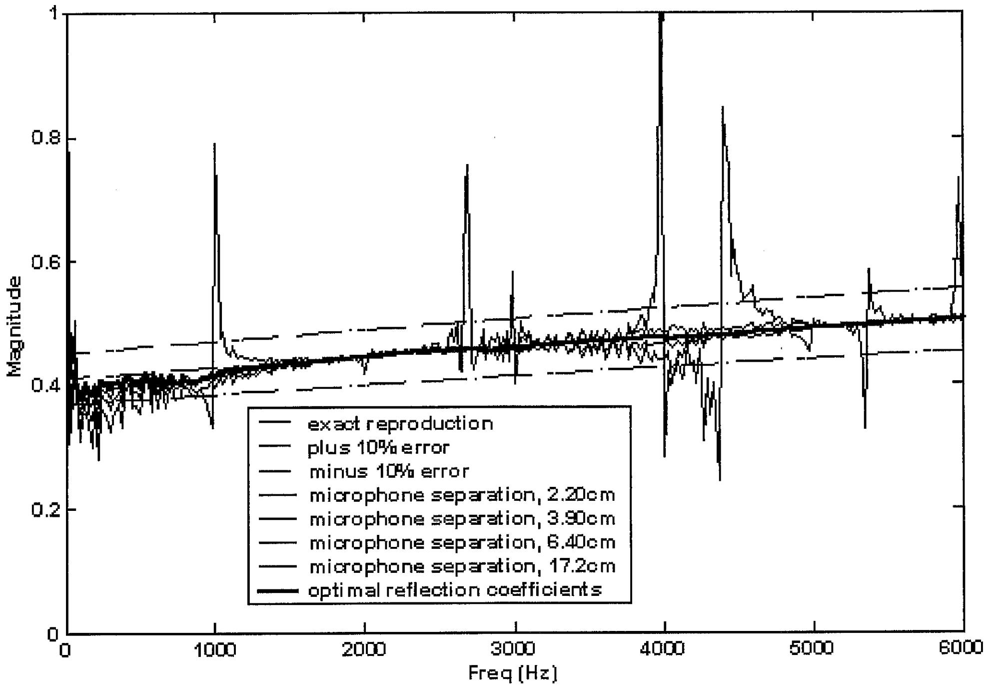

FIG. 4. Simulation with multiple microphone positions.

As shown in Figure 4, the final result does not have any peaks. Adding other separate results elimi nated poles at specific frequencies in each separate result. This is achieved by applying Eq. (22) where the sum of the numerator is divided by the sum of the denominator. The optimal reflection c oefficient is nearly the same as the exact values down to 1.5kHz. From 1.5kHz down to 100Hz the optimally estimated values follow the results obtained from the longest microphone separation dist ance.

This is only a computer-based simulation result and it is not certain that the practical measurement will be as good as this. If it is, it could mean that the effective frequency range can be extended m uch lower frequencies, i.e., down to 100Hz as shown in Figure 5. Within this frequency range, the optimally estimated reflection coefficient is within ±10% errors of the exact values.

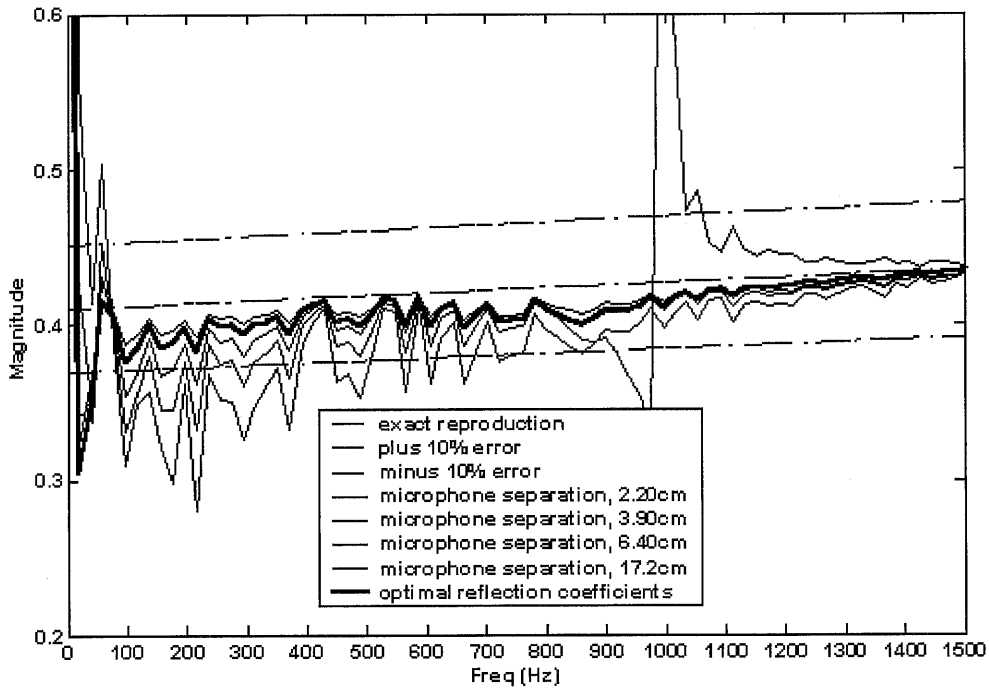

FIG. 5. Enlarged from Figure 4.

3. MEASUREMENT RESULTS

A standing wave tube was built complying with the relevant International Standard.[2] For micropho ne positioning at 5.3cm, 7.5cm, 9.2cm, 11.7cm and 22.5cm away from the test sample, 1.5mm dia meter holes were made along the tube for use of electret microphones. For the measurement only one microphone was used to avoid microphone calibration. Each measurement point is labelled as 1, 2, 3, 4 and 5 with the ascending order from nearest one to the test sample which consisted of o pen cell plastic foam. Measurement of a pair of five impulse responses in the time domain using th e MLSSA system resulted in four transfer functions, H_{12}, H_{13}, H_{14} and H_{15}. Substituting these tran sfer functions into the Eq. (22) resulted in Figure 6.

In the measurement results, the optimal reflection coefficient appears to behave well down to 100 Hz. As predicted in the computer simulation, summing all the separate results eliminated peaks du e to long microphone separations. It is promising to see that the low frequency performance has le ss tendency to "blow up" with longer microphone separation distances. The optimally estimated val ues follow the result from the longest microphone separation, which is clearly shown in the frequen cy range below 500Hz. It seems that the test sample has a resonance frequency at around 2kHz, i.e., the strongest absorption of acoustic energy by the test sample seems to occur at this frequenc y.

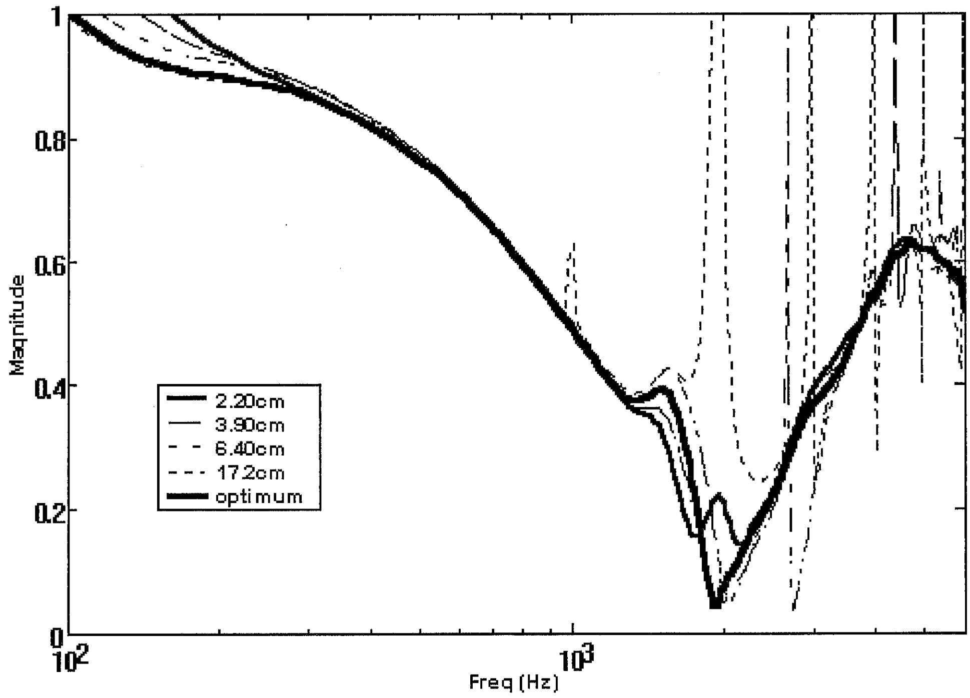

FIG. 6. Results of measured reflection coefficient.

4. CONCLUSION

The method of estimating acoustic reflection coefficient showed a promising aspect of improving lo w frequency performance for both in computer simulation and measurement. This technique need s to be compared with the approved method such as SWR method or Multipoint method (MPM)[1] i n order to confirm the validity of the results.

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