

COUPLING OF ACOUSTIC AND ENTROPIC WAVES IN THE RIJKE TUBE

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1. INTRODUCTION

In this study, we have investigated the effects of the entropic mode of fluctuation on the acoustic propagation, occurring when an unsteady heat transfer takes place in a fluid in movement in which propagates the wave. This study is conducted within the Rijke's tube phenomenon, constituting a typical case where acoustic and entropic modes of perturbation are present because of the mean flow convected heat flux ("hot spot"). This involves important modifications of the acoustic field, such as resonant frequencies and spatial distributions of acoustic pressure and velocity. Such a situation is in fact very close to those encountered in industrial burners where thermoacoustic instabilities occur spontaneously, leading to important damage, and needs a specific analysis.

2. ACOUSTIC PROPAGATION AND THERMAL SOURCE

Rijke's thermoacoustic phenomenon results from the interaction between a heat source and a steady acoustic field in an open (or semi-open) cavity. The acoustic wave is superimposed on a convected mean flow generally due to the mean heat transfer between the fluid and the thermal source. Acoustic velocity fluctuations release an unsteady heat flux which, if it has a component in phase with acoustic pressure fluctuations, produces acoustic energy.

It is understood that the acoustic wave propagation in such a situation can't be described from the usual equations of the isentropic acoustics, even if considering the mean field inhomogeneities due to spatial variations of temperature. A convected mean flow, even very low, is always present inside the tube leading to important phenomena in our situation. At first, it affects the propagation velocity of the longitudinal acoustic mode (the only one of interest here), which propagates at $c+U$ downstream and $c-U$ upstream (c and U are

respectively the velocities of sound and of the mean flow). Secondly, and more importantly, it admits a second kind of linear perturbation, that is hydrodynamic and entropic. These are convected by the flow, involving vorticity and entropy ("hot spot") transport.

Consequently, conservation of mass, momentum and energy equations are strongly coupled, and it is impossible to write a single equation for the acoustic propagation in the Rijke's tube. This implies strong disturbances of the resonant frequency and of associated spatial distributions of acoustic pressure and velocity that we are trying to show here.

Resonant equations : Let us take into account that we are not concerned by the hydrodynamic mode of disturbance in the unidimensional situation considered here still the velocity fields, as well as mean than acoustic, are supposed uniform and have only a longitudinal component simply deriving from a scalar potential.

The acoustic field determination is then obtained from the numerical resolution of the whole system of the linearized fluid dynamics equations (conservation of mass, momentum in one direction and energy) integrating velocity and temperature inhomogeneities of the mean flow (these ones are related from the conservation of mass flow relation), in which convective terms are conserved [1]. The heat release is taken into account from the energy equation.

Boundary conditions are such that we consider the acoustic pressure null at each extremity of the tube (open ended tube), or the acoustic pressure null at one end of the tube and the acoustic velocity null at the other end (semi open ended tube).

Unsteady heat release : It is therefore not simple to give a theoretical definition of unsteady heat release. The thermal transfer must not be seen here as a real source quantity still we are not concerned by the amplitude of the acoustic field. In this study, we only want to define the acoustic propagation conditions when an unsteady heat transfer takes place, that is the consequence of the entropic mode of perturbation on the acoustic field. In a certain manner, we consider that the heat source exchanges energy with the mean flow only, without releasing the acoustic field : It only modifies its propagation.

The heat release is then modelled in considering that the heating process reaction time is quicker than the oscillations of the resonant acoustic mode [2]. We consider that the heat release is constant per unit mass, leading to an increase in the fluid temperature. The quantity of the heat supplied Q' by the thermoacoustic source is then proportional to the unsteady flow mass through the heated area of unit length d , and can be expressed by the following

$$Q'(t) = C_p \frac{T_2 - T_1}{d} \rho_1 U_1 (1 - \tau)$$

where the indexes 1 and 2 represent the state of the variables upstream and downstream of the heat source respectively (with T the temperature and ρ the density), and where τ is the time delay between the flow mass and the heat flux. In the case of a heat transfer from a hot grid, τ is given by $\tau = 0.2 d/U$ [3].

In practice, the system of equations describing the acoustic field is numerically resolved by the Runge-Kutta method. Together with the boundary conditions, we are in fact confronted by an eigen-function problem, where the eigen-value is the resonant frequency of the phenomenon and the eigen-vector represents the associated spatial distribution of the acoustic variables.

3. NUMERICAL AND EXPERIMENTAL RESULTS

Resonant frequencies : The figure 1 gives the resonant frequency of the Rijke's tube with and without unsteady heat release for different values of the ratio of the temperature upstream and downstream of the heat source being located at $x=L/4$ (L is the length of the tube). We notice that the unsteady heat transfer increases throughout the resonant frequency in comparison to the natural corresponding mode. On the other hand, we shown that a source localized at $x=3L/4$ decreases the same resonant frequency. These last two results collaborate Rayleigh's observations which indicate the frequency increases (respectively decreases) if the heat is supplied one quarter of a period before (respectively after) the phase of maximum compression [4].

The previous results are again confirmed by those presented on figure 2, which shows the resonant frequency of a semi-open ended duct with and without a heat source located in the center of the duct for different values of the temperature ratio. In this case, it appears that the resonant frequency of the system decreases importantly. This agrees with Rayleigh's observations since the pressure is in advance by a quarter of period with the velocity in this last situation.

The comparison of our results with those of Dowling [2] confirms this behaviour. In spite of this, there exists some discrepancies between our results and those of Dowling quite probably due to the fact that this author made use of the geometric acoustic concept in order to give a general description of the phenomenon.

Spatial distribution : The figure 3 gives the numerical and experimental (obtained from a Laser Doppler Anemometry technique) spatial distributions of the acoustic velocity at the resonant frequency with an unsteady heat release. We can observe the distortion of the acoustic field in the tube, especially in the heat source area, due to the unsteady heat release involving an increase of the velocity according to the fluid dilatation. Furthermore, we notice that the velocity mode is displaced upstream and that the amplitude of the oscillations is more important in the hot zone of the tube. Numerical simulations have shown that this spatial behaviour is strongly different to this of the natural corresponding mode of resonance, even in taking into account of the inhomogeneities of the mean field.

4. CONCLUSION

In this paper, we have shown that the presence of the mean flow and taking into account the entropic and its coupling with the acoustic mode strongly affects the acoustic field. It involves its distortion and more importantly changes notably the resonant frequency of the phenomenon. In practice, it signifies that this coupling can't be neglected, even for a rough calculation, which could give an error value in a range of 25%.

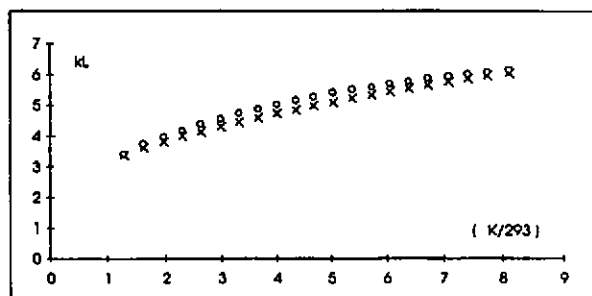


Figure 1 - Resonant frequency of an open ended tube as a function of the ratio of temperatures without (x) and with (o) unsteady heat release at $x = L/4$

Figure 2 - Resonant frequency of a semi-open ended tube as a function of the ratio of temperatures with (○ : numerical results; ● : Dowling's results) and without (□ : numerical results; X : Dowling's results) unsteady heat release

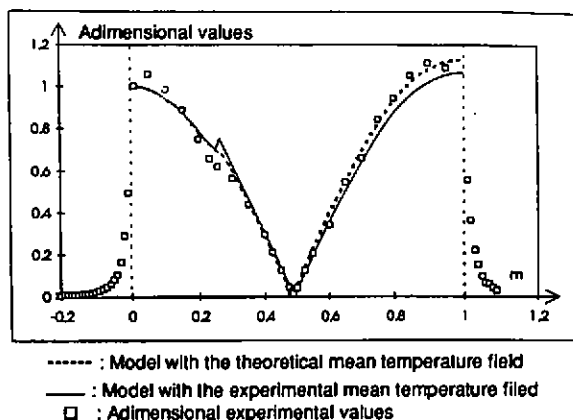
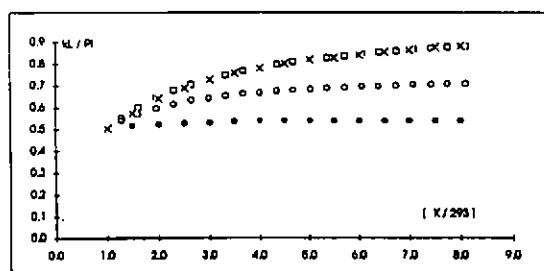


Figure 3 - Longitudinal distribution of the acoustic velocity in the Rijke's tube

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