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COMPARING PREDICTIVE METHODS FOR A RING PROJECTOR

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1. INTRODUCTION

It is important for transducer designers to have good modelling tools to predict the performance of transducer designs as well as the interaction effects encountered when several transducers form an array. In this study two different and independently developed finite element/boundary element codes are used and compared. One is MAVART (Model to Analyse the Vibrations and Acoustic Radiation of Transducers) developed for Defence Research Establishment Atlantic (DREA), Canada and the other is PAFEC (Program for Automatic Finite Element Computation) partially funded by the Defence Research Agency, UK. These two numerical methods are used to predict the performance of the Sparton Free-Flooded Ring Projector (FFR)[1]. This transducer is characterized by high power over a large bandwidth and essentially unlimited depth capability. An axisymmetric representation of the Sparton FFR projector is analysed using MAVART, while a three dimensional model is analysed using PAFEC. The Sparton FFR is not strictly axisymmetric so the 3D model is a more accurate representation, but at the expense of more computational time requirements. The axisymmetric model runs faster but the ring must be represented by a fictious composite material to make the problem axisymmetric.

In the next section, some of the features of MAVART and PAFEC are discussed, followed by a description of the Sparton free-flooded ring projector. Then, MAVART and PAFEC models of a simplified FFR are described, models which do not account for waterproofing of the projector. The results presented focus on parameters important to the transducer designer: the projector sensitivity, directionality and the admittance. The next section describes two approaches used to model the waterproofing (oil-filled boot) and the results obtained. Finally, an array of four elements at half wavelength spacing is modelled.

2. MAVART AND PAFEC

MAVART is an axisymmetric finite element code. It can only model axially symmetric structures, although loading and vibration modes that are not axisymmetric can be analysed. To model electroacoustic transducers with piezoelectric drivers two types of piezoelectric elements are available, an axially/radially poled piezoelectric element and a tangentially poled piezoelectric element. The later is employed in the present study. The finite element technique is used as a mathematical approximation for the elastic and piezoelectric solids, and all fluids. To model acoustic radiation problems, the transducer must be enclosed within a region of acoustic fluid. The enclosing acoustic fluid must occupy either a sphere or a hemisphere, as symmetry conditions in the model dictate. The boundary elements on the spherical fluid surface model the immersion of the transducer in an infinite medium. A Legendre polynomial series solution matched to the boundary pressures is used to solve for the pressures outside the sphere of fluid surrounding the transducer.

PAFEC is a three dimensional finite element/boundary element code with axisymmetric modelling capability. The axisymmetric problems that PAFEC can model are limited to structures where the piezoelectric elements are axially/radially poled. For the analysis of the free-flooded ring, which is tangentially poled, the axisymmetric mode of PAFEC could not be used and a three dimensional model had to be constructed.

In PAFEC the surrounding fluid is not explicitly specified but the boundary elements are based on an integral equation with the implicit Sommerfeld radiation condition.

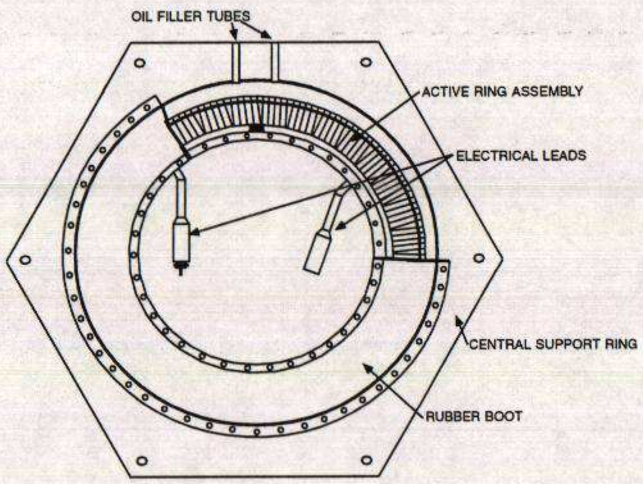

Figure 1: *Sparton of Canada Free-Flooded Ring.*

3. DESCRIPTION OF SPARTON FREE-FLOODED RING

The Sparton Free-Flooded Ring is shown in Figure 1. The ring is made of rectangular slabs of Navy Type I ceramic interspersed with aluminium wedges to provide the curvature. The construction results in the ring being tangentially poled, which gives a higher coupling factor than would be possible with radial or axial poling. The staves are wired in parallel, with the wiring embedded in the fiberglass roving around the ring. The roving is used to prestress the ceramic and prevents the ring from failing in tension. An oil-filled boot is used to waterproof the projector. The transducer is .72 m in diameter, has a height of .29 m and weighs 270 kg. Note that most of the volume of the projector is made of active material.

This projector has the potential to radiate high power (>50 kW) at low frequency over a wide band covering the frequency span between approximately 1 to 2 kHz at a high electroacoustic efficiency($\approx$75 %). The large bandwidth is the result of closely coupled structural and acoustic modes, namely the radial breathing mode of the ring and the mode of the water column (cavity) within the ring. Also, the projector is free-flooded, giving it essentially unlimited depth capabilities.

The frequency of resonance(f_R) is defined as the maximum of the conductance. For the Sparton Model 28FC1000 free-flooded ring this occurs at a frequency of 960$\pm$2.5 Hz. Experimental measurements[2] and finite element modelling[3] show that this resonance is the cavity mode. The radial projector sensitivity (or transmitting voltage response (TVR)) at f_R is 154.87 dB re 1 μPa/V @1 m. The bandwidth of this resonance is 214 Hz as measured at the -3 dB points on the projector sensitivity curve. The ring mode resonance occurs at 1600 Hz with a TVR of 151.54 dB re 1 μPa/V @1 m. The -3 dB point for the ring resonance gives a bandwidth of $\approx$1120 Hz.

4. SIMPLE MODEL

4.1 MAVART

The FFR simple model consists of the active material surrounded by the fiberglass roving. The structure has midplane reflective symmetry in the z-direction and rotational symmetry about the z-axis. Thus it is only necessary to model the top half of the ring and the top half of the fluid sphere and apply the symmetry boundary condition of setting to zero the z-displacements of the ring in this plane. The model is shown in Figure 2. The solid structure as well as the fluid sphere are divided into 9-noded isoparametric quadrilateral elements. The solids and fluid are linked by 6-noded isoparametric line elements. The outer spherical boundary

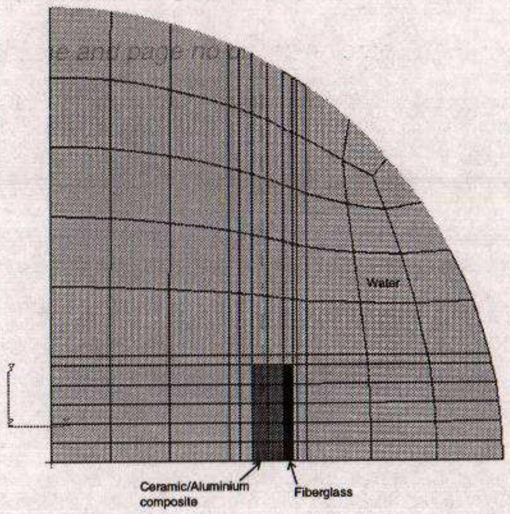

Figure 2: *Simple MAVART model for the Free-Flooded Ring.*

	Measured	MAVART	% error
f_R	1510 Hz	1515±5 Hz	.3
f_A	1810 Hz	1780±5 Hz	1.7
C(@120 Hz)	2260 nF	2255.6 nF	.2

Table 1: In-air resonant frequency f_R, anti-resonant frequency f_A, and low frequency capacitance C.

of the fluid is described by 3-noded isoparametric line elements with a pressure degree of freedom at each node.

As described in the previous section the ring is not really an axisymmetric structure because the active portion of the ring is made up of piezoelectric ceramic interspersed with metal wedges. To model this type of structure using MAVART the ring must be considered as a composite material. To obtain the material constants of this composite the proportions of ceramic, metal and epoxy used are computed from the volume each material occupies in the ring. In the tangential direction, the component stiffnesses are in series, meaning that the compliance of each component is added. In the radial and axial directions stiffnesses are in parallel, meaning that the stiffness of each component may be added. In the tangential direction the compliance of the glue joint is important but in the radial and axial directions this is not the case. The d_{31} and d_{33} values were book values and K_{33}^T is divided by the proportion of the ceramic in the ring. The material properties for the fiberglass roving were estimated from the amount of epoxy and glass fibers used in the construction. In-air admittance and capacitance measurements were made during various phases of the construction. From these measurements the estimates for s_{33}, d_{33} and K_{33}^T were fine tuned. The final material constants differed from book values as little as 2 % and as much as 40 %. The finite element results obtained with these material constants, and the corresponding experimental values are shown in Table 1. Note that no damping factors were applied in the model. Given that the values obtained for the model are relatively close to the measured values, the material matrix constructed for the composite structure was deemed to be a fair representation of the actual device.

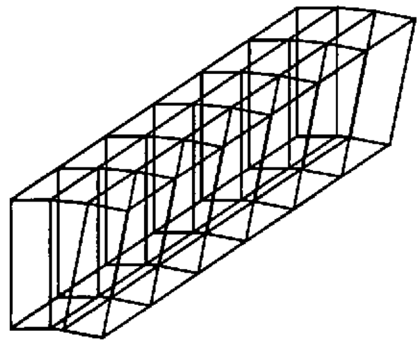

Figure 3: *PAFEC mesh for model of Sparton Free-Flooded Ring.*

4.2 PAFEC

A three dimensional model of the FFR was constructed for modelling by PAFEC. The symmetry of the ring was utilised to keep the problem to an acceptable size. This reduced the model to a 9 degree wedge section shown in Figure 3. It would have been possible to exploit the midplane symmetry and model only the top half of the ring as was done with MAVART, but to enable reuse of the model in arrays, it proved expeditious to model the full height of the ring. The solids are 20-noded isoparametric brick elements. The fluid is represented by 9-noded acoustic boundary elements, with one pressure degree of freedom, on the fluid-structure interface. The mesh density was chosen partly from the geometry of the components and partly by the frequency range of interest. The PAFEC geometry checking routine requires that the aspect ratio of sides of the elements should not exceed 5:1. To comply with the limitations imposed in PAFEC the radial direction of the wedge was divided into two elements. The PAFEC geometry checking routine was later turned off to remove this limitation. The number of elements in the radial direction was then reduced to a single element in an attempt to make the problem size smaller. There were no differences between the results of the finer mesh and those of the coarser mesh. In the tangential thickness of each ceramic plate only one element was used so that the electric boundary conditions would be exactly those of the actual device. The frequency range to be modelled extended up to 5 kHz and this dictated the number of elements along the longest dimension to be of the order of six. The material properties used for the PAFEC model were handbook values for PZT-4 ceramic and aluminium.

4.3 RESULTS FROM SIMPLE MODELS

The projector sensitivity in the radial direction predicted by both models compared to the experimental data is shown in Figure 4. MAVART predicts f_R=980±2.5 Hz and TVR(@f_R)=158.37 dB re 1 μPa/V @1 m, both of which are 2 % higher than the measured values. The uncertainty in the frequency of the model results are half the frequency increment used in the analysis. PAFEC predicts f_R=1000±25 Hz and TVR(@f_R)=158.4 dB re 1 μPa/V @1 m, an overestimate of 2.3 %. The predicted projector sensitivity being higher than the measurements is most likely a result of using an idealized lossless model. This is also evident in the magnitude of the admittance (not shown here) which is overestimated by both MAVART and PAFEC.

The second peak in the experimental TVR in Figure 4 at 1600 Hz corresponds to the ring mode. This resonance is predicted by MAVART to occur at 1630±15 Hz with a TVR of 149.8 dB re 1 μPa/V @1 m and a bandwidth in excess of 1230 Hz. PAFEC estimates the frequency for this resonance to be 1700±100 Hz with a projector sensitivity of 150.67 dB re 1μPa/V @1 m and a bandwidth of the order of 1200 Hz. In both models the ring mode frequency is slightly higher than the measured value and the projector sensitivity for

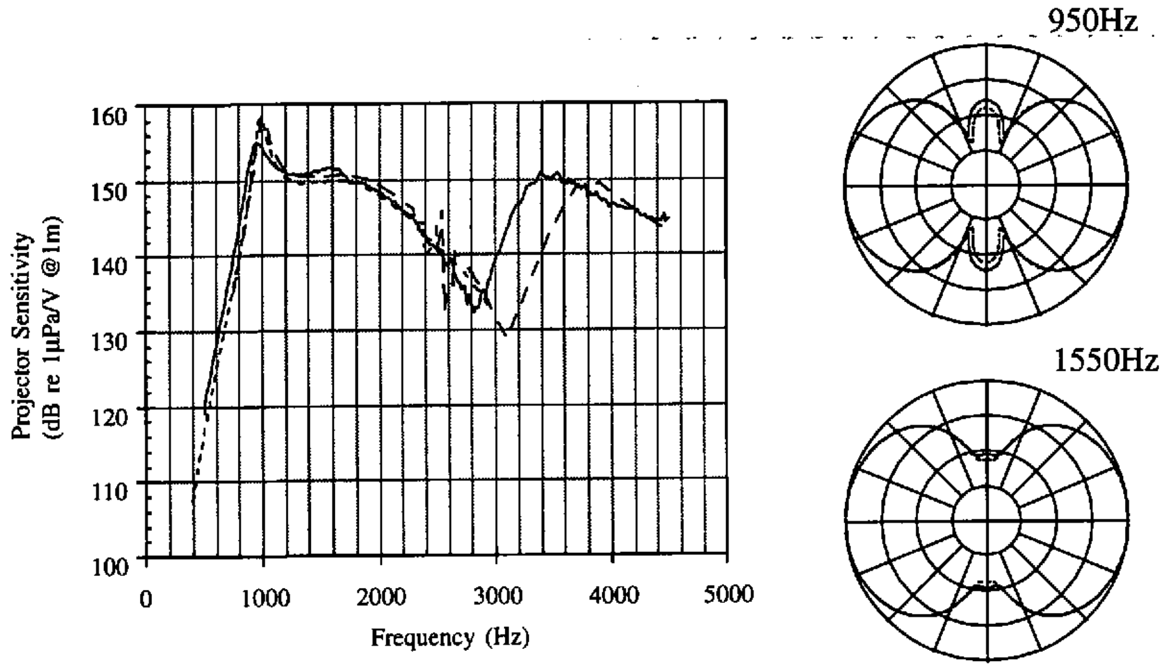

Figure 4: *Left: Projector Sensitivity for the simple ring model of the Sparton FFR. Experimental measurements: solid line; MAVART data: short dash line; PAFEC data: long dash line. Top Right: Directivity patterns at 950 Hz. MAVART data: solid line; PAFEC data: dashed line. Bottom Right: Directivity patterns at 1550 Hz. MAVART data: solid line; PAFEC data: dashed line.*

this mode is lower. The dip at ≈1750 Hz in the measured TVR data is not evident in either MAVART or PAFEC results. At both the cavity resonance and the ring resonance the predicted frequency is higher than the experimental value indicating that the models describe a stiffer transducer than the actual device.

The experimental projector sensitivity data in Figure 4 has a very weak dip at approximately 2400 Hz. This feature appears as a very pronounced glitch in the PAFEC results. The MAVART results also show a feature near this frequency in the projector sensitivity and in the admittance data. The displacement of the ring as a function of time was plotted from the MAVART results and this established that the 2400 Hz resonance corresponds to the first bending mode of the ring.

To evaluate the directivity of the transducer the far field pressure was calculated at various angles in the vertical plane. On the right side of Figure 4 the directivity patterns obtained from MAVART and PAFEC are shown for two frequencies on the low side of the cavity and ring resonances. Each circle represents a 10 dB increment in the pressure amplitude. The two models compare very well. The directivity index (DI) calculated[4] by PAFEC from the directivity results at 950 Hz is about 3 dB, and at 1550 Hz it is approximately 2.4 dB.

5. ADDITION OF WATERPROOFING

5.1 MODEL MODIFICATIONS

The Sparton FFR projector is waterproofed by an oil-filled rubber boot which was not analysed in the simple model of the previous sections. The oil and rubber boot were added explicitly to the MAVART model as shown in Figure . The oil is silicon based and it was modelled using manufacturer's data for density, with an estimate for compressibility. The rubber is a special formulation developed by Defence Research Establishment

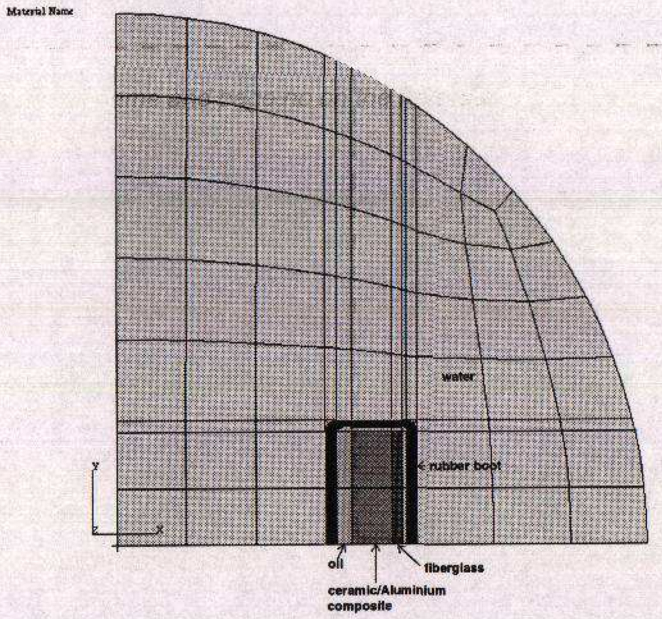

Figure 5: *MAVART model for the Free-Flooded Ring including oil and boot.*

Ottawa, Canada. For modelling purposes, estimates for the rubber properties including damping, were used.

To simulate the effect of the boot in the three dimensional PAFEC model, a damping term was added. Previous experience in modelling small axisymmetric piezoelectric rings coated with polybutadiene showed that reasonably good agreement could be achieved by this method rather than specifically adding elements for the waterproofingl[5]. With these two approaches the performance of the free-flooded ring was recalculated.

5.2 RESULTS FROM ADDITION OF WATERPROOFING

The estimated projector sensitivity in the radial direction is shown with the measured data in Figure 6. MAVART predicted the frequency of resonance to occur at 970±2.5 Hz. It is lower than in the simple model and is only 1 % higher than the measured value of 960 Hz. The reduction in frequency is probably due to the increase in the apparent height of the ring in the model, because of the addition of oil and boot. The projector sensitivity at resonance is 155.38 dB re 1 μPa/V @1 m which is in very good agreement with experiment being only .3 % from the measured value. The addition of waterproofing has also increased the bandwidth of the resonance peak. The ring resonance is now predicted to be at 1600 Hz with a TVR at this frequency 1.8 dB lower than the experimental measurements and close to the same value as previously predicted.

PAFEC predicts the same frequency of resonance for the cavity mode as without consideration of the waterproofing and the level of the projected sensitivity is .1 % less than the measured value, which is better agreement than obtained with the simple model. For the ring resonance PAFEC estimates a frequency of 1650 Hz which is closer to the actual value. The TVR at this frequency, 149.88 dB re 1 μPa/V @1 m, is now slightly lower than the estimate obtained with the simple model and is slightly worse when compared with the experimental data. Neither MAVART or PAFEC show a peak in the TVR at the ring resonance, and both sets of results are below the experimental result. Adding damping only increases the discrepancy.

The inclusion of the waterproof coating by simply adding some damping, as was done with PAFEC, has a negligible effect on the directivity of a single ring. With MAVART this was not the case. The vertical axis side-lobes in the directivity patterns for 950 Hz were lower than without the added boot and oil elements. Referring to Figure 4 the axial lobes predicted by MAVART from this more elaborate model would be lower than the PAFEC estimate, which is the dashed curve in this figure. At 1550 Hz MAVART predicts a deeper

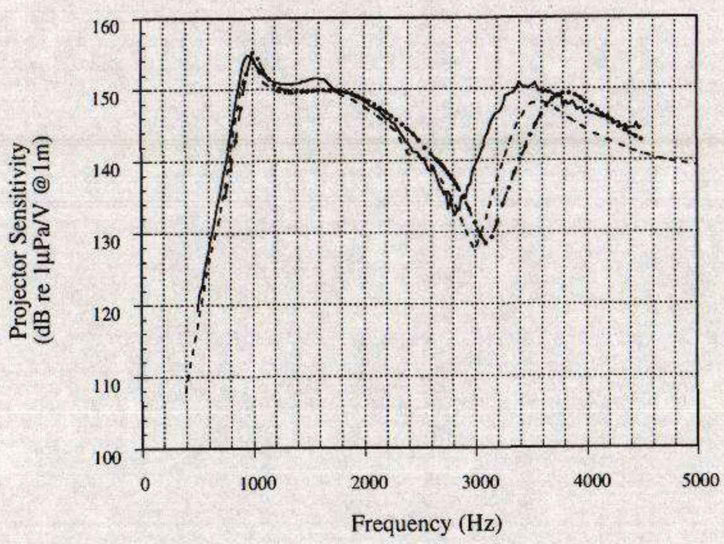

Figure 6: *Predicted and measured Radial Projector Sensitivity of Sparton FFR including waterproofing effects. Experimental data: solid line; MAVART data: short dash line; PAFEC data: long dash line.*

Referring to Figure 4 the axial lobes predicted by MAVART from this more elaborate model would be lower than the PAFEC estimate, which is the dashed curve in this figure. At 1550 Hz MAVART predicts a deeper cusp than with the simple ring model.

Substantial improvements in the fit to the experimental admittance data were observed with both MAVART and PAFEC when waterproofing was included in the models, but some discrepancies between MAVART and PAFEC clearly exist. Figure 7 shows these differences. Conventionally the admittance loop of a transducer is measured by applying a unit voltage at intervals over the required frequency range and measuring the magnitude and phase of the current. This measurement process can be simulated by MAVART and by PAFEC. The PAFEC piezoelectric elements have 4 degrees of freedom (DOF) at each node, 3 translational and one is the electric potential. A charge can be applied to an electric DOF to define a load. Alternatively a voltage can be specified at a node, which is the electrical equivalent of specifying the displacement of a purely structural DOF. One electrode of each piezoelectric element is earthed by restraining all the electrical degrees of freedom. The electrical DOF of the other electrodes are linked and a sinusoidally varying charge load is applied. This load is weighted with respect to frequency over the frequency range of interest and phase shifted by 90°. This simulates the application of a unit alternating current over the frequency range of interest. The voltage is then calculated at the same electrodes. From this the impedance, and consequently the conductance and susceptance, can be found. In MAVART the tangentially poled piezoelectric elements also have three translational DOF and one electrical DOF. The charge on an electrode surface is calculated by integrating the (complex) electric displacement over a surface midway between the electrodes. The complex impedance is then the ratio of the driving voltage and the time derivative of the charge (or current). The conductance and susceptance are obtained from the impedance. The two techniques should be equivalent but in this study yielded significantly different results. The calculated admittance loops are very sensitive to the choice of damping, and this is the most likely explanation for the discrepancies.

When structural damping was included in both models this greatly reduced the feature at ≈2400 Hz identified to be a bending mode of the ring, which is in better agreement with the observed behaviour of the actual transducer.

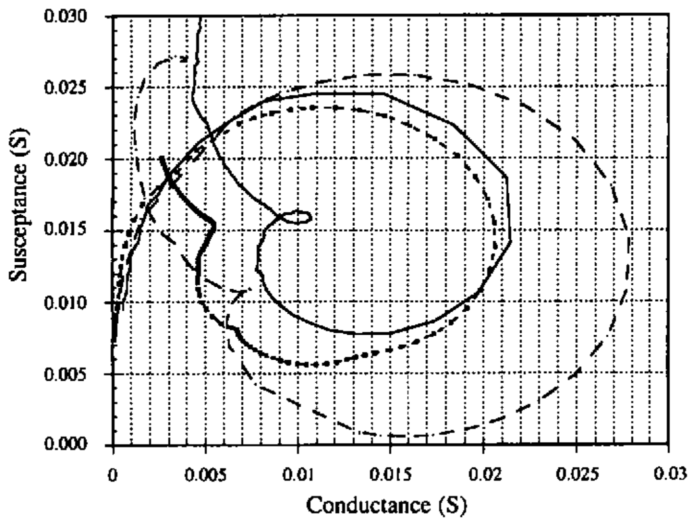

Figure 7: *Predicted and measured admittance loops of Sparton FFR including waterproofing effects. Experimental data: solid line; MAVART data: short dash line; PAFEC data: long dash line.*

6. FOUR RING ARRAY

6.1 THE MODELS

To form an array the symmetry of the free-flooded ring suggests a coaxial line geometry. A four element array was considered with the interelement spacing set at 790 mm from centre to centre.

A four element line array of FFR projectors has a midplane reflective symmetry. Both MAVART and PAFEC can use this symmetry condition to limit the size of the problem. This means that only two rings, as modelled in the previous sections, needed to be explicitly specified. Since adding the boot and oil in the axisymmetric case increases the model complexity significantly and the improvements to the projector sensitivity are only of the order of 2 to 3 % for a single ring, the simple model of the FFR was used to model the 4 element array.

6.2 RESULTS FOR RING ARRAY

MAVART and PAFEC were used to compute the projector sensitivity up to 3000 Hz. Figure 8 shows the projector sensitivity as a function of frequency for the line array of 4 free-flooded rings. This is the idealized lossless model so the bending mode at ≈2500 Hz is not damped out. At the cavity resonance, the radial projector sensitivity has increased by 7.9 dB and 8.2 dB for MAVART and PAFEC respectively. For N uniform point sources in a line array the expected increase is $20\log(N) \approx 12$ dB, which is not observed here because the directionality and interactions of the individual sources come into play. An experimental study by McMahon[2] where the number of elements in a coaxial array of rings was varied from 1 to 6 showed an increase of the projector sensitivity at the cavity resonance for a 4 element array to be about 8 dB.

The right side of Figure 8 displays the MAVART and PAFEC predictions for the vertical directivity patterns for the array at 950 Hz and 1550 Hz. The predicted patterns are very similiar. The maxima of the patterns are in the directions at right angles to the axis of the array. The directivity indices predicted by PAFEC are 6.3 dB and 7.4 dB respectively.

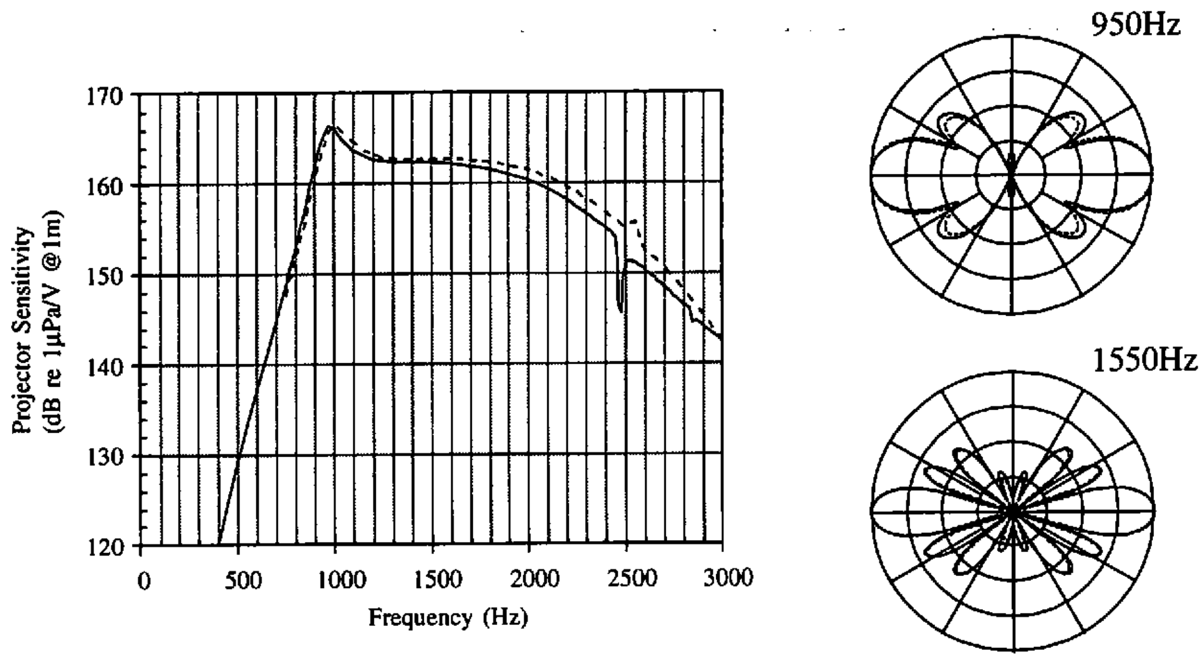

Figure 8: *Left: Projector Sensitivity for array of 4 Sparton FFR. MAVART data: solid line; PAFEC data: dashed line. Top Right: Directivity patterns at 950 Hz: MAVART data: solid line; PAFEC data: dashed line. Bottom Right: Directivity patterns at 1550Hz.*

7. COMPARING EXECUTION TIMES

One of the major considerations when using computationally intensive numerical techniques such as the finite element method is the amount of execution time. During this study we were unable to run the two programs on the same computer platform, so a precise comparison of execution times was not possible. As benchmarks for future work, we can state that for a model of a single ring, using PAFEC in three dimensions with a total of 541 DOF, it takes 21 minutes for each frequency in addition to an initial startup time of 21 minutes on a SUN 4/260 with 32 MB RAM. The axisymmetric model in MAVART without boot and oil has 836 DOF and takes 18 seconds for each frequency. This figure includes the initial startup time which is distributed among all frequencies. MAVART was run on a Silicon Graphics Iris Elan workstation with 70MB of RAM, which is considered to be 20× faster than the SUN 4/260.

8. CONCLUSION

The performance of the axisymmetric finite element code MAVART and the 3D hybrid finite element/boundary element code PAFEC were compared by modelling a tangentially poled free-flooded ring projector built by Sparton of Canada. It has been shown that both models can predict the performance of the FFR quite well and that the differences in the results between the two models are small. This is in spite of having to model with MAVART the complex structure of the ring as a material with uniform properties.

There were features visible in the experimental results, such as the peak in the TVR at the ring resonance, that neither MAVART or PAFEC predicted. This could be due to the omission from both sets of models of some small details such as the mounting flanges. Further refinement of material properties, especially damping, may also resolve these discrepancies.

The simple model without waterproofing, agrees surprisingly well with measured data. The effect of adding the oil-filled boot is most pronounced in the admittance results. It appears that for a structure that is as symmetric as the Sparton FFR there is little advantage to using a 3D model.

9. REFERENCES

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