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VIBRATORY STUDY OF THIN ISOTROPIC PLATES WITH EDGES RESTRAINED BY POINTS

Y Murer, F Barret, F Simon & S Pauzin

CERT - ONERA DERMES, 2 avenue E. Belin BP 4025, 31055 Toulouse Cedex, France

1. INTRODUCTION

The vibrational study of thin plates is of interest for many industrial applications, and particularly for acoustical insulation. If a great deal of papers has been dedicated to usual geometries with classical boundary conditions (e.g. : edges totally clamped, free, simply supported or a combination of these lasts) since the first publications of Warburton [1], the treatment of elastic and discontinuous boundary conditions is more recent and still developping. The aim of this paper is to present an analytical thin isotropic plate model compatible with complex conditions at the edges; its results are then compared with the experimentation in the case of the modal behaviour of a square plate restrained by points against translation and rotation on twelves mounts regurlarly distributed along the edges.

Different approaches has been followed in the treatment of complex boundary conditions. Gorman [2,3,4] proposed the superposition method as a first solution of vibrational problems for plates with classical or elastic discontinuous edges conditions. Bapat [5] suggested a model for a thin plate with two opposite sides elastically restrained, and other edges simply supported. Both of these authors used usual and hyperbolic trigonometric basis to describe the displacement fields. This kind of functions provides an orthogonal eigenvectors set, but have the important drawback to impose a geometrical approximation of the modal shapes. Berry and al. [6], and later Woodcock [7], developped solutions for a rectangular plate with elastic boundary conditions using a Taylor functions basis which allows not to freeze the modal shapes geometry. Liew and al. [8] applied the Gram-Schmidt process to orthogonalize such functions in the case of rectangular plates with mixed clamped, simply supported or free edge conditions.

2. VIBRATIONAL MODEL

Plate model

Let consider a rectangular, isotropic, homogeneous thin plate. Its lenght *a* and width *b* are directed by the x-axis and y-axis respectively, and its thickness *e* by the z-axis. Each point of the plate contour is submitted to a forces tensor restraining the edges motions against both deflection and rotation; this tensor is defined by the following equations :

$$\begin{cases} \vec{F}(P,t) = -K(P)w(P,t)\vec{z} \\ \vec{M}(P,t) = -C(P)\dfrac{\partial w(P,t)}{\partial n}\vec{\tau} \end{cases}$$

with : $(\vec{n},\vec{z},\vec{\tau})$ local orthonormal coordinate system for a point P on the plate contour;

K(P) elastic restrain against deflection at P ($N.m^{-1}$);

C(P) elastic restrain against rotation at P ($N.rad^{-1}$).

Such a formulation allows varied and discontinuous boundary conditions.

The displacement field of the plate will follow the Love-Kirschoff theory : only pure flexural vibrations will be taken into account. Thus, calling *w* the unknown flexural displacement along the z-axis :

$$U_1 = -z\frac{\partial w}{\partial x},\ U_2 = -z\frac{\partial w}{\partial y},\ U_3 = w$$

The strains and stresses fields are calculated according to linear elasticity theory for isotropic undamped materials.

The variational principle is used to determine the plate motion. The free vibrations problem is described in energetical terms using the Hamiltonian of the plate :

$$H(w) = \int_{t0}^{t1}\left[KE\left(\frac{\partial w}{\partial t}\right) - PE(w)\right]dt$$

where *KE* is the kinetic and *PE* the potential energies of the plate.

As mentionned in the introduction, the flexural motion is developped on a polynomial basis :

$$w(x,y,t) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}(t)\left(\frac{2x}{a}\right)^m\left(\frac{2y}{b}\right)^n$$

Using the differential form of the Hamiltonian leads to the Lagrange's equations; assuming a sine dependence in time for the plate motion (pulsation w), we obtain the usual eigenvalue system :

$$\left(-\omega^2\left[M_{mnpq}\right] + \left[K_{mnpq}\right]\right)\left[a_{mn}\right] = [0]$$

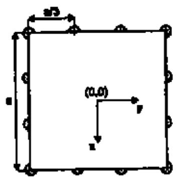
Note that the stifness matrix $[K_{mnpq}]$ include both contributions of the plate deformation and of the edges conditions. The four dimension matrices are turned into two dimension matrices, which allows an easier calculation of the determinant; the eigenfrequencies and the eigenvectors give the modal behaviour of the plate through a very reasonnable CPU time.

Validation of the model

The model is first validated on classical cases found in the bibliography : totally free or clamped plate (Gibert [8]). The convergence occurs for M = N = 10, for mode under (8,8). Values of 10^7 for K and 10^5 for C are sufficient to ensure clamped conditions.

3. EXPERIMENT AND COMPARISON

Modal analyses were carried out on a thin square aluminium plate fixed on twelve mounts regularly attached along the edge of the plate (see figure 1). The mounts are themselves clamped on a supposed infinitly rigid basis.



$a = 0.9$ m
$e = 0.0016$ m
density : 2800 kg/m^3
Young modulus : 72000 Mpa
Poisson ratio : 0.3

figure 1

Two types of mounts were used : rigid steel ones, and elastomeric ones. The longitudinal and flexural stiffnesses K and C of these lasts were found applying two differents methods (modal analysis and static deformation measurements) to twelve similar mounts :

K = 154 600 +/- 3% N/m C = 1.08 +/- 4% N/rad

The error percentages show the dispersion of results for the mounts population.

The modal frequencies and shapes are calculated with the analytical model for the tested plate, with local stiffnesses introduced in the edge conditions describing the locally clamped or restrained cases (respectively $K = 10^7$ N/m, $C = 10^5$ rad/m and K = 154600 N/m, C = 1.08 N/rad). The problem of the discontinuity of $\vec{\tau}$ is solved introducing diagonal normal external vectors at the corners.

	plate with elastic mounts			plate locally clamped		
mode (x,y)	numerical freq. (Hz)	experiment freq. (Hz)	error (%)	numerical freq. (Hz)	experiment freq. (Hz)	error (%)
(1,1)	16.3	16	2	20.2	24.4	-17
(1,2)	20.8	30.7	-32	35.2	39.8	-11.5
(2,1)	20.8	27.2	-23.5	35.2	33.7	4.5
(2,2)	30.5	39.8	-23	-	49.3	-
(0,3)	38.8	-	-	53.2	60.8	-12.5
(3,0)	38.8	48.2	-19.5	53.2	58.1	-8.5
(1,3)	40.3	53.6	-25	56	66.5	-15.5
(3,1)	40.3	51.2	-21	56	63.9	-12
(3,3)	46.7	-	-	78.6	75.8	3.5

table 1

Table 1 presents the results compared with the experimental measurements, for the modes up to (3,3). Figure 2 shows experimental and numerical modal shapes for the (3,1) mode in the local clamped points case :

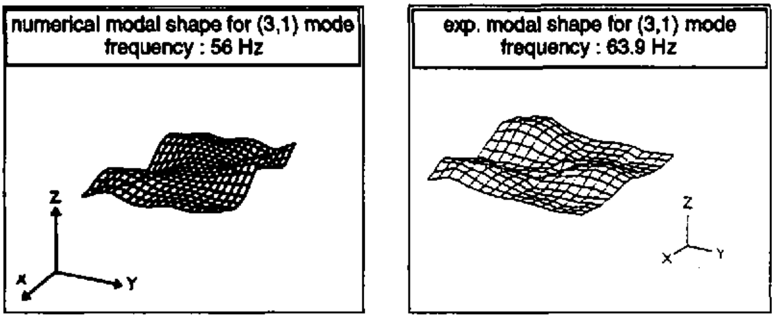


figure 2 : numerical and experimental modal shapes for mode (3,1)

All the found modal shapes match with the experimental results; but the frequency values are shifted of about 10% for the clamped case and 20% for the elastomeric mounts case. The error tends to reduce for higher modes and frequencies. The difference between experimentation and model is partly due to experiment uncertainties (realization of the boundary conditions, modal extraction), underlined by the discrepancies between experimental frequencies of symetric modes. A parametric study has been carried out on the stiffnesses values, and shows that it is necessary to increase C to obtain a modal sketch closer to the experimental reality. For example, taking K= 154600 N/m and C = 108 N/rad leads to discrepancies lower than 7,5% for modes up to (3,3), except for the (2,2) mode which is 10% over the experimental frequency. The locally clamped case is more problematic because the totally clamped case is validated for K = 10^7 N/m and C = 10^5 N/rad. So location and surface of mounts as well as numerical method are to be considered..

4. CONCLUSION

A vibratory model applied to a square thin plate with local elastic boundary conditions on the edges is compared with the associated experiment; good agreement is found for the modal shapes, but the boundary conditions need modifications to reach a good approximation in frequency.

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