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A TIME DOMAIN ANALYSIS OF PERFORATED TUBE EXHAUST SILENCERS

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INTRODUCTION

Silencers in the exhaust systems of internal combustion (I.C.) engines are broadly of two types, 'dissipative' and 'reactive'. Dissipative attenuators exhibit some degree of reactive behaviour, particularly at low frequencies, while reactive silencers can involve a substantial degree of acoustic energy dissipation by flow/acoustic interaction. The topic of interest in this paper is the acoustic modelling of reactive silencers involving perforated tubes.

Various workers have described theoretical models of reactive silencers, either with or without perforated tubes. In the work of Davis, Stokes, Moore and Stevens [1], for example, a variety of silencer configurations was examined, but the effects of mean gas flow and high amplitude sound were ignored. Later Alfredson and Davies [2,3] reported improved linearized models of expansion chamber silencers that allowed for the presence of gas flow, though perforated tube elements were not incorporated in the models. Sullivan [4,5] developed rather more comprehensive models that involved perforated tubes and allowed for possible gas flow through the perforations as well as grazing flow. Some consideration was given to high amplitude effects at the orifices in this work, though a combination of mean flow and nonlinear effects was not fully treated.

To the author's knowledge, the theoretical models of reactive silencers reported in the literature are exclusively in the frequency domain. Frequency domain modelling is quite in order if the dynamics of the systems involved are linear but in the case of nonlinear systems, it is difficult to devise satisfactory frequency domain models. In the case of perforated tube silencers, the orifices would exhibit a nonlinear hydrodynamic behaviour when exposed to typical sound pressures in I.C. engine exhaust systems. One cannot take account of this in a straightforward way by the use of frequency domain treatments, and an alternative approach is required; this need can be met by time domain modelling. In the present investigation, the quasi-one dimensional time-dependent finite difference solution reported by Cummings and Chang [6] in the case of a flow duct lined with nonlinear resonators is extended to the case of a perforated tube silencer.

THEORY

The geometry of a perforated tube silencer is shown in Figure 1; the perforations in the central tube are taken to be cylindrical. Mean flow is assumed to exist only in region 2, and an arbitrary transverse velocity profile $\overline{V}_x(y,z)$ is allowed (y and z being transverse coordinates, and the overbar denoting a time average). A locally space-averaged acoustic volume flow per unit length of tube, $\langle Q_w' \rangle$, may be defined, representing the

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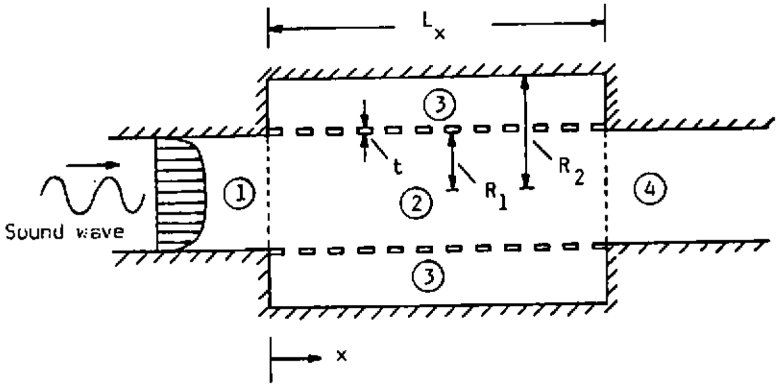

Figure 1 Geometry of a perforated tube silencer

fluctuating incompressible volume flow outward from region 2 (or inward into region 3), through the perforations in the pipe wall. The orifice flow velocity $\langle V_h' \rangle$ (averaged across the orifice area) may be related to $\langle Q_w' \rangle$ via the product of the area porosity σ and the perimeter L of the central tube,

$$\langle Q_w' \rangle = \sigma L \langle V_h' \rangle . \tag{1}$$

Cummings and Chang [6] have utilized a quasi-one dimensional linearized continuity equation to describe the acoustic perturbations in a perforated pipe at low frequencies and this may be written in a form applicable to region 2 as follows:

$$\partial \rho'/\partial t + \langle \overline{V}_x \rangle_A \, \partial \rho'/\partial x + \overline{\rho} \partial V_x'/\partial x + \overline{\rho} \langle Q_w' \rangle / A = 0 \quad , \tag{2}$$

where ρ represents fluid density, V_x is the axial velocity component, the prime denotes a perturbation, the overbar a time-averaged quantity, A is the cross-sectional area and $\langle \ \rangle_A$ signifies an average over A. Equation (2) applies to region 3 if the sign of the last term is made negative. In this case, of course, $\langle \overline{V}_x \rangle_A = 0$.

A quasi-one dimensional linearized Euler equation,

$$\partial V_x'/\partial t + \langle \overline{V}_x \rangle_A \, \partial V_x'/\partial x + (1/\overline{\rho}) \partial p'/\partial x = 0 \; , \tag{3}$$

also applies [6], and here p' is the sound pressure. If the acoustic process is taken to be isentropic, $p' = \rho' c^2$ (c being the adiabatic sound speed) and equations (2) and (3) may be combined to yield the quasi-one dimensional linearized wave equation for regions 2 and 3,

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$$(1 - \langle M_K\rangle_A{}^2)\partial^2 p_K'/\partial x^2 - (2\langle M_K\rangle_A/c)\partial^2 p_K'/\partial x\partial t - (1/c^2)\partial^2 p_K'/\partial t^2 = (-1)^K(\bar{\rho}/A_K)(\partial/\partial t + c\langle M_K\rangle_A\partial/\partial x)\langle Q_w'\rangle , \quad (4)$$

where K = 2 or 3 (signifying regions 2 and 3) and $\langle M_K\rangle_A$ is the average mean flow Mach number over the duct cross-section (this being zero in region 3). It is assumed that $\bar{\rho}$ and c are the same in regions 2 and 3. Areas A_2 and A_3 are respectively equal to $\pi R_1{}^2$ and $\pi[R_2{}^2 - (R_1 + t)^2]$.

Since the numerical procedure used in the solution of the governing differential equations has both the x coordinate and time as independent variables, it is necessary to specify both appropriate spatial boundary conditions and initial conditions in the time domain, when the acoustic signal is 'switched on'. These are as follows.

Boundary condition at the inlet

Elementary plane wave theory may be used to equate sound pressures and particle velocities on both sides of the inlet plane and yields the relationship

$$p_2' = 2p_{1i}' - \bar{\rho}cV_{x2}' \quad \text{at } x = 0 , \quad (5)$$

where p_{1i}' is the incident sound pressure at the inlet plane in region 1, p_2' is the sound pressure in region 2 at the inlet plane and V_{x2}' is the particle velocity at the inlet plane in region 2. Equations (1), (2), (3) and (5) may be combined to give

$$\partial p_2'/\partial t = [2/(1 + \langle M_2\rangle_A)]\partial p_{1i}'/\partial t + c(1 - \langle M_2\rangle_A)\,\partial p_2'/\partial x - \bar{\rho}c^2[\langle M_2\rangle_A/(1 + \langle M_2\rangle_A)]\sigma L\langle V_h'\rangle/A_2 \quad (6)$$

where, a priori, σL is the product of area porosity and perimeter of the perforated tube. Equation (6) is used in the numerical scheme to match the sound fields in regions 1 and 2.

Boundary condition at the outlet

For the purposes of transmission loss computation, an anechoic termination is chosen here and the dimensionless specific acoustic impedance in the exit plane is given by

$$\zeta_e = (1/\bar{\rho}c)(p_2'/V_{x2}')_{x=L_x} = 1. \quad (7)$$

The numerical scheme requires an expression for $\partial p_2'/\partial x$ that does not involve V_{x2}'. Provided ζ_e is not a function of time (which is certainly the case for an anechoic termination), equations (2), (3) and (7) may be combined to give

$$\partial p_2'/\partial x = -(1/\zeta_e - \langle M_2\rangle_A)(\partial p_2'/\partial t)/c(1 - \langle M_2\rangle_A{}^2) + \langle M_2\rangle_A \sigma L\bar{\rho}c\langle V_h'\rangle/A_2(1 - \langle M_2\rangle_A{}^2). \qquad (8)$$

Values of $\langle V_h'\rangle$ must, of course, be available, and these are obtained from the solution to the differential equation governing the orifice hydrodynamics.

Boundary conditions in region 3

Because there are rigid walls normal to the x axis at $x = 0$ and $x = L_x$ in region 3, the boundary conditions to be imposed in this region are simply

$$\partial p_3'/\partial x = 0 \qquad \text{at } x = 0 \text{ and } x = L_x. \qquad (9)$$

Initial conditions

To comply with the causality condition, a state of quiescence is imposed in regions 2 and 3, prior to the arrival of the acoustic signal at the inlet plane. In this, acoustic pressures and particle velocities are taken to be zero.

The response of the hydrodynamic flow - in the neighbourhood of the orifices - to the pressure differential that forces the flow must be found and it will be treated here in essentially the same way as that reported by Cummings and Chang [6]. A differential equation is written, relating $\langle V_h'\rangle$ to $p_2' - p_3'$, the radial pressure differential:

$$\bar{\rho}\ell\, d\langle V_h'\rangle/dt + r\langle V_h'\rangle = p_2' - p_3'. \qquad (10)$$

The effective length ℓ of the orifice is taken to be a function of the friction velocity in the turbulent pipe flow, the fundamental signal frequency and orifice dimensions as described by Cummings [7, 8]. The orifice resistance r is based on the steady flow orifice resistance data of Rogers and Hersh [9] at sufficiently high orifice velocities and on the low amplitude resistance data of Cummings [8] at lower velocities (see reference [7] for a discussion of how the two sets of data were applied). Because of the relatively wide spacing of the orifices, it was not necessary in this investigation to take account of inter-orifice interaction, though this could readily have been allowed for, for instance by the use of the data of Hersh, Walker and Bucka [10].

NUMERICAL SOLUTION TO THE GOVERNING EQUATIONS

The acoustic wave equations in regions 2 and 3 were written in finite-difference form and solved, subject to the appropriate initial conditions and boundary conditions, by much the same method as that employed by Cummings and Chang [6]. The orifice equation, (10), was solved by using the fourth-order Runge-Kutta method as the solution to the wave equation proceeded. The incident sound pressure p'_{1i} has, of course, to be specified and a complex periodic time-dependence was chosen. In all the numerical computations that were carried out, suitable tests were implemented to ensure the stability and convergence of the solution scheme. The spectra of the incident and transmitted pressure signals were found by using standard integral relationships for Fourier coefficients.

A variant of the expansion chamber silencer has extended inlet and outlet pipes: that is, these pipes protrude into the chamber, beyond the end-plates of the chamber itself. Alfredson and Davies [3] have analysed this type of muffler in the frequency domain. The advantages of this configuration is that the annular cavities between the end-plates and the ends of the inlet and outlet pipes act as quarter-wave filters and their lengths may be chosen so as to increase the transmission loss of the silencer selectively at frequencies where troublesome exhaust noise components exist. In the case of perforated tube mufflers, the extended inlet and outlet arrangement may be realized by perforating the central tube only over the middle part of its length. The lengths of the extended portions of the inlet and outlet pipes are denoted L_i and L_o respectively. It is straightforward to modify the present numerical scheme to incorporate extended inlet and outlet pipes. This is done simply by putting the orifice velocities equal to zero over the lengths of the extended pipe sections.

EXPERIMENTAL TESTS

The sound transmission loss (TL) of an experimental silencer was measured in a circular flow duct of diameter 26.6 mm. Bursts of a periodic signal were generated by a high-power sound source, and the maximum sound pressure attainable was about 5000 Pa (168 dB). Because signal bursts were used, the incident and transmitted pressures could be independently determined without the use of an anechoic duct termination. The incident pressure-time history was measured in the flow duct without the silencer present, and then the silencer was fitted and the transmitted pressure was recorded.

The waveform of p'_{1i} was adjusted so that it resembled a train of triangular pulses similar to the exhaust pressure signature of a typical I.C. engine. The frequency of the signal was varied from 112 Hz to 178 Hz.

The dimensions of the test silencer were as follows: L_x = 254 mm, R_1 = 13.3 mm, R_2 = 31.8 mm, t = 3.4 mm. The central perforated tube was drilled with 120 holes of 3.18 mm diameter, in eight staggered rows around the circumference; the area porosity was 3.97%. Extended inlet and outlet pipes pipes were achieved by filling some of the holes near the ends of the perforated tube with epoxy adhesive. The lengths of the extended inlet and outlet used in the experiments were: L_i = 104 mm, L_o = 59 mm.

NUMERICAL RESULTS COMPARED TO EXPERIMENTAL DATA

Comparison was made between the measured and predicted transmitted pressure signals, their spectra and also measured and predicted transmission losses calculated on the basis of the incident and transmitted spectra, as

$$TL = 20\log(C_n^{\,i}/C_n^{\,t}) ,$$

$C_n^{\,i}$ and $C_n^{\,t}$ being the amplitudes of the nth Fourier components in the incident and transmitted pressure spectra respectively. Because of the non-linear response of the silencer and the discrete nature of the incident and transmitted spectra, it was not possible to compute the TL at frequencies other than those of the Fourier components of the incident and transmitted signals. This fact is, of course, of no practical consequence since these other frequencies are absent from the transmitted signal and would therefore be of no interest.

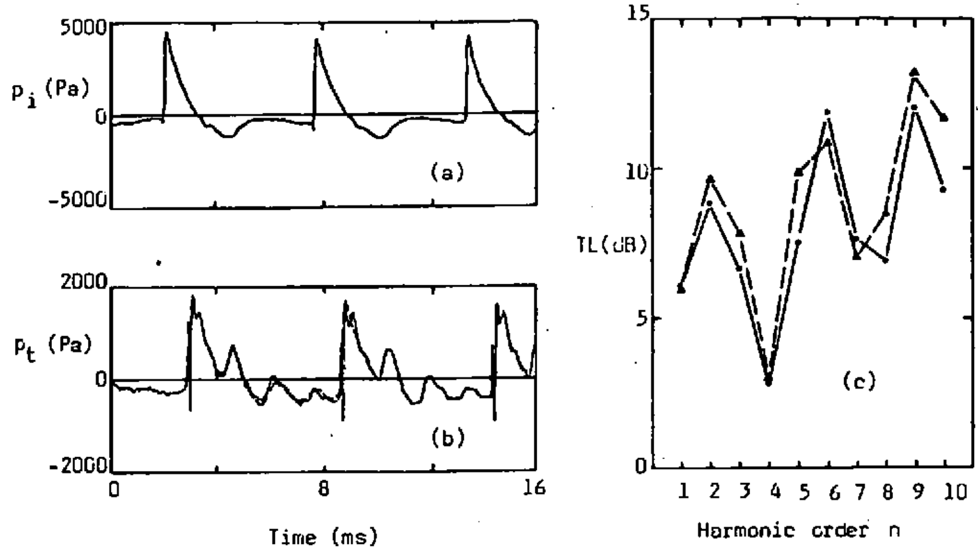

Figure 2 Incident and transmitted pressures (a,b), TL (c) in experimental silencer with $\langle M_2\rangle_A = 0.193$, fundamental frequency = 178 Hz, $L_i = L_o = 0$; ———, measured data; — — —, predicted data.

Figures 2 (a-c) show comparison between prediction and measurement for an intense periodic signal with a peak level of 167 dB, a fundamental frequency of 178 Hz, and $\langle \overline{V}_x\rangle_A$ = 66.7 m/s ($\langle M_2\rangle_A$ = 0.193). The full length of perforated tube was incorporated here, with $L_i = L_o = 0$. In Figure 2(a), the time history of the incident sound pressure p_i is shown and in Figure 2(b), comparison is made between the predicted and measured transmitted pressure p_t. Excellent qualitative and quantitative agreement is observed between the predicted and measured transmitted pressure. The most striking feature of the p_t curves is the multiple reflections of the main pressure peak. These occur as the incident pressure pulse reflects from one end of the chamber to the other, losing energy upon each reflection. The interval between successive reflections is about 1.5 ms, corresponding to the time taken for the wave to traverse the chamber twice. The TL, shown in Figure 2(c), is similarly well predicted, the maximum discrepancy between numerical and measured data being just over 2 dB. The mean flow speed at which the data shown in Figure 2 were taken was relatively high. By contrast, TL data were taken for a similar waveform, at a similar peak pressure and a slightly lower fundamental frequency (151 Hz), but with a much lower mean flow Mach number of $\langle M_2 \rangle_A$ = 0.018. Figure 3 shows these data, together with corresponding numerical predictions of the TL. It can be seen that the TL is predicted to within 2 dB by the numerical scheme. Because the mean flow Mach number of the data in

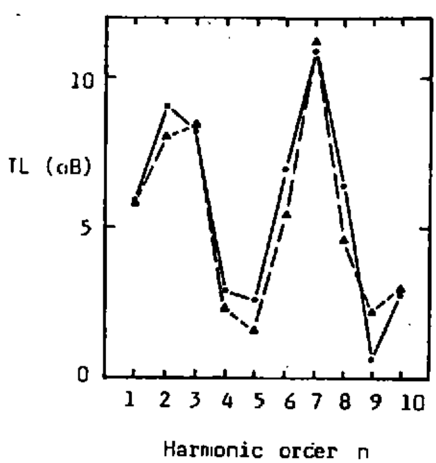

Figure 3 TL of experimental silencer with $\langle M_2 \rangle_A = 0.018$, fundamental frequency = 151 Hz, $L_i = L_o = 0$; ———, measured data; — — —, predicted data.

Figure 3 is so low, and the peak pressure in the incident wave is high, one might anticipate a nonlinear behaviour of the silencer. A numerical study was carried out to assess nonlinear effects on the TL; in this, the incident wave was assumed to have the same form as that in Figure 2 (though with a fundamental frequency of 115 Hz), but the peak level was varied. The mean flow Mach number was 0.02. Figure 4 shows the results. Peak levels of 147, 167 and 187 dB were chosen and one can see that indeed the TL does change with the peak level in the incident wave. Apart from a general increase in TL with increasing peak level, there is also a change in the shape of the TL curve, particularly at a level of 187 dB. This is thought to be caused partly by

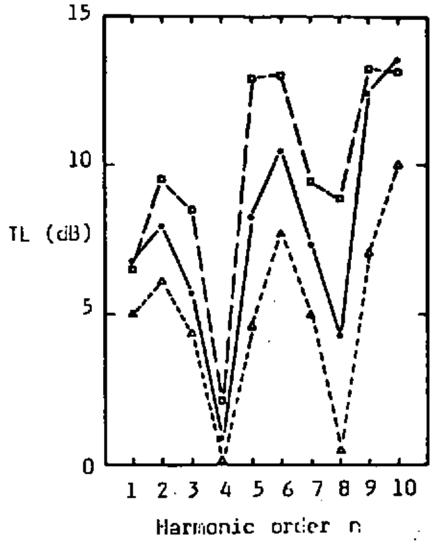

Figure 4 The predicted effect of differing peak incident sound pressure level (L_{pi}) on TL of experimental silencer with $\langle M_2 \rangle_A$ = 0.02, fundamental frequency = 115 Hz, incident waveform that of Figure 2 (a), $L_i = L_o = 0$, △– – –△, L_{pi} = 147 dB; ●———●, L_{pi} = 167 dB; □— —□, L_{pi} = 187 dB.

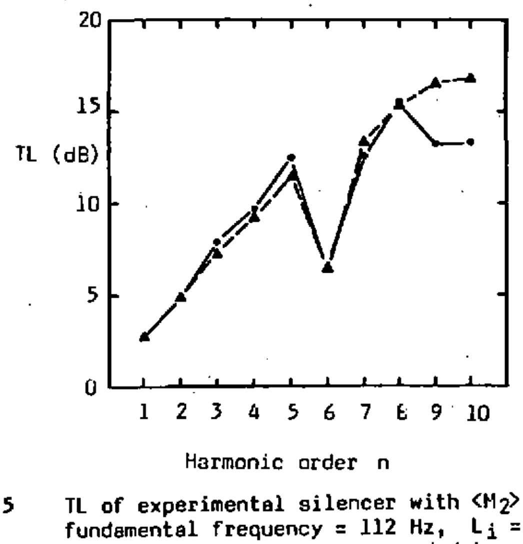

Figure 5 TL of experimental silencer with $\langle M_2 \rangle_A$ = 0.197, fundamental frequency = 112 Hz, L_i = 104 mm, L_o = 59 mm; ———, measured data; — — —, predicted data.

nonlinear interharmonic interaction; Cummings [11] also noted that nonlinear interaction between the harmonics of an intense complex periodic signal, upon transmission through an orifice plate, was significant. Computations for levels of less than 147 dB revealed little change in the TL with level.

Tests were carried out on the silencer with extended inlet and outlet pipes, and TL data for $\langle M_2 \rangle_A = 0.197$ and a fundamental frequency of 112 Hz are shown in Figure 5. The numerically predicted TL is in excellent agreement with the measured data up to $n = 8$, and agreement is within 3 dB for $n = 9$ and 10. If one compares these curves to corresponding TL data for the silencer without extended inlet and outlet pipes (these data are not shown here), the shape of the TL curves may be seen to be drastically altered by the presence of the inlet and outlet extensions.

COMPARISON BETWEEN TIME DOMAIN AND FREQUENCY DOMAIN ANALYSES

Although the time domain analysis described in this paper has given results that are in good agreement with experiment, it is not obvious whether this method has any significant advantage over the more commonly-used frequency-domain analyses.

Comparison were made, in this work, between time-domain predictions and corresponding frequency-domain results. It was found (as, perhaps, one would expect) that in cases where nonlinear effects should be noticeable (at high sound pressures and with relatively low flow speeds), the time domain analysis gave substantially better TL predictions than the frequency domain method. On the other hand, at moderate sound pressures and higher flow speeds, there was much less difference between the two approaches.

CONCLUSIONS

It has been demonstrated that a quasi-one dimensional time domain solution for the TL of a perforated tube silencer gives excellent results in comparison to experimental data. Moreover, it has also been shown that this method can give significantly better predictions than those from a frequency domain model. This would, perhaps, be expected since the time domain solution is much more satisfactory inasmuch as it can incorporate details of the orifice hydrodynamics that cannot be allowed for in a frequency domain solution. But it is important to know whether it is worthwhile to involve the additional computation effort inherent in a time domain solution. It would seem from the results presented here that this effort would, in general, be justified. Clearly, the quasi-one dimensional treatment used here has an inherent upper frequency limit of validity imposed by the propoagation of higher order radial modes in the silencer chamber. This is not considered to be a serious restriction in practical terms, since the frequency range of a one dimensional analysis would normally encompass that part of the noise spectrum which is of major interest from the subjective point of view. And the upper frequency limit of this part of the spectrum would tend to scale roughly in inverse proportion to the size of the I.C. engine and hence the diameter of the silencer chamber.

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