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MEASUREMENT OF INSTANTANEOUS AMPLITUDE DISTRIBUTIONS OF AUDIO FREQUENCY SIGNALS

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INTRODUCTION

Investigations into the long-time amplitude distributions of speech and music have been carried out previously by several workers [1-5] but much of the work has given rather imprecise results due to the difficulties in quantifying the measured distribution densities. Therefore, the aim of the present investigation has been to produce measurements which permit detailed statistical analysis. The system which has been developed enables signals such as speech or music to be analysed simply and accurately so that relatively small differences in particular statistical parameters can be resolved.

AMPLITUDE DISTRIBUTIONS AND STATISTICAL PARAMETERS

A truly random (noise) signal has a normal (Gaussian) distribution of instantaneous amplitudes and is therefore unbounded in range.

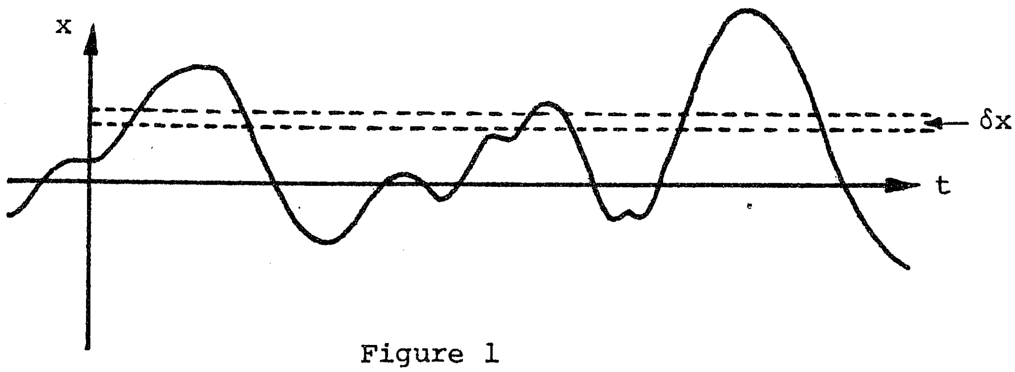

Figure 1

A continuous random signal such as that shown in Figure 1 has the probability density function p(x) associated with it. i.e. $p(x)\delta x$ is the probability that the signal occupies the range $x \rightarrow x + \delta x$ as δx tends to zero. The probability that the signal has a value below the given level 'a' (the cumulative distribution function) is denoted by P(a), while the probability that the signal has amplitude between levels 'a' and 'b' where b>a is given by P(b)-P(a), the amplitude distribution function. The amplitude probability density curve is obtained from a plot of p(x) versus x, while the plot of P(x) versus x gives the cumulative probability density (or the probability distribution curve).

It is usual to discuss the nature of signals in terms of their moments. For a continuous signal the simple or uncorrected r^{th} moment, μ'_r, is defined by

$$\mu'_r = \int_{-\infty}^{\infty} x^r p(x)\,dx \qquad (1)$$

Alternatively, a signal presented by a large number of samples of m discrete values, u'_r is defined by

$$\mu'_r = \sum_{n=1}^{m} x_n^r p(x)_n \delta x \qquad (2)$$

Thus the first uncorrected moment is simply the mean, μ.

The central or corrected moments are then defined by

$$\mu_r = \int_{-\infty}^{\infty} (x-\mu)^r \, p(x)\,dx \text{ for the continuous case.} \qquad (3)$$

or

$$\mu_r = \sum_{n=1}^{m} (x_n-\mu)^r \, p(x)_n \delta x \text{ for the discrete case.} \qquad (4)$$

When r=2 we have the second central moment normally referred to as the variance σ^2, where σ is the standard deviation or root mean square (rms) value. For distributions which are symmetrical about the mean level μ, all odd order central moments are zero. Thus the third order central moment is usually calculated to determine any possible asymmetry present in the distribution. Although higher odd order moments may also be used to describe this asymmetry, they are more sensitive to errors produced by random effects in the usually sparsely populated amplitude extremes.

Similarly, it is usually only necessary to go to the fourth central moment to identify the spread of the 'tail-ends' of an amplitude distribution or its departure from normality.

In order that comparisons may be made between the distributions of different signals, it is necessary to normalise the moments by a factor representing the overall magnitude of the signal. The factor chosen is the standard deviation of the signal, σ, raised to the power of the moment. This gives rise to the statistical parameters of skewness and kurtosis which are derived from the third and fourth central moments respectively.

ie. skewness = $\dfrac{\mu_3}{\sigma^3}$ and kurtosis $\dfrac{\mu_4}{\sigma^4}$

An alternative to the kurtosis measurement of signal 'dynamic range' is the peak to mean (rms) ratio or crest factor which gives a clearer description of the form of a signal than kurtosis. However its use is limited by the need for a strict definition of the peak value of a signal. Thus in the case of a random signal, for instance, the value of the crest factor is dependent on some arbitary definition of 'peak value'. The kurtosis is not limited in this way and so is more widely applicable.

The crest factor and kurtosis values of various different signals are shown in Table 1.

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Table 1

Signal	Crest Factor	Kurtosis
Square	1.0	1.0
Sine	$\sqrt{2}$ (1.414)	1.5
Triangular	$\sqrt{3}$ (1.732)	1.8
Random noise	strictly Indeterminate	3.0

DATA ACQUISITION SYSTEM

While it is relatively straightforward to capture digitally and process a sufficiently short-term signal, the problem becomes difficult for very long signal durations. Normally, hardware limitations necessitate the use of an alternative method for the measurement of probability density functions over long term intervals. A dedicated multi-channel amplitude analyser is usually required for this purpose. A variation on this approach has been made by using a multi-channel pulse-height analyser (PHA) normally employed for radio-active decay analysis. By means of an analogue gate coupled to the PHA, a relatively low sampling rate may be used, enabling signals of up to two or more hours duration to be analysed.

The system used is that shown in Figure 2.

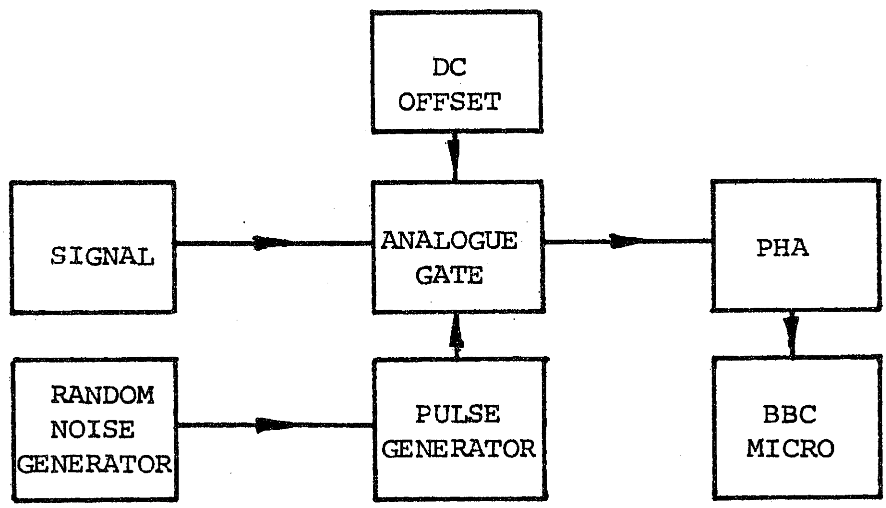

Figure 2

The analogue gate passes the input signal for the duration of an applied gating pulse. Thus a pulse amplitude modulated signal is fed to the PHA input. Since the analyser responds only to positive pulses, however, a d.c. offset has to be applied to the input signal in order to record a bipolar amplitude distribution. A resolution mode of 256 channels is used covering an amplitude range of 0-6 volts, each channel having a capacity of 13 x 10^6 counts, thus giving scope for long time measurements.

Although sampling at a relatively low rate, problems associated with periodic sampling do not arise for a truly random signal, provided that the 'sampling' is near instantaneous. Speech and music signals, however, can demonstrate some degree of periodicity in the short term, and this may lead to errors due to the sampling of the equivalent points in a periodic waveform. This potential problem is eliminated by using random sampling of the signal. This is achieved by using a white noise source to trigger the sampling pulse generator. In practice, the optimum pulse width has been found to be 1μs.

Once acquired, the data can be stored on floppy disc and processed by means of a microcomputer to give the required statistical parameters and plots of various curves.

CALIBRATION

Calibration of the system may be carried out using test signals with well defined statistical parameters. In particular a triangular input waveform has been used and the system adjusted until the flat amplitude density curve predicted theoretically for this waveform has been realised in practice. Due to the narrow pulse widths being used it has been found necessary to pay special attention to the matching of the impedance of the analogue gate circuit with that of the pulse height analyser. This calibration has been validated further by analysing sine waves and Gaussian signals whose theoretical statistical parameters are known precisely. Table 2 gives the expected parameter values for the particular waveforms used together with the measured values.

Table 2

Signal	Skewness measured	Kurtosis measured	Kurtosis theoretical
1kHz triangular	0.0003	1.800	1.800
20kHz triangular	0.003	1.807	1.800
40kHz triangular	0.0143	1.801	1.800
1kHz sinusoidal	0.003	1.500	1.500
20kHz sinusoidal	0.004	1.503	1.500
White noise	0.007	3.007	3.000

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PRESENTATION OF DATA

In addition to providing the values of the various statistical parameters previously mentioned, the system gives a graphical representation of the data. In the first instance, the information is displayed as the amplitude probability density curve. which is normalised in terms of σ to give unity area under the curve, i.e. $p\left(\frac{x}{\sigma}\right)$ is plotted against $\frac{x}{\sigma}$. This provides a good overall picture of the amplitude behaviour of the signal, and permits a useful comparison of data. For random signals, however, the nature of large amplitude signal peaks is usually of major interest so that a logarithmic probability axis is more suitable. Hence a plot of log $p\left(\frac{x}{\sigma}\right)$ (scaled in dB) against $\frac{x}{\sigma}$ is preferable. Both single-, and double-sided density curves may be produced, the latter being useful in demonstrating bipolar asymmetry in the data. In most cases, however, the single-sided distribution is more concise, this being produced by 'folding over' the distribution about its zero amplitude position, thus giving the modulus of the amplitude density function. Amplitude distribution curves i.e. $P\left(\frac{x}{\sigma}\right)$ against $\frac{x}{\sigma}$ are not considered to offer any significant advantages over the density curves as described.

SPEECH ANALYSIS

While the system has been designed primarily for the analysis of music, it may also be used for the analysis of other suitable long term signals such as speech.

To illustrate the latter use, the speech of each of six different subjects (three male and three female) was sampled over a measurement period of five minutes. The amplitude probability density curves obtained were all similar in form and the values for the various associated statistical parameters measured are summarised in Table 3 while typical curves are shown in Figures 3, 4 and 5.

Table 3

Subject	Skewness	Kurtosis	P(x) for $x>4\sigma$ in %	rms in dB SPL
A (female)	0.493	8.354	0.514	70.1
B (female)	0.713	8.257	0.491	68.5
C (female)	0.245	8.958	0.578	70.1
D (male)	0.544	7.418	0.441	70.8
E (male)	0.697	10.243	0.665	72.6
F (male)	0.413	10.063	0.631	74.7

From these curves it may be seen that the probability densities follow an exponential relationship for amplitudes greater than the rms value but for smaller amplitudes a 'peak' in the log $p\left(\frac{x}{\sigma}\right)$ curve is obtained. Similar results have been found in previous investigations [1, 2].

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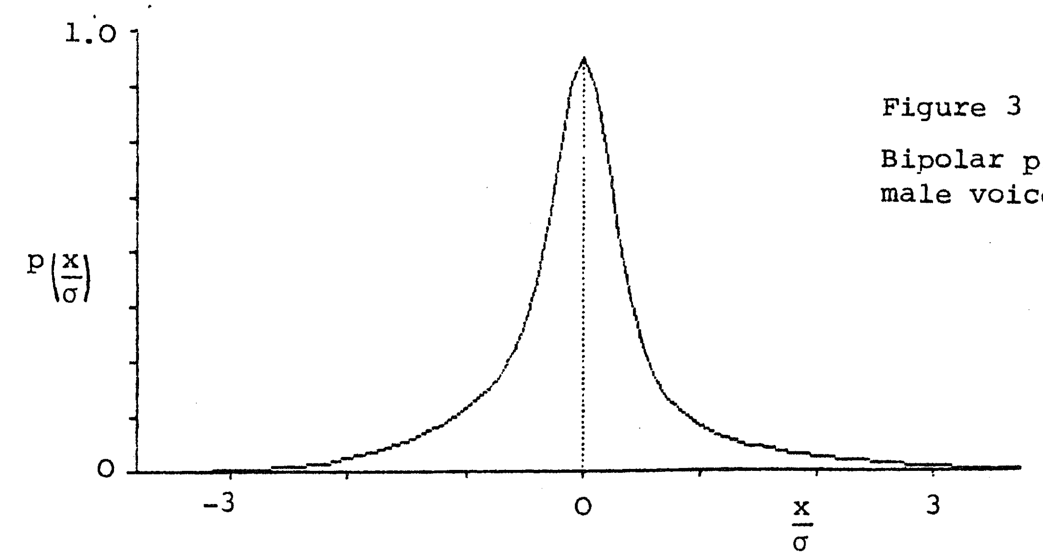

Figure 3

Bipolar plot of male voice E

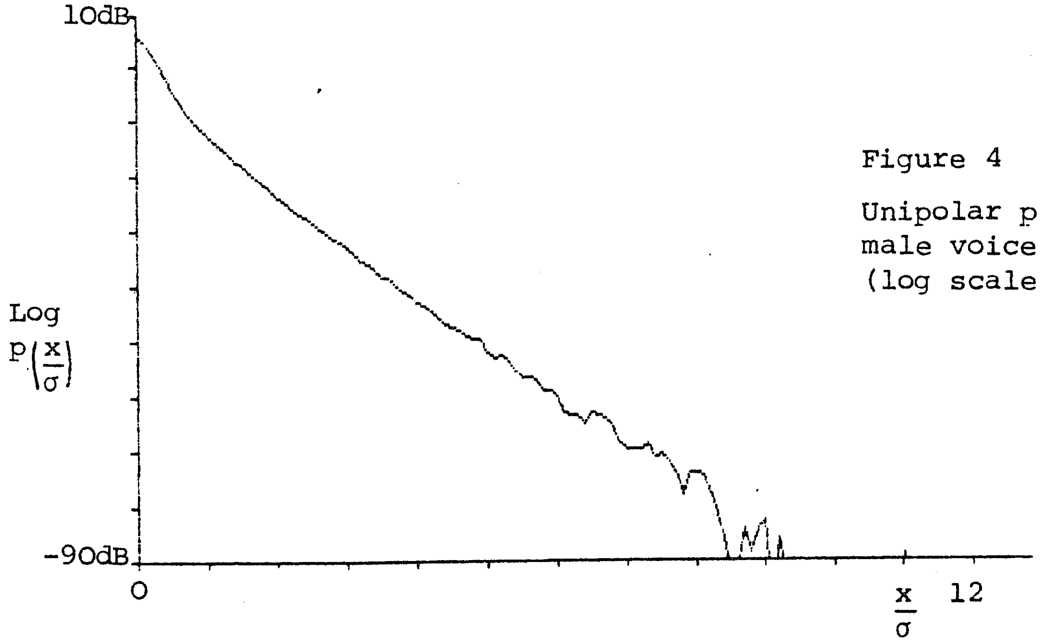

Figure 4

Unipolar plot of male voice E (log scale)

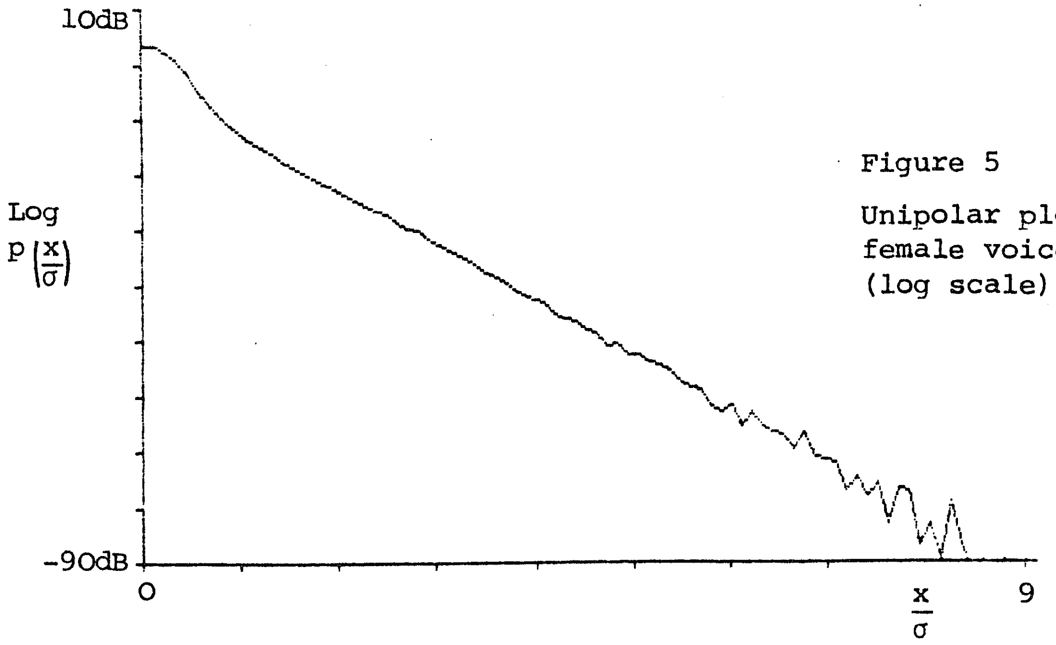

Figure 5

Unipolar plot of female voice C (log scale)

MEASUREMENT OF INSTANTANEOUS AMPLITUDE DISTRIBUTIONS OF AUDIO FREQUENCY SIGNALS

A figure of 12dB 'headroom' (a ratio of approximately 4:1) is often quoted as representing the crest factor for speech (for example, for hearing aid induction loop systems). Putting $\frac{x}{\sigma} = 4$ for speech, it may be seen qualitatively from Figure 3 that the probability of speech amplitudes exceeding this limit is very small. In fact, analysis of the data, for the six subjects tested, yielded a range of probabilities of exceeding this limit between 0.4% and 0.7%. Thus the results confirm that the 12dB headroom figure assumed for speech is quite justifiable in practice.

CONCLUSIONS

A system for the measurement of the instantaneous amplitude density of audio signals has been described. Results obtained using this system have shown excellent agreement with those theoretically predicted for various test signals. The statistical parameters of skewness and kurtosis have been shown to provide concise descriptions of measured random signals. To demonstrate possible applications of the system, the results of the analysis of several samples of speech have been presented.

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