Sound source directivity considering source movement

Yusuke MAKINO[1] and Yasushi TAKANO[2]
Graduate School of Engineering, Kyoto University
C1-4, Nishikyo-ku, Kyoto 615-8540, JAPAN

ABSTRACT

When sound sources move, frequency modulation (Doppler effect) and amplitude modulation occur in radiated sound field. We investigated these effects on directivity of radiated sound represented by three mathematical models: a monopole volume source, a monopole pressure point source and a dipole pressure point source. Assuming that a single tone point source is moving at constant velocity, we calculated sound pressure level from the source. When Mach number increases, radiated noise level increases if the radiation angle, which is the angle between source moving direction and a line from the source to a receiving point, is within a range from 0 to 120 degrees, and decreases if the angle is above 135 degrees. Amplitude modulation of radiated sound from the dipole pressure point source is greater than that from the monopole volume source when Mach number exceeds 0.5. Calculated acoustic power levels of all the three models increase as Mach number increases. The calculated power level variation of the dipole pressure point source is larger than that of the monopole volume source and it is larger than that of the monopole pressure point source.

1. INTRODUCTION

There are many studies on predicting about transportation noise caused by automobiles, aircraft, railways, etc. conducted using sound propagation analysis, numerical simulations of radiated sound, for drawing noise map, sound source localization with microphone arrays to identify dominant sound sources, etc. In these studies, it is important to model the propagation of radiated sound from moving sources mathematically and represent sound field by some formulas. Multiple types of mathematical models or formulas representing radiated sound from moving sources have been proposed as volume sources [1] or pressure point sources [2].

Sound pressure can be calculated as a product of time derivative of velocity potential and air density analytically. When a single tone source with source frequency ω is stationary, time derivative of velocity potential $\phi(t) = \phi_0 \exp(-i\omega t)$ at the time t is $\partial\phi/\partial t = -i\omega\phi(t)$. Therefore sound pressure from the stationary source is proportional to velocity potential.

However, when a single tone volume source is moving at constant velocity, velocity potential and sound pressure of radiated sound are not proportional because of the source movement [3]. When sound sources move, frequency modulation (Doppler effect) occurs in radiated sound field. Directivity

[1]makino.yusuke.77e@st.kyoto-u.ac.jp

[2]takano@archi.kyoto-u.ac.jp

of radiated sound also changes because amplitude modulation occurs when sound source moves. Zhang showed that horizontal directivity of rolling and aerodynamic noise sources of high-speed railway cars is changed significantly by these modulation effects [4]. Makino *et al.* investigated the difference of radiated sound from a monopole volume source and pressure point source considering source movement [5]. However, the effect of there source movement on directivity on the radiated sound is not clarified.

In this paper, we introduced three mathematical models representing radiated sound from a moving dipole pressure point source as well as a moving monopole volume source and a moving monopole pressure point source. Then we calculated sound pressure directivity of radiated sound represented by three kinds of models. We also discussed the effect of directivity of the radiated sound from moving source due to source movement.

2. PRESSURE AND VOLUME SOURCE ON BOUNDARY SURFACE

Radiated sound field from acoustical sources on a closed surface F can be expressed as a solution of Kirchhoff-Helmholtz integral equation which the integration over the surface. Velocity potential $\phi(P)$ at a receiving point P outside the surface can be expressed as Equation 1 [6]

$$\iint_F \left[\phi(q) \frac{\partial G}{\partial n_q} - \frac{\partial \phi(q)}{\partial n_q} G \right] dS = \phi(P) \tag{1}$$

where q is a point on the surface F and n_q is the unit vector normal to the surface at q. G is the free-space Green's function for Helmholtz' equation, i.e.

$$G(P, q) = \frac{\exp\left(ikr_{Pq}\right)}{4\pi r_{Pq}} \tag{2}$$

where r_{Pq} is distance from q to P. Space derivative $\partial G/\partial n_q$ means Green's function of a single dipole source.

Provided that $\phi(q) = \phi_0(q)\exp(-i\omega t)$, sound pressure at q is $p(q) = \rho\partial\phi(q)/\partial t = -i\rho\omega\phi(q)$ where ρ is air density. Particle velocity $v_n(q)$ at q in the direction n_q is $\partial\phi(q)/\partial n_q$. In addition, volume velocity $a(q)$ of a point volume source at q with radius r_q is $4\pi r_q^2 v_n(q)$. Then the Equation 1 can expressed as the following.

$$\iint_F \left[\frac{p(q)}{-i\rho\omega} \frac{\partial G}{\partial n_q} - \frac{a(q)}{4\pi r_q^2} G \right] dS = \phi(P) \tag{3}$$

First term of the integrated function in Equation 3 indicates a dipole pressure source because it is proportional to a product of sound pressure at q and Green's function of a dipole source. The second term indicates a monopole volume source because it is proportional to a product of volume velocity of a volume source at q and Green's function of a monopole source. Therefore the integrated function to be integrated means sound field radiated from a monopole source and a dipole source on a closed surface. This indicates that sources on finite surface area can be regarded as distributed monopole volume sources and dipole pressure point sources.

3. RADIATED SOUND FROM A MOVING SOURCE

It is supposed that a single tone point source is moving at velocity V [m/s] on a straight track in parallel with the x axis (see Figure 1). t[s] is time, and the position of the point sound source is $(x, y, z) = (Vt + x_S, y_S, z_S)$.

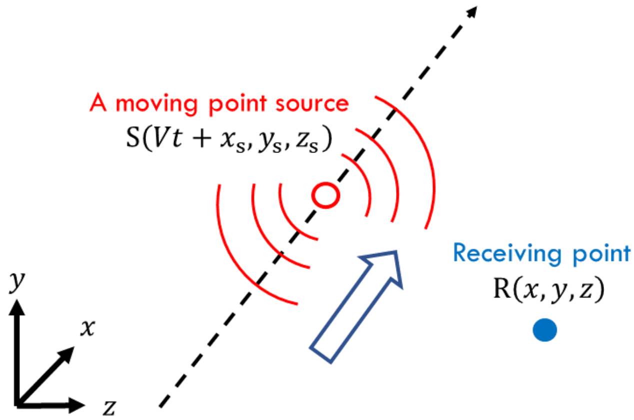

Figure 1: A moving point source on a straight track

3.1. Monopole Volume Source

Morse *et al.* [1] derived a wave equation for a moving source as in Equation 4 where $\phi_{\mathrm{MV}}(x, y, z, t)[\mathrm{m}^2/\mathrm{s}]$ is velocity potential and c [m/s] is sound velocity. $a(t) = \mu_{\mathrm{M}} \exp(-i\omega_{\mathrm{S}} t)\,[\mathrm{m}^3/\mathrm{s}]$ is volume velocity of a monopole source. $\mu_{\mathrm{M}}[\mathrm{m}^3/\mathrm{s}]$ is the amplitude of the volume velocity, and ω_{S} is angular frequency of the source.

$$\nabla^2 \phi_{\mathrm{MV}} - \frac{1}{c^2}\frac{\partial^2 \phi_{\mathrm{MV}}}{\partial t^2} = -a(t)\delta\left[x - (Vt + x_{\mathrm{S}})\right]\delta(y_{\mathrm{S}})\delta(z_{\mathrm{S}}) \tag{4}$$

Velocity potential ϕ_{MV} and sound pressure p_{MV} can be calculated as

$$\phi_{\mathrm{MV}} = \frac{a\,(t - R/c)}{4\pi R\,(1 - M\cos\theta)} = \frac{\mu_{\mathrm{M}} \exp\left[-i\omega_{\mathrm{S}}\,(t - R/c)\right]}{4\pi R\,(1 - M\cos\theta)} \tag{5}$$

$$\begin{aligned} p_{\mathrm{MV}} &= \rho \frac{\partial \phi_{\mathrm{MV}}}{\partial t} \\ &= \frac{\rho a'\,(t - R/c)}{4\pi R\,(1 - M\cos\theta)^2} + \frac{\rho V\,(\cos\theta - M)\,a\,(t - R/c)}{4\pi R^2\,(1 - M\cos\theta)^3} \\ &= \frac{-i\rho\omega_{\mathrm{S}}\mu_{\mathrm{M}} \exp\left[-i\omega_{\mathrm{S}}\,(t - R/c)\right]}{4\pi R\,(1 - M\cos\theta)^2} + \frac{\rho V\,(\cos\theta - M)\,\mu_{\mathrm{M}} \exp\left[-i\omega_{\mathrm{S}}\,(t - R/c)\right]}{4\pi R^2\,(1 - M\cos\theta)^3} \end{aligned} \tag{6}$$

where variables which appear in Equation 5 and Equation 6 mean the following list.

- $\rho[\mathrm{kg/m^3}]$: air density
- R [m]: length of sound propagation path (distance from the source to the receiving point when sound was emitted, see Figure 2)
- θ [rad]: an angle between the direction of sound radiation and source movement (see Figure 2)
- $M = V/c$[-]: Mach number

If we define L_w [dB] as acoustic power level of a stationary point source, μ_{M} can be calculated as [7],

$$\mu_{\mathrm{M}} = \frac{p_0}{f_{\mathrm{S}}\rho}\sqrt{\frac{2}{\pi}10^{\frac{L_w}{10}}}, \tag{7}$$

where $p_0 = 20\mu\mathrm{Pa}$, and $f_{\mathrm{S}} = \omega_{\mathrm{S}}/2\pi$ [Hz] is source frequency.

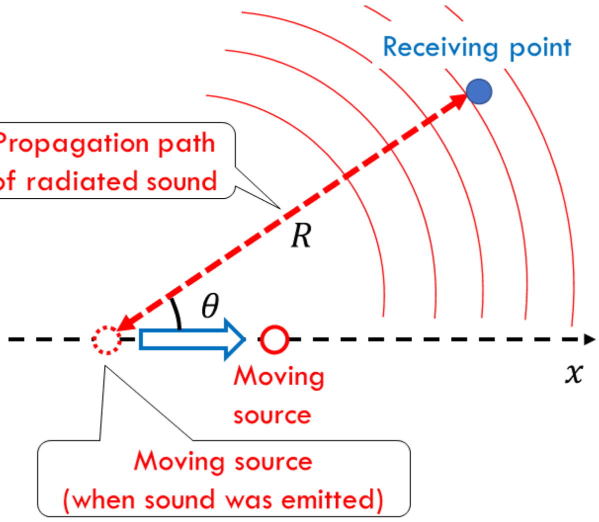

Figure 2: Propagation path of radiated sound R and propagation direction θ

3.2. Monopole Pressure Point Source

Another form of wave equation of radiated sound from a monopole source such as Equation 8 appears by replacing ϕ_{MV} and $a(t)$ in Equation 4 by sound pressure p_{MP} and source strength $S(t)[\mathrm{kg/s^2}]) = S_{\mathrm{M}} \exp(-i\omega_{\mathrm{S}} t)$.

$$\nabla^2 p_{\mathrm{MP}} - \frac{1}{c^2}\frac{\partial^2 p_{\mathrm{MP}}}{\partial t^2} = -S(t)\delta\left[x - (Vt + x_{\mathrm{S}})\right]\delta(y_{\mathrm{S}})\delta(z_{\mathrm{S}}) \tag{8}$$

This equation can be solved as follows [2].

$$p_{\mathrm{MP}} = \frac{S\,(t - R/c)}{4\pi R\,(1 - M\cos\theta)} = \frac{S_{\mathrm{M}} \exp\left[-i\omega_{\mathrm{S}}\,(t - R/c)\right]}{4\pi R\,(1 - M\cos\theta)} \tag{9}$$

When p_{MP} equals p_{MV} in Equation 6 at $V = 0$, $S_{\mathrm{M}} = -i\rho\mu_{\mathrm{M}}\omega_{\mathrm{S}}$. Then p_{MP} can be calculated as follows.

$$p_{\mathrm{MP}} = \frac{-\,i\rho\mu_{\mathrm{M}}\omega_{\mathrm{S}} \exp\left[-i\omega_{\mathrm{S}}\,(t - R/c)\right]}{4\pi R\,(1 - M\cos\theta)} \tag{10}$$

3.3. Dipole Pressure Point Source

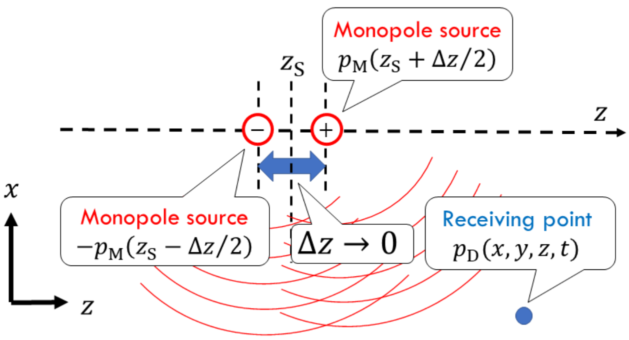

Figure 3: Monopole source configurations to approximate a single dipole source

A dipole source can be defined by two monopole sources along the dipole axis z whose signal amplitude values are opposite to each other [8]. A moving dipole source can be modeled by the monopole sources moving at same velocity with a separation distance Δz [m] (much smaller than the wavelength) and a 180 degree phase difference (see Figure 3).

We define the source amplitude of each point source as $S(t) = S_{\mathrm{D}} \exp(-i\omega_{\mathrm{S}} t)$. Then sound pressure from the dipole source $p_{\mathrm{DP}}(x, y, z, t)$ [Pa] can be calculated as

$$\begin{aligned} p_{\mathrm{DP}} &= \lim_{\Delta z \to 0} \left[p_{\mathrm{MP}}(z_{\mathrm{S}} + \Delta z/2) - p_{\mathrm{MP}}(z_{\mathrm{S}} - \Delta z/2) \right] \\ &= \Delta z \lim_{\Delta z \to 0} \frac{p_{\mathrm{MP}}(z_{\mathrm{S}} + \Delta z/2) - p_{\mathrm{MP}}(z_{\mathrm{S}} - \Delta z/2)}{\Delta z} \\ &= \Delta z \frac{\partial p_{\mathrm{MP}}}{\partial z_{\mathrm{S}}} \\ &= \frac{S_{\mathrm{D}}(z - z_{\mathrm{S}}) \Delta z \exp[-i\omega_{\mathrm{S}}(t - R/c)]}{4\pi R^2 (1 - M\cos\theta)^2} \left(-\frac{1 - M^2}{R(1 - M\cos\theta)} + i\frac{\omega_{\mathrm{S}}}{c} \right) \end{aligned} \tag{11}$$

where $p_{\mathrm{MP}}(z_{\mathrm{S}})$ is sound pressure of radiated sound from the monopole source at $z = z_{\mathrm{S}}$ with source strength $S(t) = S_{\mathrm{D}} \exp(-i\omega_{\mathrm{S}} t)$.

We calculate source strength S_{D} so as to make sound pressure from a stationary monopole and dipole source equal along the dipole axis z where the sound pressure is the highest. Then $p_{\mathrm{DP}} = p_{\mathrm{MP}}$ where $V = 0$, $\theta = \pi/2$, and $R = z - z_{\mathrm{S}}$. That equation can be solved as follows.

$$S_{\mathrm{M}} = S_{\mathrm{D}} \Delta z \left(-\frac{1}{z - z_{\mathrm{S}}} + i\frac{\omega_{\mathrm{S}}}{c} \right) \tag{12}$$

Substituting Equation 12 to Equation 11, p_{DP} can be calculated as follows.

$$p_{\mathrm{DP}} = \frac{S_{\mathrm{M}}(z - z_{\mathrm{S}}) \exp[-i\omega_{\mathrm{S}}(t - R/c)]}{4\pi R^2 (1 - M\cos\theta)^2} \left(-\frac{1}{z - z_{\mathrm{S}}} + i\frac{\omega_{\mathrm{S}}}{c} \right)^{-1} \left(-\frac{1 - M^2}{R(1 - M\cos\theta)} + i\frac{\omega_{\mathrm{S}}}{c} \right) \tag{13}$$

Substituting $S_{\mathrm{M}} = -i\rho\mu_{\mathrm{M}}\omega_{\mathrm{S}}$ as well as Equation 10, p_{D} can be calculated as below.

$$p_{\mathrm{DP}} = \frac{-i\rho\mu_{\mathrm{M}}\omega_{\mathrm{S}}(z - z_{\mathrm{S}}) \exp[-i\omega_{\mathrm{S}}(t - R/c)]}{4\pi R^2 (1 - M\cos\theta)^2} \left(-\frac{1}{z - z_{\mathrm{S}}} + i\frac{\omega_{\mathrm{S}}}{c} \right)^{-1} \left(-\frac{1 - M^2}{R(1 - M\cos\theta)} + i\frac{\omega_{\mathrm{S}}}{c} \right) \tag{14}$$

4. NUMERICAL CALCULATION

In this paper, we calculated numerically the radiated sound pressure using the following three types of mathematical models.

1. Sound pressure from a monopole volume source p_{MV} [Pa]: calculated by Equation 6
2. Sound pressure from a monopole pressure point source p_{MP} [Pa] : calculated by Equation 10
3. Sound pressure from a pressure point dipole source p_{DP} [Pa] : calculated by Equation 14

In order to make sound pressure level along axis z at farfield of all the mathematical models equal when a source is stationary, μ_{M} was calculated using Equation 7. In this paper, we calculated as $f_{\mathrm{S}} = 100$ Hz, $z_{\mathrm{S}} = 0$ m, $L_w = 100$ dB, $\rho = 1.293$ kg/m^3, and $c = 340$ m/s.

5. WAVEFRONT

Figure 4 shows sound pressure p_{MV}, p_{MP} and p_{DP} in the plane $y = y_{\mathrm{S}}$ at $V = 100$ m/s and $t = 0$ s. From the wavefront intervals for all cases in Figure 4, we can confirm frequency and amplitude modulation of radiated sound due to source movement.

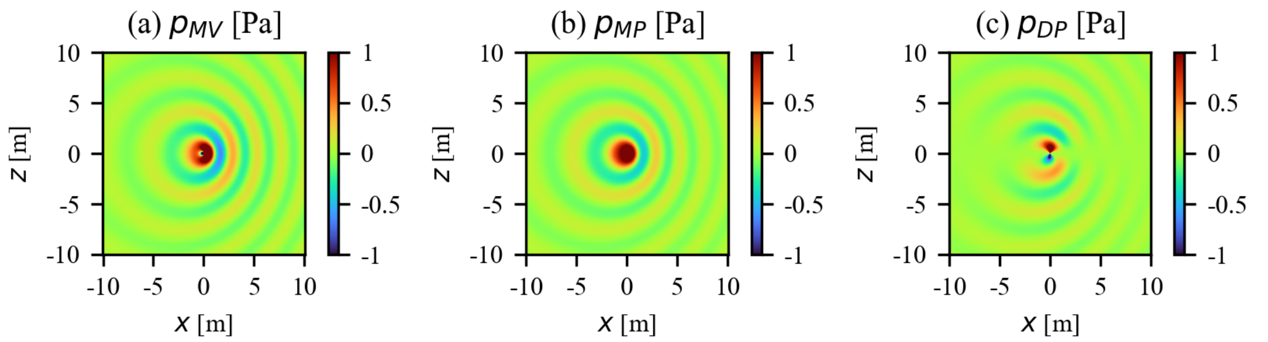

Figure 4: Sound pressure around a moving source in the plane $y = y_S$: (a)p_{MV} [Pa], (b)p_{MP} [Pa] and (c)p_{DP} [Pa] ($V = 100$ m/s, $t = 0$ s)

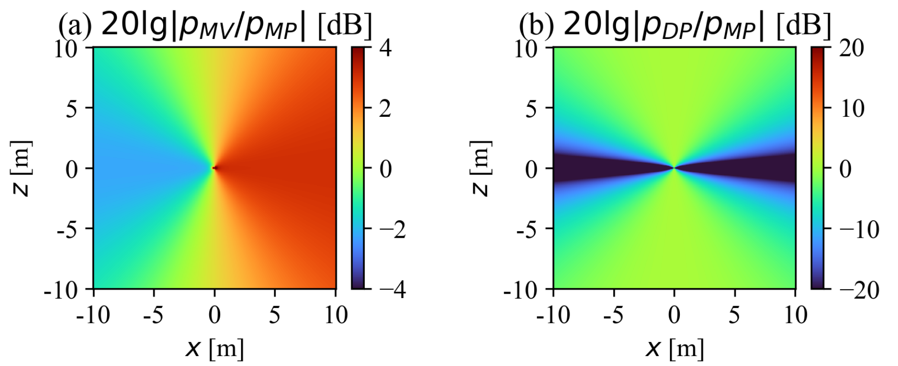

Figure 5: Difference of amplitude of sound pressure level around a moving source in the plane $y = y_S$: (a)$20\lg|p_{MV}/p_{MP}|$ [dB] and (b)$20\lg|p_{DP}/p_{MP}|$ [dB]($V = 100$ m/s, $t = 0$ s)

Figure 5 shows difference of amplitude of sound pressure level $20\lg|p_{MV}/p_{MP}|$ [dB] and $20\lg|p_{DP}/p_{MP}|$ at $V = 100$ m/s and $t = 0$ s. From Figure 5(a), the level exceeds 0 dB in positive direction of x-axis and falls below 0 dB in the reverse direction. In other words, amplitude modulation of radiated sound from a monopole volume source appears more prominent due to source movement than radiated sound from a monopole pressure point source. From Figure 5(b), we confirmed that the radiated sound pressure of a dipole source in direction along z-axis equal to that of a monopole source, and the pressure is minimized along x-axis.

6. DIRECTIVITY OF SOUND PRESSURE

In the plane $y = y_S$, we defined X [m] and ψ [rad] as length of a vector $\overrightarrow{SR}$ from a source to a receiving point and an angle between x-axis and $\overrightarrow{SR}$ (see Figure 6). We calculated sound pressure in the plane $y = y_S$ for the three mathematical models mentioned previously: $p_{MV}(\psi)$, $p_{MP}(\psi)$ and $p_{DP}(\psi)$.

In calculating directivity of sound pressure, calculating R and θ is required using X and ψ. A triangle is formed by three points: a receiving point, a position of moving source and a position of moving source when sound was emitted. From the relationship of a circumscribed circle of the triangle, the following equation is derived.

$$\frac{X}{\sin\theta} = \frac{R}{\sin(\pi - \psi)} = \frac{V(t - \tau)}{\sin(\psi - \theta)} \tag{15}$$

Using $R = c(t - \tau)$ where τ [s] is sound emission time, the equation can be solved as Equation 16-17.

$$\theta = \psi - \arcsin(M\sin\psi) \tag{16}$$

$$R = X \frac{\sin\theta}{\sin\psi} \tag{17}$$

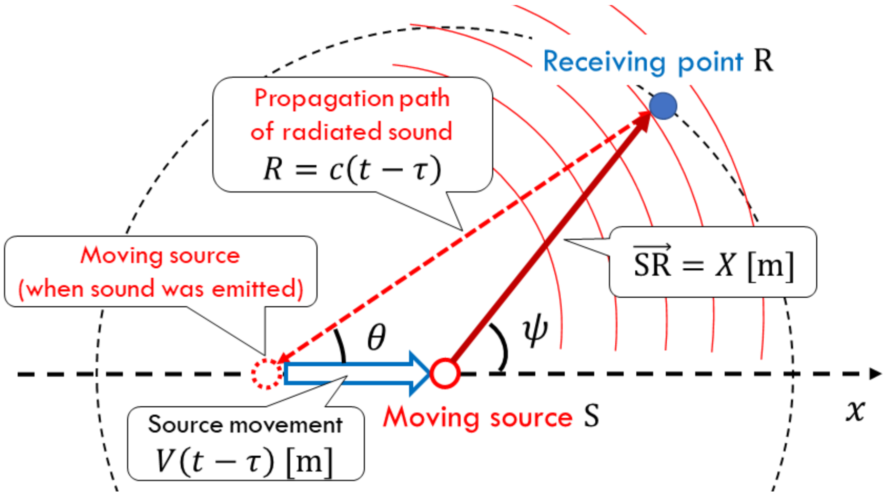

Figure 6: length of a vector from a source to a receiving point X and an angle ψ between x-axis and the vector. When a source is moving, X and ψ can be unequal value from R and θ.

Figure 7, Figure 8 and Figure 9 show directivity of sound pressure level around a moving source: $20\lg|p_{\mathrm{MV}}(\psi)/p_0|$ [dB], $20\lg|p_{\mathrm{MP}}(\psi)/p_0|$ [dB] and $20\lg|p_{\mathrm{DP}}(\psi)/p_0|$ [dB]. These figures show the calculation results for $X = 100$ m. For all the source models, there is an angle ψ which sound pressure level doesn't change as the source moves faster within a range from 120° to 135°. When ψ is smaller than that, the level increases as the source moves faster. When ψ is larger than that, the level decreases as the source moves faster.

Figure 10 shows directivity of sound pressure level around the three models. When Mach number exceeds 0.5 and ψ is smaller than the angle, sound pressure level from a dipole pressure point source is greater than a monopole volume source. In other words, amplitude modulation of radiated sound of a dipole pressure point source due to source movement is greater than that of a monopole volume source when Mach number exceeds 0.5.

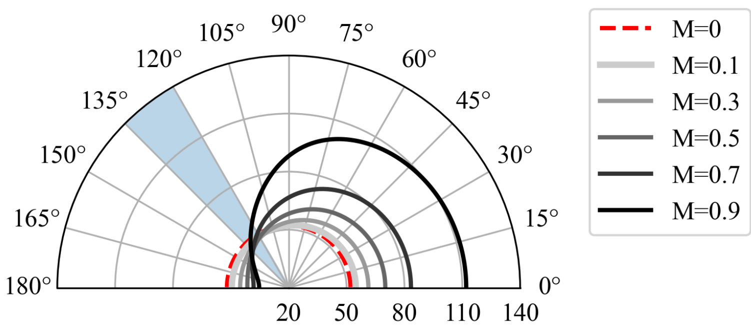

Figure 7: Directivity of sound pressure level around a moving monopole volume source $20\lg|p_{\mathrm{MV}}(\psi)/p_0|$ [dB] ($X = 100$ m)

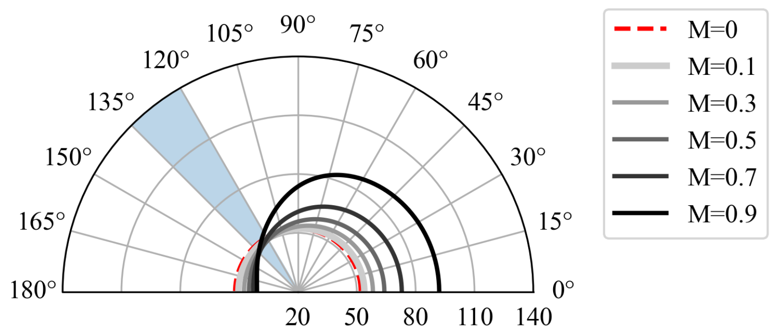

Figure 8: Directivity of sound pressure level around a moving monopole pressure point source $20\,\lg|p_{\mathrm{MP}}(\psi)/p_0|$ [dB] (X = 100 m)

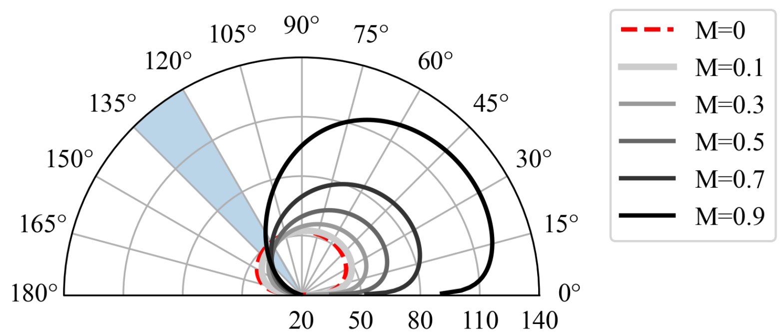

Figure 9: Directivity of sound pressure level around a moving dipole pressure point source $20\,\lg|p_{\mathrm{DP}}(\psi)/p_0|$ [dB] (X = 100 m)

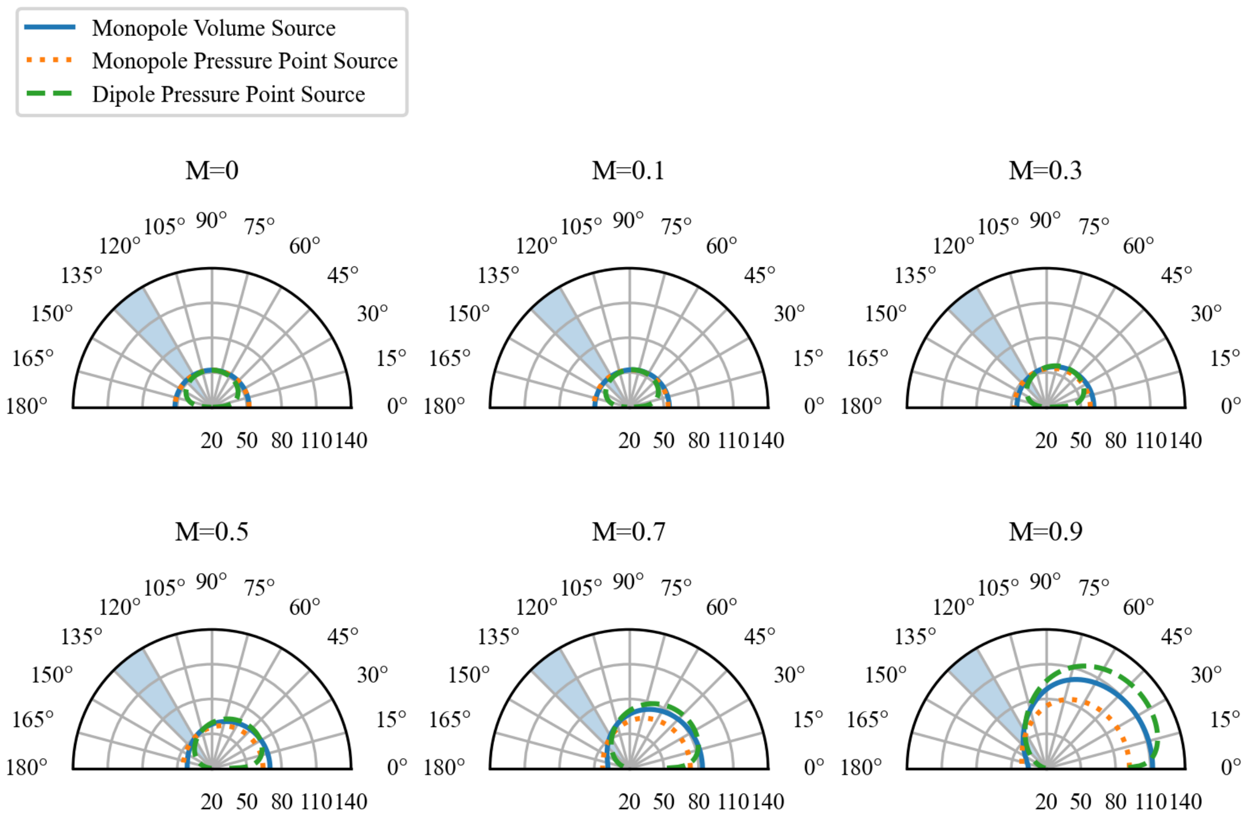

Figure 10: Directivity of sound pressure level around a moving source $20\,\lg|p(\psi)/p_0|$ [dB] (X = 100 m)

7. SOUND POWER LEVEL

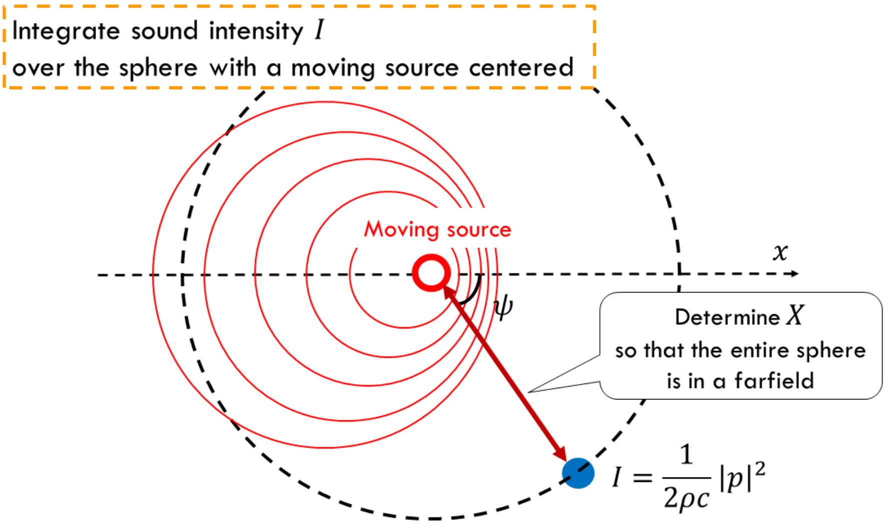

Figure 11: A sphere with radius X and a moving source centered

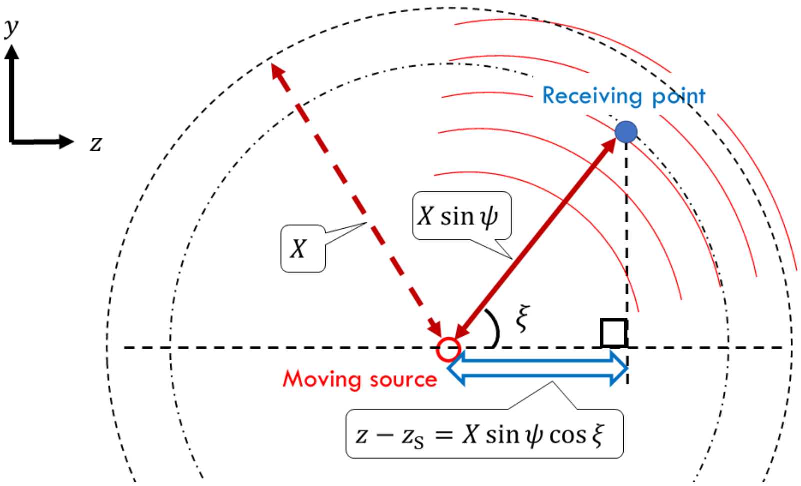

Figure 12: An angle ξ between a vector from a moving source to a receiving point and z-axis in yz plane

We set a sphere with a radius X centered on a moving source (see Figure 11). If the entire sphere is in a farfield, acoustic intensity I [W/m^2] at a certain point on the sphere can be calculated as below using sound pressure on the point p[Pa].

$$I = \frac{1}{2\rho c}|p|^2 \tag{18}$$

After that, we obtained the sound power W[W] by integrating I over the sphere as

$$W = X^2 \int_0^{\pi}\int_0^{2\pi} I \sin\psi d\psi d\xi \tag{19}$$

where ξ [rad] is an angle between a vector from a moving source to a receiving point and axis z in yz plane (see Figure 12).

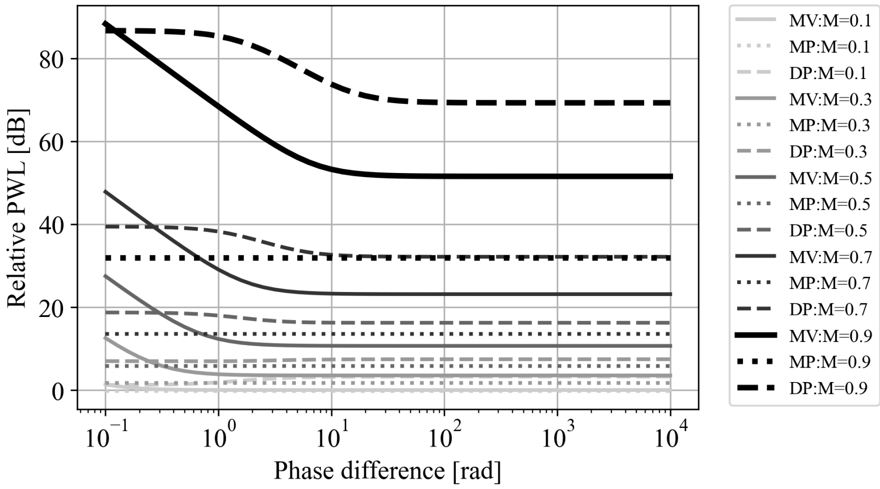

Figure 13: Acoustic power levels relative to a stationary monopole source [dB] (MV: monopole volume source, MP: monopole pressure point source, DP: dipole pressure point source). All the power levels converge at about $\varphi = 10^2$.

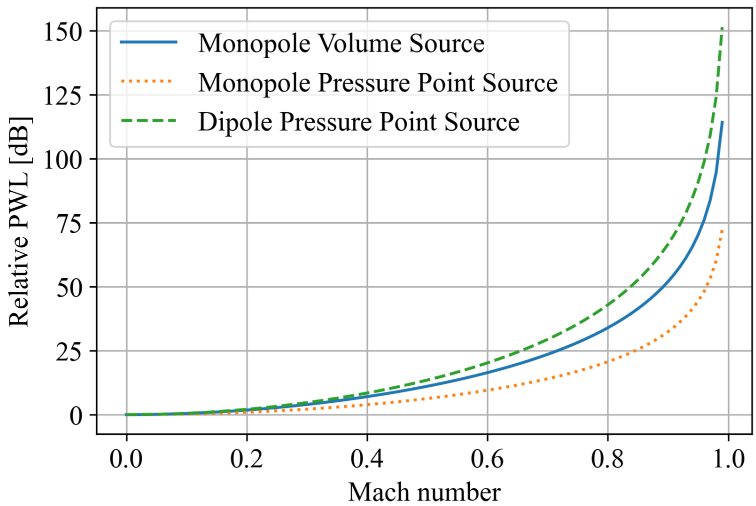

Figure 14: Acoustic power levels relative to the levels when sources are stationary [dB] at $\varphi = 10^2$ [rad]

Figure 13 shows calculated acoustic power levels (PWLs) relative to those of a stationary monopole source. The horizontal axis represents a spacial phase difference $\varphi = 2\pi f_S X/c$ [rad] between a stationary source and receiving points on the sphere. From the figure, when $\varphi = 10^2$, the entire sphere is in a farfield of a moving source because all the PWLs in the figure converge at about $\varphi = 10^2$.

Figure 14 shows calculated PWLs relative to the levels when sources are stationary at $\varphi = 10^2$ ($X \approx 54.1$ m). All the calculated PWLs of the three models increase as Mach number increases. The calculated relative PWL of a dipole pressure point source is larger than that of a monopole volume source. And that of a monopole volume source is larger than that of a monopole pressure point source.

8. SUMMARY

We investigated the effect of source movement on directivity of radiated sound represented by three mathematical models: a monopole volume source, a monopole pressure point source and a dipole pressure point source. Assuming that a single tone point source is moving at constant velocity, we calculated sound pressure level of radiated sound from a moving source. Then we investigated the relationship between the sound pressure level and an angle between source moving direction and a direction from a source to a receiving point.

As a result, the following results were obtained.

- When Mach number increases, radiated noise level increases if the radiation angle, which is the angle between source moving direction and a line from the source to a receiving point, is within a range from 0 to 120 degrees, and decreases if the angle is above 135 degrees.
- Amplitude modulation of radiated sound from a monopole volume source appears more prominent due to source movement than radiated sound from a monopole pressure point source.
- Amplitude modulation of radiated sound of a dipole pressure point source due to source movement is greater than that of a monopole volume source when Mach number exceeds 0.5.
- All the calculated acoustic power levels of the three models increase as Mach number increases.
- The calculated power level of a dipole pressure point source relative to the level when it is stationary is larger than the level of a monopole volume source and it is larger than the one of a monopole pressure point source.

Future issues include the following.

- The experimental verification on about the influence of source movement on the calculated acoustic power level of a sound source
- Appropriate estimation method of acoustic power level of a moving source at high-speed

REFERENCES

[1] P. M. Morse and K. U. Ingard. *Theoretical Acoustics*. McGraw-Hill, New York, 1968.

[2] M. S. Howe. *Acoustics of Fluid-Structure Interactions*. Cambridge University Press, Cambridge, 1998.

[3] H. Nakajima, H. Tsuru, and S. Ogata. Analysis and error investigation using fourier transform of sound field produced by moving source. *Journal of Acoustic Society of Japan (in Japanese)*, 60(12):717–726, 2004.

[4] X. Zhang. The directivity of railway noise at different speeds. *Journal of Sound and Vibration*, 329(25):5273–5288, 2010.

[5] Y. Makino and Y. Takano. Examination of accuracy of mathematical models that represent moving source. *Proceedings of the Spring Meeting of Acoustic Society of Japan (in Japanese)*, pages 487–490, 2022.

[6] Y. Kawai and T. Terai. The application of integral equation methods to the calculation of sound attenuation by barriers. *Applied Acoustics*, 31(1-3):101–117, 1990.

[7] Y. Makino and Y. Takano. The influence of frequency and amplitude modulation due to the sound source movement on radiated sound characteristics. *Transactions of the Japan Society of Mechanical Engineers (in Japanese)*, 87(899):21–00052, 2021.

[8] Y. H. Kim and J. W. Choi. *Sound Visualization and Manipulation*. John Wiley & Sons, Singapore, 2013.