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THE RESPONSE OF A RESONATOR UNDER A TURBULENT BOUNDARY LAYER TO A HIGH AMPLITUDE NON-HARMONIC SOUND FIELD

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INTRODUCTION

Many types of sound attenuating device incorporate perforated plates or tubes in which the perforations are exposed to grazing gas flow on one side. In jet aircraft engines, for example, arrays of small Helmholtz resonators (each consisting of a hole in the duct wall and a backing cavity) are employed as sound-absorbing liners. In internal combustion (I.C.) engine exhaust silencers, expansion chambers frequently embody a central perforated tube to carry the gas flow. In both the examples cited, the incident sound pressures are high and the particle speeds in the orifices can, in some cases, be equal to or greater than the free-stream grazing flow speed. Moreover, the sound fields in such situations usually involve complex periodic excitation, perhaps with a significant degree of superimposed random noise. Thus the effects of grazing mean flow, high amplitude and non-harmonic excitation all need to be accounted for in the modelling of the acoustical behaviour of the perforate holes.

Rice [1] tackled this problem by writing a differential equation governing the motion of the fluid in an orifice, involving a variable resistance coefficient which was dependent on both grazing flow velocity and particle velocity in the orifice. This equation was solved numerically in the time domain. Unfortunately, the experimental data available to Rice were not suitable to enable a detailed comparison between experiment and theory to be made. Additionally, the modelling of the mean flow and the instantaneous flow was rather rudimentary, and better methods are currently available.

The present investigation arose from a need to model the behaviour of orifices exposed to grazing flow and high pressure sound waves in the context of I.C. engine silencers. In this situation, the sound field is essentially periodic, but rich in harmonics. A differential equation formulation and time domain solution, similar to those of Rice, are used, but the grazing flow effects at low amplitude are specifically those of fully-developed turbulent pipe flow, and the orifice flow at high amplitude is represented by a comparatively detailed semi-empirical quasi-steady model reported by Rogers and Hersh [2]. The response of a single Helmholtz resonator, exposed to grazing, fully developed turbulent pipe flow, and complex periodic sound waves of medium to high amplitude is studied and comparison is made between experimental and theoretical data.

THEORY

The motion of the fluid in and around the orifice is taken to be described (at frequencies where the acoustic wavelength >> cavity dimensions) by the equation

$$\rho_o \ell \ddot{x} + r\dot{x} + \rho_o c_o^2 A_o x/V = p_\infty \qquad (1)$$

where ℓ is the effective length of the orifice, x is the space-averaged fluid displacement in the orifice (toward the cavity), ρ_o is the fluid density, r is the orifice resistance (a variable coefficient), c_o is the acoustic speed, A_o is the orifice area, V the volume of the cavity and p_∞ the forcing sound pressure in the grazing flow, away from the vicinity of the orifice. Goldman and Panton [3] have shown that, for $u_o/U_* < 4$ (U_* being the friction velocity in the mean flow boundary layer, $= \sqrt{(\tau_w/\rho_o)}$, where τ_w is the wall shear stress and u_o, the space-averaged velocity perturbation in the orifice, is equal to $\dot{x}$), r is independent of u_o, while ℓ is independent of u_o up to much higher values of u_o/U_*. It is thus appropriate to use a linear orifice resistance and inertance where $u_o/U_* \leq 4$. Cummings [4] gives empirical formulae expressing these quantities for an orifice with grazing fully-developed turbulent pipe flow:

$$\theta_f\, c_o/fd = [12.52\ (t/d)^{-0.32} - 2.44](U_*/fd) - 3.2, \qquad (2a)$$

$$\varepsilon/\varepsilon_o = 1 \quad \text{for } U_*/ft \leq 0.12\ d/t,$$

$$\varepsilon/\varepsilon_o = (1 + 0.6\ t/d)\exp[-(U_*/ft - 0.12\ d/t)/(0.25 + t/d)] - 0.6\ t/d \quad \text{for } U_*/ft > 0.12\ d/t, \qquad (2b)$$

where θ_f is total orifice resistance (pressure/velocity) in ρc units minus the (relatively small) viscous orifice resistance, f is frequency, d is orifice diameter, t is orifice length, ε is orifice mass end correction with flow, and ε_o is orifice end correction without flow. The quantities U_*/ft and U_*/fd are inverse Strouhal numbers. From equations (2), r and ℓ are specified in the linear region. The frequency is taken to be the fundamental frequency of the complex periodic signal p_∞. For $u_o/U_* > 4$, the steady-flow resistance data of Rogers and Hersh [2] are used, and equation (2b) is still used to find ℓ (= $t + d$); any resulting errors in ℓ, at very high values of u_o/U_*, will have an imperceptible effect on x since the inertial term in (1) is dominated by the resistive term. The resistance is given in terms of a discharge coefficient C_D, as

$$r = \rho_o\, u_o/2C_D^2. \qquad (3)$$

where

$$C_D = a(\delta/d)^{0.1}(u_o/u_\infty)^b, \qquad (4)$$

a and b being constants which differ between the outflow and inflow phases of the cycle and also depend on the ratio u_o/u_∞, u_∞ is the instantaneous velocity (mean plus fluctuating components) at the edge of the boundary layer in the approach flow to the orifice, and δ is the boundary layer thickness. It many readily be shown (provided the acoustic wavelength is greater than about twice the pipe width) that for inflow

$$u_\infty = U_\infty + p_\infty/\rho_o c_o + u_o A_o/A, \qquad (5a)$$

and for outflow

$$u_\infty = U_\infty + p_\infty/\rho_o c_o. \tag{5b}$$

U_∞ being the free-stream mean velocity and A the area of the pipe. Rogers' and Hersh's C_D data may be represented by three-part curves:

inflow	$a = 0.85,$	$b = 0.491,$	$u_o/u_\infty < 0.2$	(6a)
	$a = 0.61,$	$b = 0.301,$	$0.2 \le u_o/u_\infty < 0.6$	
	$a = 0.56,$	$b = 0.146,$	$u_o/u_\infty \ge 0.6$	
outflow	$a = 0.97,$	$b = 0.5,$	$u_o/u_\infty < 0.35$	(6b)
	$a = 0.82,$	$b = 0.312,$	$0.35 \le u_o/u_\infty < 0.7$	
	$a = 0.74,$	$b = 0,$	$u_o/u_\infty \ge 0.7$	

for $t/d \simeq 1$. For other values of t/d, these figures change somewhat. Figure 1 shows typical steady flow patterns for outflow and inflow. It is assumed here that the instantaneous flow patterns in oscillating flow are quasi-steady and therefore substantially similar to these.

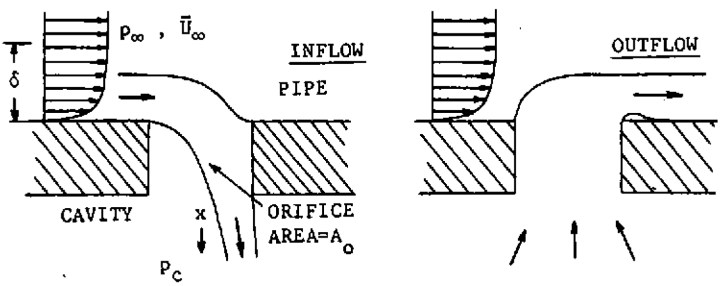

Figure 1. Steady orifice flow patterns for inflow and outflow

Equations (2) - (6) are used, together with (1), to give a differential equation that describes the fluid motion in the orifice. (The acoustics of the cavity are considered to be insensitive to the mean flow). This equation is solved by the Runge-Kutta-Nyström method in the time domain, for an arbitrary time dependence of p_∞ . The acoustic signal is suddenly switched on at time = 0, and x, $\dot{x}$ are put equal to 0 at time = 0 (satisfying the causality condition). Equations (2) or (3) - (6) are used where appropriate in the solution scheme.

In the experimental tests the sound pressure, p_c, in the cavity was measured as a representation of the resonator's response to the incident sound field. Then

$$p_c = x A_o \rho_o c_o^2/V, \tag{7}$$

and p_c may be found as a function of time, for a specified time history of p_∞.

A refinement of the above method - incorporating a three point, second order, finite difference representation of the sound field in the cavity - was used to check whether the above lumped-parameter representation of the cavity acoustics was valid in the frequency range of interest.

EXPERIMENTAL TESTS

Measurements were carried out on a single, square-edged, circular orifice (with t/d = 0.972) in the wall of a square pipe. The orifice was backed by a cylindrical cavity as shown in Figure 2. Fully-developed turbulent airflow passed along the pipe, and a burst of complex periodic sound wave (with peak pressures of up to 155 dB) was supplied by four JBL type 2482 acoustic drivers. The acoustic signals were detected in the pipe by a flush-mounted Bruel and Kjaer type 4136 1/4 in. microphone (M1) and in the cavity by a similar microphone (M2). A Scientific Atlanta type SD375 two-channel analyzer was used to capture the time histories of the signals from M1 and M2, which were representative of p_∞ and p_c respectively. Only the incident signal burst was recorded; axial reflections within the pipe were ignored.

The mean flow velocity in the pipe was measured by traversing a Pitot tube across the open pipe termination and the average wall shear stress $\bar{\tau}_w$ was found from data of Fujita [5]. The local value of τ_w was also found from

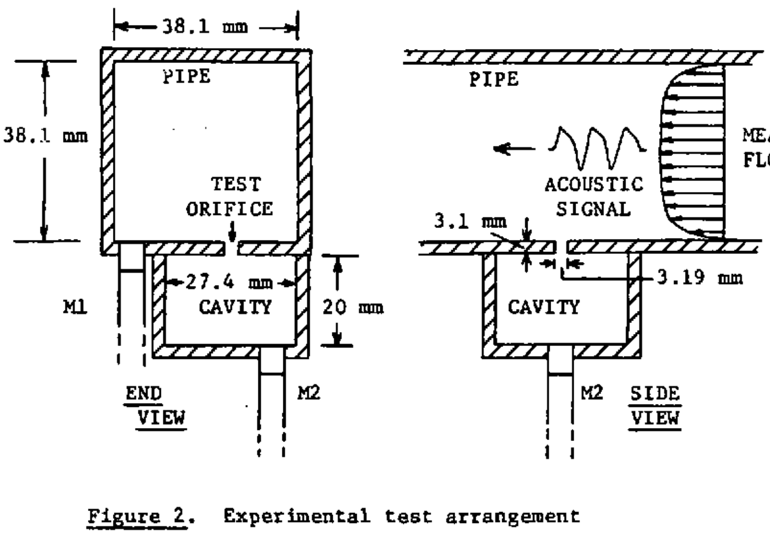

Figure 2. Experimental test arrangement

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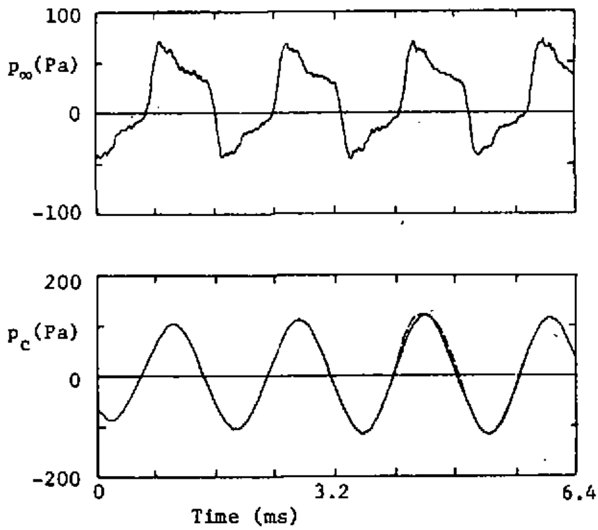

Figure 3. Pipe and cavity pressures, $\bar{U}$=14.8 m/s; ——— , measured; — — , predicted.

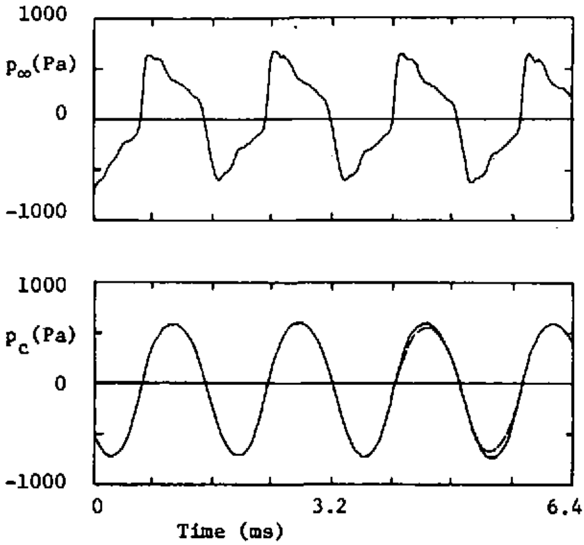

Figure 4. Pipe and cavity pressures, $\bar{U}$=14.8 m/s; ———, measured; — —, predicted.

Fujita's data. The boundary layer thickness δ was put equal to half the tube width.

MEASUREMENTS AND COMPARISON WITH THEORY

Complex periodic test signals of two fundamental frequencies were recorded: the first frequency was 588 Hz, close to the zero-flow resonance frequency of the resonator, and the second was 297 Hz, approximately half of this. Figures 3 to 7 show the measured and predicted results. In Figures 3 and 4 data at 588 Hz are given, with $\bar{U}$ (the mean flow velocity on the pipe cross-section) equal to 14.8 m/s, for relatively low and high pressures respectively. Defining R as the ratio of the peak-to-peak value of p_c to that of p_∞, one can see that in Figure 3, R = 2.0, whereas in Figure 4, R = 1.054; thus a distinctly nonlinear cavity response is noted, since the mean flow is the same in both cases. Because the frequency of the excitation signal is not too far removed from the resonance frequency, the cavity response is almost sinusoidal. Theory and measurement are in good agreement. (In both computation and measurement, one cycle of the steady state portion of the signal was chosen for comparison.) Figure 5 shows data at 588 Hz for a fairly high signal pressure and a flow velocity of $\bar{U}$ = 70.6 m/s. Here, the faster mean flow has increased the orifice resistance still further and R = 0.386. Agreement between experiment and theory is again quite good.

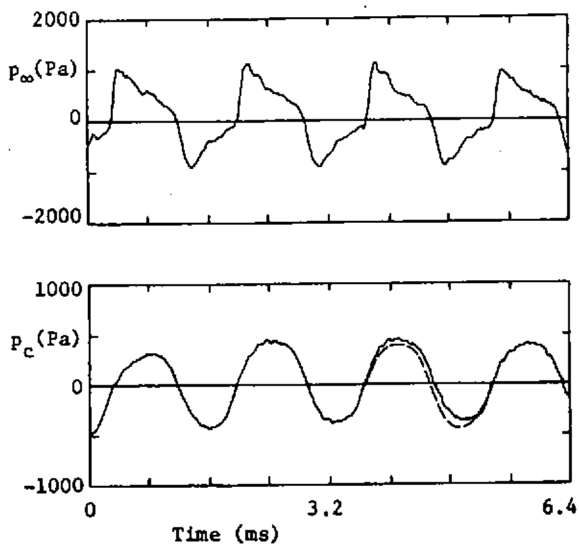

Figure 5. Pipe and cavity pressures, $\bar{U}$=70.6 m/s; ———, measured; — —, predicted.

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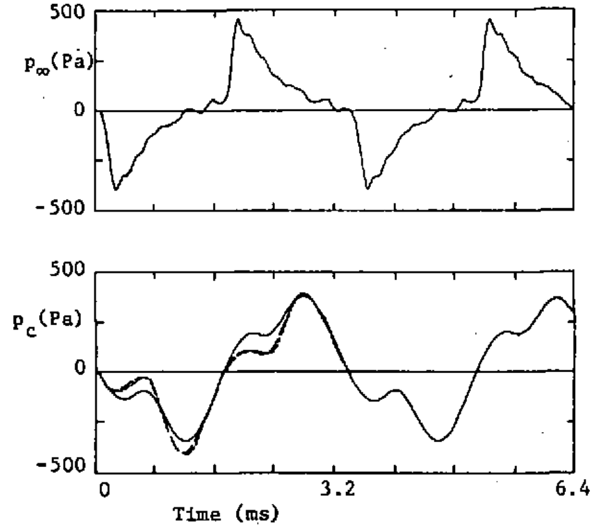

Figure 6. Pipe and cavity pressures, $\bar{U}$=14.6 m/s; ——, measured, — —, predicted (lumped constant cavity model); ------, predicted (finite difference cavity model).

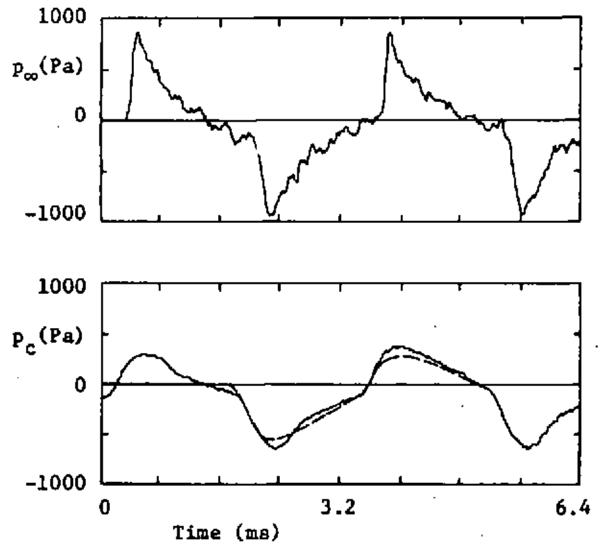

Figure 7. Pipe and cavity pressures, $\bar{U}$=70.5 m/s; ——, measured, — —, predicted.

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Figure 6 shows data at 297 Hz, with $\overline{U}$ = 14.6 m/s. Because the excitation frequency is of the order of half the resonance frequency, the second harmonic component of p_∞ introduces a corresponding component in p_c, as observed from the higher frequency undulations superimposed on the 297 Hz signal. The theory still predicts the measured response tolerably well. The finite difference solution is also plotted here, and this can be seen to be virtually indistinguishable from the lumped-constant prediction; thus the latter method is taken to be sufficiently accurate. Figure 7 shows data at a higher sound pressure and flow velocity ($\overline{U}$ = 70.5 m/s). Again, theory and measurement are in quite good agreement. Much less sensitivity of R to both U_* (via $\overline{U}$) and u_o is observed in Figures 6 and 7 because the predominant component of the response is of a frequency away from resonance.

DISCUSSION

The numerical time-domain solution scheme for the response of a resonator with grazing fully developed turbulent pipe flow to a high pressure sound field gives acceptable results, when appropriate models for the orifice flow (at both low and high values of u_o/U_*) are incorporated: a frequency-domain method of solution would seem much more difficult in consideration of the hybrid and rather complicated form of the governing equation. Although the details of vortex roll-up and convection, which are a part of the oscillatory flow process, are absent from the present model, the principal features of the orifice flow-field have evidently been embodied in the quasi-steady resistance formulae. No doubt, refinements in the orifice flow modelling could be introduced: it is not clear whether the present method of estimating the "low amplitude" impedance parameters solely from the fundamental frequency of the signal is sufficiently accurate, and improvements in the quasi-steady resistance model could, perhaps, also be considered. It should be reasonably straightforward to adapt the present approach to the more complex situation of an I.C. engine exhaust silencer, with its distributed impedance properties and multiple orifices.

ACKNOWLEDGEMENT

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