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MODELLING OF A STIFF PANEL BACKED BY A CLOSED CAVITY : A STUDY OF A PLATE RESONATOR

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INTRODUCTION

Machinery rooms are sometimes involved with high sound pressure levels at low frequencies in a narrow band of the frequency spectrum. In this case, acoustic resonators can be used to reduce the sound level in the room. If the available construction depth is small and the disturbing frequency very low, the best solution is certainly to use a type of resonator with a stiff panel backed by a closed cavity. We now propose a numerical method to qualify the acoustical performances of such a resonator. This method takes into account the most relevant design parameters. The comparison of numerical results to impedance measurements on real resonators shows good agreement.

THEORETICAL PROBLEM

For a given resonator,the best way to achieve an effective acoustic treatment of a room is to cover an area as large as possible and the parallelepipedic shape is the most convenient one.
As a matter of fact, surrounding influences a lot the behavior of resonators. Our model considers the case of a parallelepipedic plate resonator baffled in an infinite rigid perfectly reflecting plane.

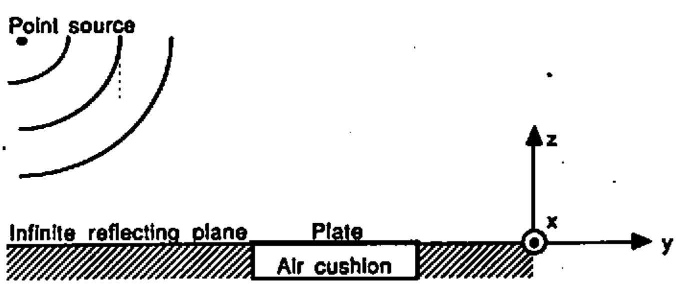

Figure 1: Model of a plate resonator baffled in an infinite reflecting plane

<u>Definition of the problem and subsequent choice of the suitable resolution techniques</u> :
The modelling of a plate resonator baffled in an infinite reflecting plane (figure 1) must simultaneously take into account:

- the acoustic wave impinging on the resonator plate,
- the vibro-acoustic coupling between the resonator plate and the semi-infinite upper region,
- the vibro-acoustic coupling between the resonator plate and the air cushion.

A pure tone analysis of this modelling problem is made because the frequency bandwith of interest is rather narrow. Therefore it is assumed that all the variables fields such as the acoustic pressure and velocity fields, p and V, as well as the plate velocity field are multiplied by $e^{-j\omega t}$, where ω denotes the excitation pulsation. The model must lead to the determination of both the pressure and velocity fields at the upper plate surface so that an estimate of the resulting absorption coefficient can be found. The Helmholtz problem in the air cushion is then discretized using a finite element method. The vibration of the resonator plate is also discretized using a finite element method for bending plate according to Kirchoff assumptions. The Helmholtz problem in the upper semi-infinite medium is solved using a boundary integral method.

Semi-infinite upper region :
The equation to be solved is the differential Helmholtz equation :

$$\Delta p + k^2 p = s \tag{1}$$

where p denotes the acoustic pressure field, k is the wave number equal to ω/c, c is the speed of sound and s is a sound source density. The following boundary conditions must be satisfied :

$$\lim_{r \to \infty} r\left(\frac{\partial p}{\partial r} + j k p\right) = 0$$

on a semi-sphere surrounding the semi infinite upper region;
$\frac{\partial p}{\partial n} = j\omega\rho V_n$ on the plate surface;
$\frac{\partial p}{\partial n} = 0$ on the infinite perfectly reflecting and rigid plane baffle.
r denotes a point on a semi-sphere surrounding the semi infinite upper region, n denotes the normal to the plane pointing downwards, $V_n = V \cdot n$ and ρ denotes the fluid density.

Plate :
The equation to be solved is the bending vibration equation :

$$\frac{Eh^3}{12(1-\nu^2)} \Delta\Delta V_n - M_s \omega^2 V_n = -j \omega f \tag{2}$$

Where E denotes the Young modulus, ν the Poisson's ratio, M_s the mass per unit area, h the thickness of the plate , f the load per unit area applied to the plate. The equation is to be solved subject to different boundary conditions such as free, simply supported or clamped edges.

Air cushion :
The equation to be solved in the cavity is the Helmholtz equation :

$$\Delta p + k^2 p = 0 \tag{3}$$

The equation must be solved according to the boundary conditions $\frac{\partial p}{\partial n} = j\omega\rho V_n$ on the lower face of the plate and $\frac{\partial p}{\partial n} = 0$ on any face of the cavity except the lower face of the plate.

DISCRETIZATION AND SOLUTION

Finite element method for the bending wave equation in the plate :
It is assumed that the plate used in real resonators is generally thin enough so that both the stress normal to the mid-surface and the shearing stresses can be neglected. The Kirchoff theory for plate bending is then used. The basic linear bidimensionnal finite element, the Adini's rectangle [4], is used. The normal velocity within an element $V_n^e(x,y)$ is expressed in terms of a 4 order polynomial :

$$V_n^e(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \dots + \alpha_{12} x y^3$$
$$= \sum_{i=1}^{12} U_i \, e_i(x,y)$$

where U denotes a vector of dimension 12 containing 4 nodal velocities, 4 derivatives of the velocities with respect to the x direction and 4 derivatives with respect to the y direction. The functions e_i are the basis functions of the finite element discretization. The principle of virtual powers leads to the resulting equation for the bending vibration of the plate:

$$[\,K - (\omega^2 + j\,\omega\,\eta)\,M\,]\;U = F\,(P^r_{(2)} - P_{(1)}) \qquad (4)$$

where K, M and F denote respectively the stiffness matrix, the mass matrix and the coupling matrix. The general elementary term of the coupling matrix is defined as follows:

$$F_{ij} = -j\,\omega \iint_{S_e} e_i(x,y)\; N_j(x,y)\; dx\; dy \qquad (5)$$

$P_{(1)}$ is the vector of the nodal pressures loading the plate on its upper face and $P^r_{(2)}$ is the restriction of the pressure vector in the air cushion to the nodal points on the lower face of the plate. N_j denote the finite element basis functions for the acoustic pressure. η accounts for the structural losses that take place in the plate as it vibrates. Its value which is usually frequency dependent can be determined experimentally by modal analysis and the classical -3 dB method.

Boundary integral method in the upper infinite region :
Let $g(r,r_o)$ be the generalized Green's function :

$$g(r,r_o) = \frac{e^{j\,k\,|r-r_o|}}{4\,\pi\,|r-r_o|} \qquad (6)$$

This generalized Green's function is a solution of the free field Helmholtz equation with a Dirac distribution source at r_o. r denotes the vector position of the observation point in the propagation medium Ω, or on its boundary $\partial\Omega$. The solution of equation (1) with the boundary conditions of the upper region can be derived from the Helmholtz-Kirchoff integral equation:

$$p(r) = -\iiint_{\Omega} s(r_o)\; G(r,r_o)\; dv_o + \iint_{\partial\Omega} [\; G(r,r_o)\,\frac{\partial p(r_o)}{\partial n} - p(r_o)\,\frac{\partial G(r,r_o)}{\partial n}\;]\; ds_o \qquad (7)$$

where $G(r,r_0)$ is an appropriate Green's function. In the case of a semi-infinite medium bounded by an infinite reflecting plane, the appropriate Green's function is:.

$$G(r,r_0) = g(r,r_0) + g(r,r'_0)$$

where r'_0 is the position vector of the image point source of r_0 with respect to the infinite reflecting plane. The derivative of $G(r,r_0)$ with respect to the normal direction to the reflecting plane cancels. The resulting integral equation expressed on the plate surface is:

$$p(r_p) = S(r_p) + 2j\omega\rho \iint_{\partial\Omega_p} g(r_p,r_o)\ V_n(r_o)\ ds_o \qquad (8)$$

where r_p denotes an observation point on the upper plate surface and $\partial\Omega_p$ is the upper plate surface. The sound source contribution is defined by :

$$S(r_p) = -\iiint_{\Omega} s(r_o)\ G(r_p,r_o)\ dv_o \qquad (9)$$

The discretization of equation (8) on the upper plate face leads to the following matrix equation:

$$P_{(1)} = S + R\ U \qquad (10)$$

where the components of vector $P_{(1)}$ are defined by the values of $p(r_p)$ on the discretization points of the plate mesh. R is a rectangular matrix whose general elementary term is defined as:

$$R_{ij} = 2j\omega\rho \iint_{S_o} g(r_i,r_o)\ e_j(r_o)\ ds_o \qquad (11)$$

The functions e_j are the plate basis functions defined above and vector U represents the nodal components of the normal velocity field at the plate surface. Vector S represents the contribution of the sound source s and its components are determined by the volume integral (9).

<u>Finite element method for the Helmholtz equation in the air cushion:</u>

We use here the classical finite element method to solve the Helmholtz equation [6]. An eight node isoparametric element is used, [2]. The acoustic pressure field can be defined in terms of basis functions of this element such as:

$$P_{(2)} = \sum_{i=1}^{8} P_{(2)i}\ N_i(x,y,z)$$

and the resulting discretized matrix equation is:

$$\begin{bmatrix} B^{rr} & B^{cr} \\ B^{rc} & B^{cc} \end{bmatrix} \begin{bmatrix} P^r_{(2)} \\ P^c_{(2)} \end{bmatrix} = \begin{bmatrix} H \\ 0 \end{bmatrix} U \qquad (12)$$

where matrix B represents the discretized differential Helmholtz operator. Its representation in four blocks allows one to separate the terms related to the lower plate face (superscript r) from the terms related to the cavity (subscript c). The rectangular matrix H which stands for the coupling between the air cushion and the vibrating plate, is defined by its general elementary term:

$$H_{ij} = j\omega\rho \iint_{S_o} N_i(x,y)\ e_j(x,y)\ dx\ dy \qquad (13)$$

MODELLING OF A STIFF PANEL BACKED BY A CLOSED CAVITY

This matrix is very similar to rectangular matrix $\mathbb{F}$ which stands for the coupling between the vibrating plate and the semi-infinite upper region.

Solving of the coupled problem :

The problem has been set into equations in each of the three regions of interest, namely the semi-infinite upper region, the plate and the cavity. The coupled problem can now be solved by gathering the three equations (4), (10) and (12) into a general system of equations. The inversion of this system provides the solution in terms of pressure vector $P_{(1)}$ at the upper plate face, nodal components of plate velocity vector U and pressure vector $P_{(2)}$ in the cavity. The contribution of each region in the global solving matrix appears in the following matrix equation :

$$\begin{bmatrix} \mathbb{I} & -\mathbb{R} & 0 & 0 \\ -\mathbb{F} & \mathbb{K}-(\omega^2+j\omega\eta)\,\mathbb{M} & \mathbb{F} & 0 \\ 0 & -\mathbb{H} & \mathbb{B}^{rr} & \mathbb{B}^{cr} \\ 0 & 0 & \mathbb{B}^{rc} & \mathbb{B}^{cc} \end{bmatrix} \begin{bmatrix} P_{(1)} \\ U \\ P^r_{(2)} \\ P^c_{(2)} \end{bmatrix} = \begin{bmatrix} S \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (14)$$

Where $\mathbb{I}$ denotes the identity matrix, $\mathbb{R}$ is the matrix which accounts for the coupling between the upper region and the plate, $\mathbb{F}$ is the plate load matrix, $\mathbb{K}$ is the plate stiffness matrix, $\mathbb{M}$ its mass matrix, $\mathbb{B}$ denotes the matrix describing the behavior of the cushion, $\mathbb{H}$ is the matrix accounting for the coupling between the cushion and the plate, U represents the nodal components of the plate velocity vector, $P_{(1)}$ is the nodal pressure vector on the upper face of the plate, $P^r_{(2)}$ is the restriction of the cavity nodal pressure vector on the lower face of the plate, $P^c_{(2)}$ stands for the nodal pressure vector within the cushion except on the plate face and S is the contribution of the harmonic source to the pressure field on the plate.

COMPARISON OF NUMERICAL AND MEASUREMENT RESULTS

Experiment strategy :

The aim of this experiment is to test various clamped plate resonators with a single design parameter varying in each configuration. The specific effect due to the change of the considered parameters can then be analysed on various measurement data such as the frequency dependance of acoustical impedance. Measurements are performed in two steps :

- first to evaluate the resonances of the clamped plate without the resonator air cushion,
- then to measure the resonances of the clamped plate backed by the airtight cushion so that the influence of the air backing is emphasized.

The measurement of the modes of the plate without the air cushion is made using a modal analysis technique. The measurement of the resonances of the plate-air cushion resonator is achieved by means of an original three microphone impedance measurement method [7]. The frequency dependance of the local complex impedance of the resonator with and without porous material partially filling the cavity has been evaluated. Various combinations are tested using five different plates made of aluminium, bakelite, plexiglass and plywood that could be clamped to three different cavities.

Comparison of computation and measurement results :

For each resonator, the specific impedance is measured in the frequency range [0-300 Hz] at different points on the plate. As expected, the sign of the imaginary part of the impedance changes from minus to plus whenever a resonator resonance is reached. At the resonance frequencies the imaginary part of the impedance is zero. They can therefore be located with accuracy. For all the resonators tested, several resonances are located in the frequency range [0-300 Hz]. The changes of the sign from plus to minus in the imaginary part of the impedance can be interpreted as the anti-resonances of the system.

MODELLING OF A STIFF PANEL BACKED BY A CLOSED CAVITY

Figure 2 displays the velocity frequency response obtained with modal analysis for a clamped aluminium plate (length 0.750 m, width 0.500 m, thickness 0.003 m) without any cushion backing it. The first measured resonance at 73.12 Hz is slightly lower than the first theoretical in vacuo resonance [3], 79.33 Hz, as expected because of the air-added mass effect. Figure 3 (a and b) presents the frequency curves of the specific impedances measured and computed at the centre of a resonator plate. For this resonator, the air cushion is 0.05 m deep and it does not contain any porous material. The analysis frequency range is [90-120 Hz] and the first resonance occurs at 103 Hz in the measurement and at 100 Hz in the numerical results.

CONCLUDING REMARKS

The main remark to be drawn from the overall study is illustrated by Figures 2 and 3. The presence of an air cushion backing a plate increases one in vacuo resonance frequency of the plate over four. The three other resonance frequencies which correspond to modal patterns with an even order in one principal direction at least remain untouched; for instance, the modal patterns (2,1), (2,2), and (1,2). The lower the in vacuo modal frequency, the greater is the ratio of the corresponding plate resonator frequency to the plate in vacuo frequency. In the case displayed in Figure 2 and 3, the ratio $\frac{f_r}{f_{(1,1)}}$ is equal to 1.41 using the measured results, and 1.38 with the calculated results.

A simplified formula for the prediction of the first resonant frequency of a plate resonator [5] is sometimes used:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{\rho\, c^2}{M_s d}}$$

where f_r denotes the first resonant frequency of the plate resonator, ρ denotes the air density, c the speed of sound in vacuo, M_s the mass of the plate per unit area and d the cavity depth. This formula is based on the simple mass-spring system where the plate acts as the mass and the cavity as the spring. It may lead to relative errors of more than 50 % in the prediction of the first resonance frequency of the resonator in some critical cases. Nevertheless the computations made with this formula and using the numerical model for an ideal resonator with a plate with free edges demonstrates that the simple formula is perfectly accurate in this unrealistic configuration. In any other case, the best way to predict the first resonance frequency of a plate resonator is to use a numerical model taking into account all the couplings between the acoustic media and the vibrating plate; futhermore, it allows one to accurately determine its overall absorption efficiency in terms of frequency.

<u>Practical design considerations :</u>

The design parameters to build a plate resonator are the following :
- the length and width of the plate
- the depth of the air cushion
- the boundary conditions at the edges of the plate
- the constitutive materials of the airbox and of the plate
- the type of porous material and the percentage of filling in the cushion

When one wants to tune a resonator at a given frequency f, the range of variation of these design parameters must be first bounded according to the following practical remarks :

- The cross dimensions of the plate : let λ be the wavelength $\lambda = \frac{c}{f}$. A typical rule is to take the cross dimensions less than $\frac{\lambda}{2}$. In order to avoid the network effects, it is also advisable that is easier to fill the room area to be treated. One also should minimize the number of units used for a given area so the cost is also minimized. The larger the surface of the plate, the lower is the cost of the treatment. All the other parameters being constant, the first resonance frequency of the plate without the air cushion tends to decrease as its surface grows. Beyond a certain value of this surface, the stiffness added by the cushion is not sufficient to shift the first resonance of the plate to f. It can be demonstrated that for a given surface, the first resonance frequency of a simply supported plate increases with the ratio of length to width.

For a given f, the choice of a length equal to two times the width can ease the construction of the resonator wall and reduce the number of units used and consequently the cost.

- Depth of the air cushion : let d be the depth of the air cushion. The value of d must be small enough so that the volume lost by the installation of the plate resonators remain small with regard to the amount of volume lost when using Helmholtz resonators.

- Boundary conditions at the plate edges : It is well known that the most efficient boundary conditions are those which approach the free conditions. A compromise should be found between the airtightness of the cushion and the freer displacement at the plate edges.

- Constitutive materials for the airbox and the plate : the airbox must 'trap' the sound and be as light as possible to ease the handling. For a given mass per unit area, the components of the airbox have their highest resonance frequencies when their material is such that $\sqrt{\frac{E}{\rho^3(1-\nu^2)}}$ is the greatest. The coupling with the airbox is then minimized. This is the best material for building an airbox of plate resonator. The plate is made of a material presenting great structural losses so that a good amount of energy is dissipated at the resonance.

- Porous material filling the cushion : the experiments showed that the presence of porous material in the air cushion decreases the plate resonator frequency of an average of 7% . It increases the pitch of absorption by increasing the real part of the specific impedance towards 1 at the resonance where the imaginary part is 0. It widens the range of absorption by decreasing the slope of the imaginary part versus frequency.

. Since the losses take place within the porous material by friction between air and fibers it is advisable to chose a dense porous material with a large friction area between air and solid. The presence of a 2.5 cm thick rockwool in a 5 cm deep cushion produces noticeable effects.

Parameters influencing the ratio $\frac{fr}{f(1,1)}$:

The use of the program for the case of simply supported plate (length 1m and width 0.5 m) provides the following information :

The ratio $\frac{fr}{f(1,1)}$ is an increasing function of the ratio $\frac{a}{d}$ where a denotes the length of the plate. It is a decreasing function of E, a decreasing function of the plate thickness h. Figure 4 presents the variation of $\frac{fr}{f(1,1)}$ as a function of $\frac{a}{d}$ for a simply supported plate (length=1m, width=0.5 m, thickness=0.005 m , plate density=900 kg/m^3, E=1.e10 N/m^2, ν = 0.35) . For $\frac{a}{d}$ in the range [1 - 10] the variation of $\frac{fr}{f(1,1)}$ is linear [9] but for stiffer cavities, i.e. d is small and for long plates, an asymptote seems to be reached.

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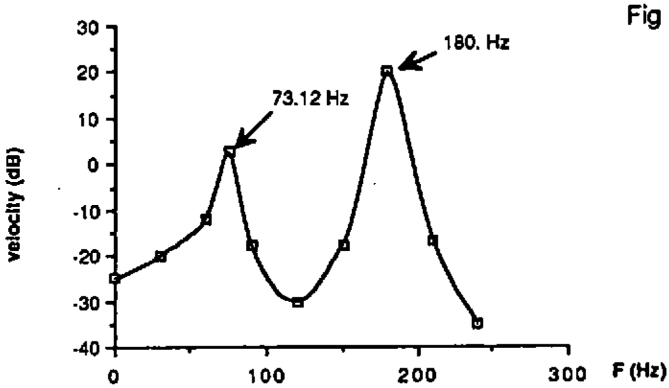
MODELLING OF A STIFF PANEL BACKED BY A CLOSED CAVITY

Fig 2.

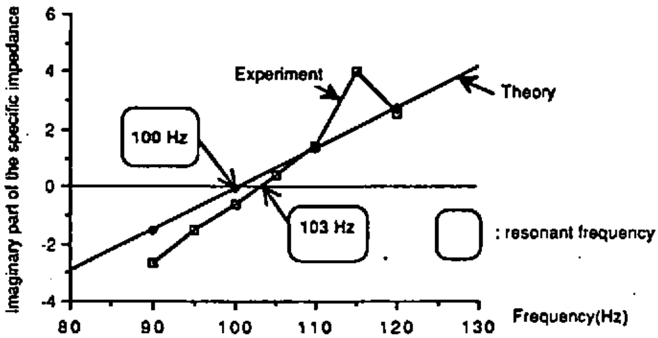


Experimental frequency response (velocity/force)

MODELLING OF A STIFF PANEL BACKED BY A CLOSED CAVITY

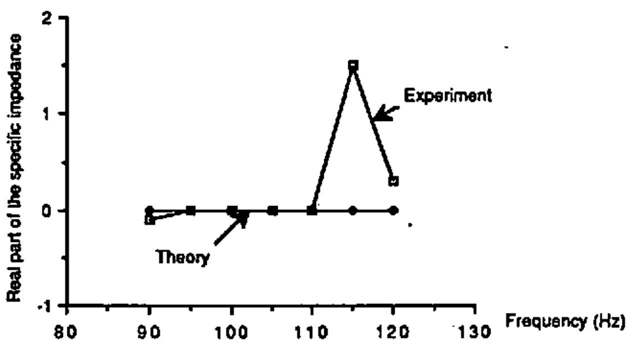
Comparison between theory and experiment

FIG 3.a



Experimental resonant frequency : 103 Hz theoretical resonant frequency : 100 Hz

Fig. 3b



Fr/F(1,1) as a function of a/d

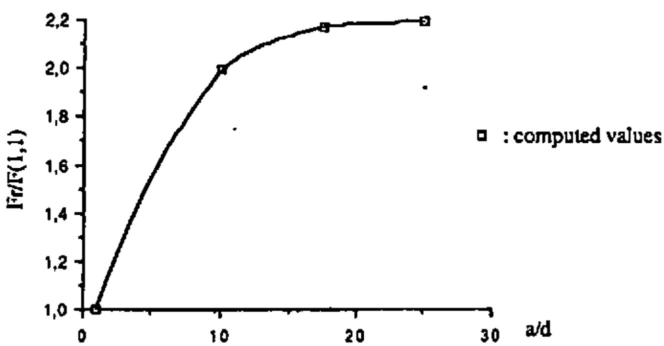


Figure 4