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# THE EFFECTS OF FLANKING TRANSMISSION ON SOUND ATTENUATION IN LINED DUCTS

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## 1. INTRODUCTION

Bulk-reacting, porous sound-absorbing material is commonly employed to reduce noise levels in air-moving ducts, and may be installed in the form of splitters or an internal wall liner. In the case of high-performance silencers, there is a possibility that flanking paths may exist, resulting in a loss of acoustic performance (see the paper by Mechel [1]). Vér [2] has also noted a limiting effect on attenuation in the case of a thin sheet metal duct with a 25 mm internal acoustic liner, as the length of the liner was progressively increased: the insertion loss exhibited a "levelling-off" beyond a certain length of liner. He "suspected that flanking through bending waves traveling in the duct wall may be partly responsible for this behavior", but did not elaborate on the details of the transmission mechanism, or mention the possibility of the breakout/breakin flanking path. Mechel [1] was aware of the possibility of flanking paths, in particular the "radiation bypass" (*i.e.* breakout/breakin) and "structure-borne sound" (transmitted by the silencer housing or the outer supporting structure of the internal baffles). The various national and international standards concerned with the laboratory measurement of silencer insertion loss make mention of flanking paths, though they are not generally discussed in great detail.

Engineers responsible for silencer design can either accept flanking as a factor limiting silencer performance, or attempt to reduce its severity. In the former case, some quantitative means of assessing its magnitude is desirable, and in the latter, a fairly thorough understanding of flanking transmission mechanisms is necessary. There appear to be no quantitative studies of flanking effects in silencers reported in the Literature, and there would seem therefore to be a need for - at least - preliminary investigations aimed at identifying flanking transmission mechanisms, formulating appropriate mathematical models with the object of quantifying the flanking transmission process, and carrying out experimental tests to validate the models. In the present investigation a simplified system, consisting of an internally lined, flexible-walled, duct passing through a reverberant chamber, was analysed with the object of predicting the axial sound pressure level distribution *via* quantification of the flanking transmission processes. Two mechanisms - structural wave transmission in the duct walls and acoustic breakout/breakin through the walls - are included in the model.

## 2. THEORY

In Figure 1, the arrangement of the duct and reverberant enclosure is shown.

The duct has a "bulk-reacting" porous, sound-absorbing liner. A length $L$ of the duct is exposed within the enclosure. Sound is generated by a point source on the axis of the duct in the plane where it enters the enclosure. It is assumed that anechoic conditions exist in both directions along the duct. The "breakout" of sound from the duct walls generates a reverberant sound field in the enclosure, and "breakin" from this surrounding sound field back into the duct may also occur. The two possible types of coupled wave combination - one in which most of the energy travels along the duct walls and the other in which most of the energy travels in the "airway" and the liner - are allowed for in this analysis. There are four stages in the theoretical development: (i) calculation of the coupled eigenmodes in the duct; (ii) construction of a modal solution to represent the sound field from a point source in the duct, in the absence of breakin; (iii) calculation of the sound power radiated by the duct into the enclosure; (iv) calculation of the sound pressure level within the duct, caused by breakin, and subsequent estimation of the total sound pressure on the duct axis. The principle of reciprocity is an essential part of stage (iv). Since the analysis is fairly lengthy, only the salient points will be outlined here.

### 2.1 The Coupled Eigenproblem in the Duct

Coupled eigenmodes are sought for the acoustic pressure in the duct and the wall displacement; the common axial wavenumber is $\alpha$. The exterior acoustic radiation load on the duct walls is ignored here, since it will be negligibly small as compared to the acoustic losses occurring in the sound-absorbing liner. We utilise a variational treatment to find $\alpha$ (rather than the alternative full finite element method), in keeping with other approximations that are made in this analysis. The variational functional used involves both the sound pressure in the airway and liner, and the structural displacement of the walls; the Euler equations arising from it are the governing acoustic wave equations, the structural equation of motion and appropriate natural boundary conditions; the anisotropy of the acoustic absorbent is allowed for in the formulation, and a structural loss factor is also included to account for damping in the walls. The functional differs only slightly from that used in a previous finite element analysis of ducts with bulk-reacting isotropic liners [3] and is similar to that used in the analysis of acoustic breakout from rectangular ducts [4]. The trial functions adopted here are illustrated in Figure 2. The sound pressure in the airway is given a doubly parabolic shape, and that in the liner is allowed to vary linearly across the thickness. Cubic expressions are used for the trial functions for the wall displacements. There are altogether six degrees of freedom: three for the sound pressure and three for the wall displacement.

A coupled algebraic eigenproblem results from minimisation of the functional $F$ with respect to the acoustic and structural coefficients in the trial functions. Minimisation of $F$ w.r.t. the acoustic coefficients yields

$$[\mathbf{E} + \alpha^2\mathbf{G}]\mathbf{p} - \mathbf{T}\mathbf{w} = 0 \qquad (1)$$

and minimisation of $F$ w.r.t. the structural coefficients gives

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$$[\mathbf{A} + \alpha^2\mathbf{B} + \alpha^4\mathbf{C}]\mathbf{w} - \mathbf{T}^T\mathbf{p} = 0. \tag{2}$$

Here, **A**, **B**, **C**, **E**, **G** and T are 3×3 matrices and **p**, **w** are column vectors containing the coefficients of sound pressure $p$ and normal wall displacement $w$ in the trial functions. Matrices **A**-**G** involve integrals relating to the trial functions and the "system parameters", while **T** is a coupling matrix. Equations (1,2) pose a coupled eigenproblem in α. Removal of **T** yields the two uncoupled problems. Two types of coupled mode are possible: in the first, most of the energy flow occurs in the fluid, and in the second, the power flow is predominantly in the duct wall. Two separate solution algorithms for equations (1,2) were developed, one for each type of mode. These both involve iterative methods and proved robust while requiring minimal computational effort.

**2.2 The Sound Field Radiated by a Point Source in the Airway**
The next stage in the theoretical development involves finding an expression for the sound field radiated by a point source located on the centreline of the duct in the airway, in the absence of breakin. Again, the external acoustic radiation load on the duct walls is ignored here. We utilise an eigenfunction expansion involving the coupled acoustico-vibrational modes found by the method described in Section 2.1. Acoustical and vibrational symmetry requirements on either side of the source plane may be specified, and a weighted residual matching procedure is used to find the modal amplitudes. This involves the orthogonality properties of the coupled modes, which have been proved in the present investigation.

**2.3 Calculation of the Sound Power Radiated from the Duct Walls**
Having solved the coupled eigenproblem in the flexible-walled duct and found the modal amplitudes generated by the point source of Section 2.2, we now have to determine the sound power radiated by the vibrating duct walls. This is done here by modelling the duct as a finite-length line source, with decaying multimodal structural wave fields travelling in both directions away from a source point located a distance εL from one end of the duct, which has length $L$ (see Figure 3). By integrating the far-field pressure contributions from the radiating elements of the duct wall, we find an expression for the angular dependence of the radiated sound field,

$$G(\theta) = \sum_{j=1}^{m} Q_j\{[1 - \exp(-i\beta_j^-\varepsilon L)]/\beta_j^- + [1 - \exp(i\beta_j^+(1-\varepsilon)L)]/\beta_j^+\}, \tag{3}$$

where the far-field pressure is expressed as $(\rho\omega/4\pi r)\exp[i(\omega t-kr)]G(\theta)$, $r$ being the radial distance from the origin $O$ and $k$ the acoustic wavenumber. In equation (3), the summation is carried out over $m$ eigenmodes, $Q_j$ is the amplitude of the volume velocity/unit length of the jth mode (obtained by integrating the wall displacement around the perimeter of the duct) and $\beta_j^{\pm}=k\cos\theta\mp\alpha_j$. The total radiated sound power is now determined by integrating the sound intensity - found from equation (3) - over a spherical surface of radius $r$. Gauss-Legendre quadrature was employed in this process.

It is convenient to define an "insertion coefficient", $I(\varepsilon)$, as the ratio of the total sound power radiated by the duct to the sound power that would be radiated by the point source radiating into a free field, in the absence of the duct.

**2.4 The Inclusion of Breakin**

The necessary elements of an approximate analysis of the effects of breakin have been discussed in Sections 2.1-2.3, and we will employ these as follows.

First, we observe that the mean-squared sound pressure in the duct *in the absence of breakin*, from a point source on the duct centreline at x=0, is $(1/2)|p^+(x,y,z)|^2$, where y,z are transverse coordinates having their origins on the duct centreline and $p^+$ denotes the pressure amplitude in a positive-travelling set of modes. Therefore, at $x=\varepsilon L$, the mean-squared pressure on the duct centreline is $\bar{p}_a^2 = (1/2)|p^+(\varepsilon L,0,0)|^2$; the breakout sound power may be calculated as outlined in Section 2.3. Next consider the mean-squared pressure ($\bar{p}_b^2$, say) at the same point, *caused by breakin only*. For the purposes of finding $\bar{p}_b^2$, it may be assumed that the effect of the duct as a source of breakout sound power is the same as that of a point source (located in the reverberant enclosure) that radiates the same acoustic power. The mean-squared volume velocity, $\bar{Q}_b^2$, of this source is equal to $I(0)\bar{Q}^2$, where $\bar{Q}^2$ is the mean-squared volume velocity of the point source located in the duct at (0,0,0).

The problem of finding the breakin sound power becomes one of determining $\bar{p}_b^2$ on the duct axis at a point *P* with coordinates ($\varepsilon$L,0,0), due to a source $\bar{Q}_b^2$ at a point *P′* in the reverberant enclosure. According to the reciprocity theorem - see, for example, the rather general formulation of Lyamshev [5] - $\bar{p}_b^2$ is unaltered if the source and observer are interchanged. In the *reciprocal* problem, the sound power, $W_b$ say, radiated into the enclosure by the source $\bar{Q}_b^2$ at *P* is equal to $(\rho\omega^2/4\pi c)I(\varepsilon)\bar{Q}_b^2$. Then $\bar{p}_b^2$ can be written as the sum of "reverberant" and "direct" field components, $W_b(4\rho c/R)$ + direct field contribution, where *R* is the "room constant" of the enclosure. The direct field contribution cannot be found without a more detailed model for the radiated sound field, and it will be assumed that it is small in comparison to the reverberant field term. We then find that $\bar{p}_b^2 = (\rho^2\omega^2/\pi R)I(0)I(\varepsilon)\bar{Q}^2$. Now, the *total* mean-squared sound pressure within the duct at point ($\varepsilon$L,0,0), from the point source $\bar{Q}^2$ at (0,0,0), is given by

$$\bar{p}^2(\varepsilon) = (1/2)|p^+(\varepsilon L,0,0)|^2 + (\rho^2\omega^2/\pi R)\ I(0)\ I(\varepsilon)\ \bar{Q}^2. \tag{4}$$

This expression gives the mean-squared sound pressure distribution along the centreline of the duct. The first term represents the energy travelling directly to the observer from the source, *via* the fluid in the duct and vibrational motion of the walls, and includes the "flanking through bending waves traveling in the duct wall" of Vér [2]; the second term embodies the breakout/breakin flanking transmission path, *i.e.* the "radiation bypass" of Mechel [1].

## 3. RESULTS

Experimental data were taken at the University of Canterbury, on a 300 mm × 300 mm galvanized steel duct with 0.575 mm walls. The duct was lined internally on all four walls with a semi-rigid glass fibre slab material 52.5 mm thick, having steady flow resistivities normal and parallel (respectively) to the surface of 21 440 and 8 870 SI rayl/m. The effective radiating length of the duct was 5.5 m, and a 1.2 m long anechoic termination was fitted at one end. The sound source, consisting of a loudspeaker within a soundproof concrete box, was mounted at the other end. The entire apparatus was supported on trestles in a reverberant chamber having a volume of 221 $m^3$ and a surface area of 236 $m^2$; the room constant was inferred from reverberation time measurements. The sound source was fed with one-third octave bandwidth frequency-modulated sinusoidal signals, in order to avoid signal-to-noise problems resulting from the very large axial variation in sound pressure level that were measured. The sound pressure level on the duct axis was detected by means of a probe microphone inserted through the duct wall, and was measured along the length of the radiating section of duct; third-octave filtering was employed in the measurement of the sound level.

The bulk acoustic properties of the lining material were predicted from the steady flow resistances in the directions normal to, and parallel to, the surface by the use of the empirical formulae of Delany and Bazley [6]. Comparisons between predicted and measured axial sound pressure level distributions in the test duct at various frequencies are shown in Figures 4(a-d). At 315 Hz, both the predicted an measured levels vary linearly up to x=4.5m, beyond which a slight fall-off in slope is evident. The predicted and measured attenuation rates agree closely. At 500 Hz, however, we observe a much steeper attenuation rate up to x=2m, and a pronounced plateau in the sound level - caused by breakin - beyond that point. Again, the predicted and measured levels agree fairly well. The sharp 6dB fall-off in level at about x=5.3m is caused by the rapid decrease in $I(\varepsilon)$, brought about by a halving of the effective radiating length of duct, as the observation point (or source point in the reciprocal problem) moves toward the end of the duct, given that the structural wave motion is reasonably heavily attenuated. At 1.25 kHz, agreement between prediction and measurement is not quite so good, but the patterns are still similar. We note that, despite breakin effects, the sound level still falls by about 87 dB from the source, before levelling out. At 1.6 kHz, additional details are plotted in the theoretical curves: the axial sound level is plotted without breakin for zero structural loss factor, and for a structural loss factor of 0.01. This is done in an an effort to illustrate the effect of flanking transmission taking place purely *via* structural wave motion (see the paper by Vér [2]). Although "structural" type modes (having most of the power flow in the walls and being relatively lightly attenuated as compared to "acoustic" type modes) are only weakly excited by the source, they persist and eventually dominate the sound field at a distance from the source. As we might expect, increasing the structural loss factor increases the attenuation rate in the plateau region, but it is certainly clear that breakin effects are of far greater significance in determining the overall flanking effects than the direct structural wave path, notwithstanding the rather modest agreement between prediction and measurement.

## 4. DISCUSSION

The relatively simple theory that has been described here contains the essence of flanking transmission mechanisms. The rôle of direct structural flanking has been identified and its physical process has been modelled. The breakout/breakin flanking path has also been modelled, and the part played by the acoustical properties of the space surrounding the duct has been made clear (see equation (4)). Parametric studies (which space does not allow us to include here) have been carried out, to demonstrate the way in which various system parameters (*e.g.* room constant, duct cross-section, liner thickness) affect the flanking process. The comparisons between experiment and theory that have been made here reveal reasonable, though by no means perfect, agreement; one does, however, have some confidence that the physical mechanisms have been correctly modelled. The main conclusion to be drawn from the case examined here, at least, is that breakout/breakin flanking appears to predominate over direct structural flanking. Better prediction accuracy could be achieved by more detailed modelling, *e.g.* by using more degrees of freedom in the variational procedure for the duct eigenproblem or by employing a full finite element scheme, by including the direct field contribution in the reciprocal problem, *et cetera*. In principle, the model could be extended to apply to more complex systems, where (for example), one might have structural energy transmitted along a perforated facing sheet covering the sound-absorbing liner.

## 5. ACKNOWLEDGEMENT

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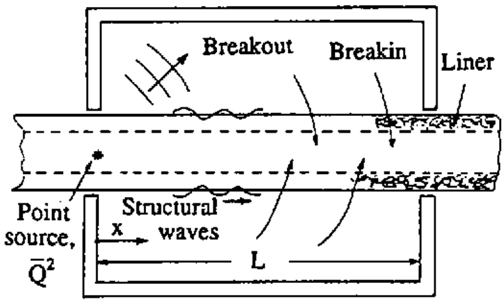


**Figure 1.**
Duct and reverberant enclosure

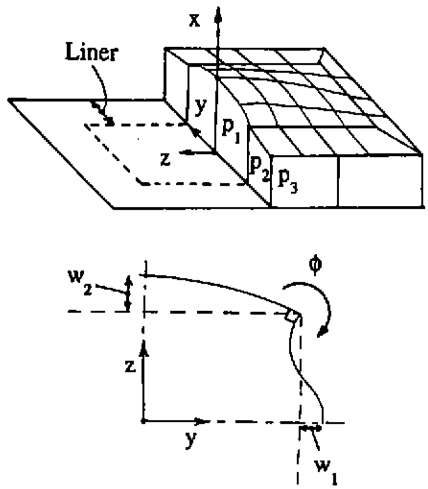


**Figure 2.**
Trial functions for pressure and displacement

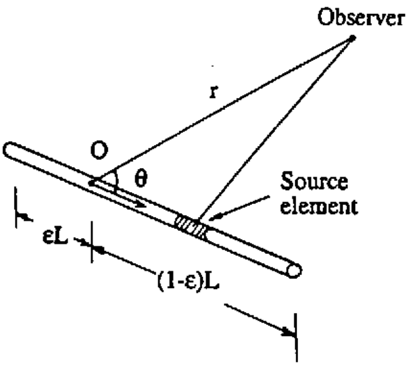


**Figure 3.** A line source

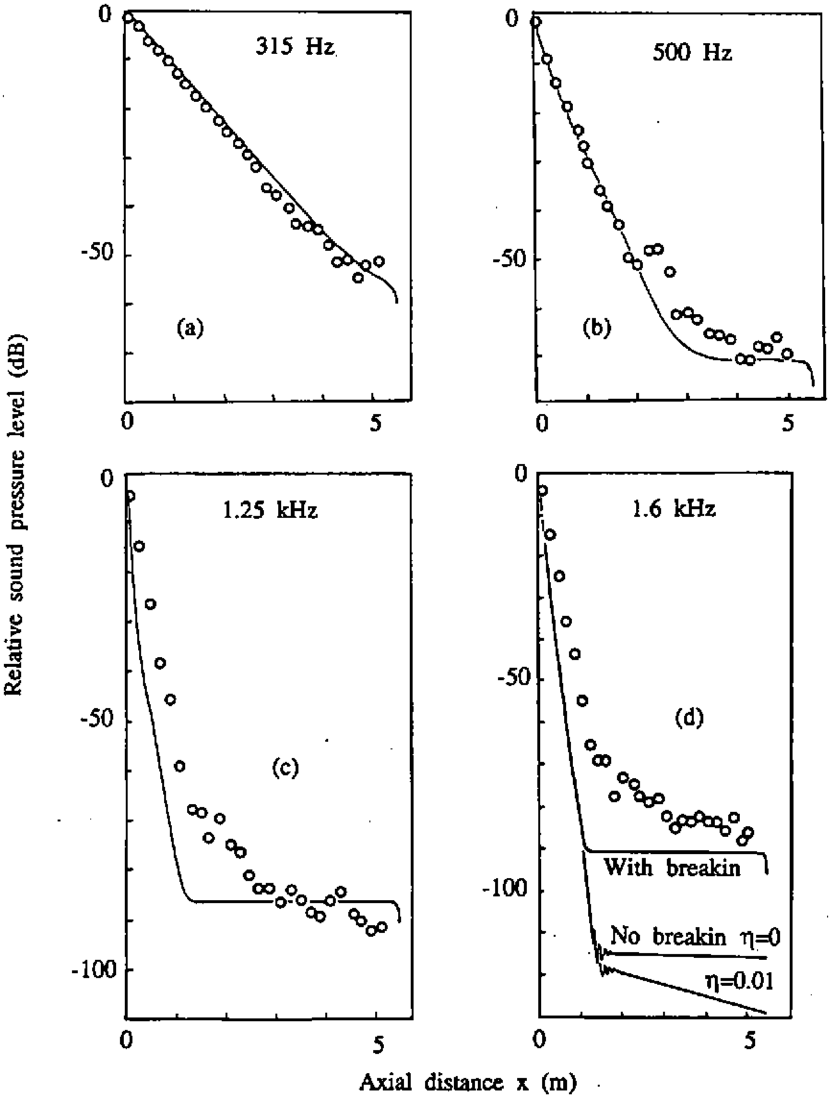


Figure 4. Predicted (———) and measured (o) axial sound pressure levels in the test duct.