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LOW FREQUENCY ACOUSTIC TRANSMISSION THROUGH THE WALLS OF RECTANGULAR DUCTS.

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Introduction

Noise "breakout" through the walls of rectangular air-conditioning ducts is sometimes a problem in acoustical design work: fan noise can be transmitted to spaces not actually served by air ducts, as the duct walls are caused to vibrate by internally-propagated sound.

This is principally a low-frequency problem, since a dissipative muffler is often used, near to the fan, to attenuate duct-borne noise, and although such silencers are effective at mid and high frequencies, the low-frequency performance is usually poor; additionally, fan noise usually contains most of its energy at low frequencies.

What appears to be the only available method of estimating breakout from rectangular ducts is a simplistic formula quoted by Allen [1], and this is, in any case, not valid at low frequencies.

The purpose of the present work is to produce a simple, but sufficiently accurate, theoretical model which can be used for design purposes to estimate breakout.

Theory

Consider an infinitely long rectangular duct with thin flexible walls, and a cartesian coordinate system with the x axis parallel to the duct's axis, and the y and z axes in the plane of the duct's cross-section. At low frequencies, a coupled structural/acoustic wave system can travel in the duct's walls and in the fluid in the duct. An appropriate acoustic wave equation in acoustic pressure is (for fairly small wall admittance)

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} + \frac{L\bar{\beta}}{cS}\frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial x^2} = 0, \qquad (1)$$

where L and S are the duct's perimeter and cross-sectional area respectively, c is the adiabatic speed of sound, and $\bar{\beta}$ is the normal wall admittance, averaged on the duct's perimeter. Equation (1) governs acoustic wave propagation inside the duct; the acoustic pressure is assumed uniform on the duct's cross-section. The axial wavenumber of the wave system is obtained from equation (1) as

$$k_x = \pm k(1 - iL\bar{\beta}/kS)^{\frac{1}{2}}, \text{ where } k = \omega/c,$$

ω = radian frequency, (for simple-harmonic time variation).

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The thin-plate structural wave equation may be solved exactly for a forcing pressure of the form $P\exp(-ik_x x)$, to a yield a solution for the normal wall displacement of the x-y walls,

$$\xi(x,y) = \left[A_1\cos(\alpha_1 y)+A_2\sin(\alpha_1 y)+A_3\cosh(\alpha_2 y)+A_4\sinh(\alpha_2 y)\right]e^{-ik_x x}+\frac{kPe^{-ik_x x}}{k_x^4-\gamma^4}, \quad (2)$$

and a similar expression for the x-z walls. The coefficients A_1 to A_4 are found from the boundary conditions, and $K=12(1-\sigma^2)/E h^3$, σ = Poisson's ratio, E=Young's modulus of plate, h=plate thickness, $\gamma^4=\omega^2 mK$, m=mass/unit area of plate, $\alpha_1=\sqrt{\gamma^2-k_x^2}$, $\alpha_2=\sqrt{\gamma^2+k_x^2}$.

The boundary conditions on the walls are that the normal displacement is zero at the corners, the corners remain right angled during an oscillation cycle, and the slope of the walls at their centres is always zero. Equations (1) and (2) may now be solved together to determine the wall admittance over the duct's cross-section; a simplifying assumption is to neglect the external radiation load on the duct walls in computing the wall response to the internal acoustic pressure.

The wall transmission loss (TL) - a quantitative measure of breakout - may be found once the wall admittance is known. The TL is defined thus:

$$TL = 10\log(W_{int}/W_{rad}), \quad (3)$$

where W_{int} is the in-duct sound power and W_{rad} is the radiated sound power per unit length of duct. The TL is calculated as follows:

$$TL = 10\log(abc/C_r k\, c_x |a\bar{\beta}_y+b\bar{\beta}_z|^2), \quad (4)$$

where $c_x=\omega/k_x$, a and b are the y and z duct dimensions, $\bar{\beta}_y$ and $\bar{\beta}_z$ are the average admittances on the x-y and x-z walls, and C_r is a "radiation efficiency" factor, representing the ratio between the sound power radiated per unit length from a finite radiating length of duct, with arbitrary axial wave speed, and the sound power per unit length radiated from an infinite duct with supersonic wave speed. Values of this quantity are given by Brown and Rennison [2], for a simple finite-length line source model incorporating a single travelling wave. This should approximate to the present system provided the acoustic wavelength is sufficiently greater than the transverse dimensions of the ducts.

Comparison between Theory and Measurements

Various measurements were made on two duct systems, for the purpose of comparison with predictions, made on the basis of the above simple model. The duct systems were: first, a square section duct, with a loudspeaker source at one end to generate internal acoustic waves (which would quickly establish a coupled wave system), and a (more or less) "anechoic" termination for both acoustic and structural waves, consisting of a large thickness of rock wool, with a wedge-shaped end to it; and secondly, a rectangular section duct, again furnished with a loudspeaker as an acoustic sources, but this time with a reflecting termination. The square duct had butt-welded seams which were beaten flat, to minimise structural discontinuities, whereas the rectangular duct had a lapped, pop-riveted seam along the length of one wall. The first duct system was intended to realise the theoretical assumptions, as nearly as possible, in practice. The second was intended as a severe test of the theory, since strong structural and acoustic reflected waves would be set up, and one of the duct's walls had a seam running along it (this arrangement is common in practical air-conditioning ducts).

Measurements of the transverse displacement profiles on the walls of both ducts were made at various frequencies and found to be in reasonable agreement with calculations. The axial phase speed of the wave in the square duct was measured as a function of frequency, and good agreement was found between measurements and predictions.

The quantity of principal interest is the TL of the ducts. Figures 1 and 2 show calculated and measured results on the square duct and on the rectangular duct.

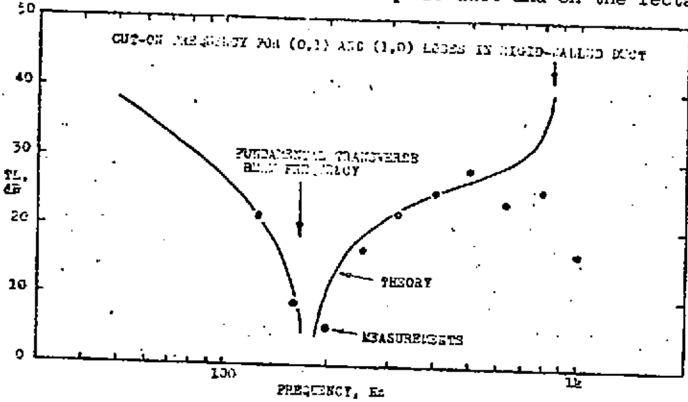

Figure 1

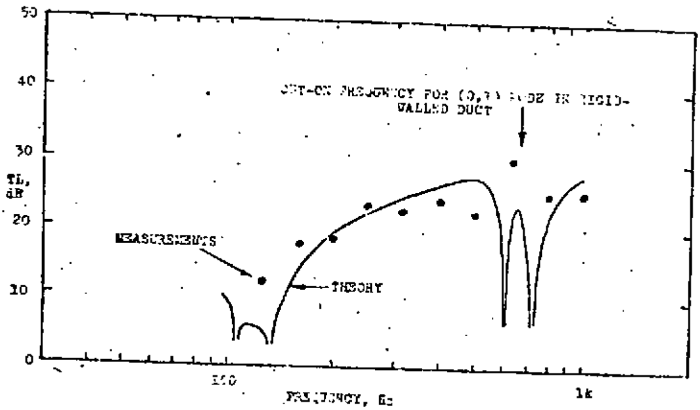

Figure 2

LOW FREQUENCY ACOUSTIC TRANSMISSION THROUGH THE WALLS OF RECTANGULAR DUCTS.

Agreement between the calculated and measured TL for the square duct is very good up to about $\frac{2}{3}$ of the cut-on frequency for the first cross-mode in the equivalent rigid-walled duct. The sharp minimum in TL at about 170 Hz is caused by a "transverse beam" type of resonance in the duct walls.

The results on the rectangular duct are surprisingly good (since the experimental system violated the theoretical assumptions), again up to about $\frac{2}{3}$ of the cut-on frequency for the first cross-mode; some discrepancy is evident at low frequencies.

Discussion

The simple theory given here for estimating low-frequency breakout appears to give satisfactory results in comparison with experiment, particularly in the case of an idealised experimental arrangement; even in a case where the theory would not be expected to be valid, reasonable results are obtained; one thus feels that the theory should be adequate for most design purposes.

References

1. C.H. Allen, Chapter 21, Noise Reduction (ed. L.L.Beranek), McGraw Hill, N.Y., 1960.
2. G.L. Brown and D.C. Dennison, 1974 Proceeding of the Noise, Shock and Vibration Conference, Monash University, Melborne, 416-425. Sound radiation from pipes excited by plane acoustic waves.