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APPLICATION OF THE ROUND TRIP THEORY FOR THE INDIRECT MEASUREMENT OF POINT IMPEDANCE BY AIRBORNE EXCITATION

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A method has recently been developed which allows the measurement of point impedances or mobilities using excitations made only at remote positions. The method utilises the recently published "Round Trip" theory which requires measurements to be made at three sets of positions (a), (b) and (c): where (a) and (b) are sets of points either side of a virtual boundary on which the points of interest (c) lie. The point impedances at points (c) on the boundary are determined by performing excitations at points (a) and (b) whilst measuring responses at points (b) and (c) thereby removing the usual requirement to excite at points (c). This makes the method particularly useful when these locations are difficult or impossible to access when instrumented. It has previously been shown that the point impedance or mobility at the interface between a vibration source and receiver can be determined using this approach and more recently the method has also been implemented successfully for continuous interfaces where discreet connection points do not exist. In these previous tests the required excitations of the system were performed using an instrumented hammer but in theory it is also possible to excite the system acoustically to obtain either structural point impedances or point impedances in air. Explored in the paper is the use of the "Round Trip" theory for the measurement of these two quantities using a calibrated volume velocity source.
Keywords: Mobility, impedance, vibration, structure-borne sound, sub-structuring

1. Introduction

When using the mobility method for transfer path analysis, source characterisation or sub-structuring, one is required to measure a matrix of point and transfer frequency response functions, or 'mobilities', which characterise the passive properties of the source and receiver (or assembly [1]) at the source-receiver interface. When attempting to perform these measurements it is often found that certain degrees of freedom are difficult to excite in a satisfactory way using an instrumented hammer or shaker, typically when transducers for response measurement prevent access or when there is no suitable surface on which to apply a force. This is particularly a problem for example when performing in-plane point mobility measurements on a plate [2,3]. In order to overcome this issue a method was recently developed for the reconstruction of Green's functions at passive locations using the 'Round Trip' theory reported in [4,5].

The round trip theory avoids the requirement to perform an excitation at the points and/or in the degrees of freedom which are difficult to access by employing additional measurements at two sets of points either side of an interface where the mobilities are required. The theory is exact for interfaces consisting of discreet points [4] providing the system is linear and time invariant. It has also been shown that good approximations can be obtained when the interface is continuous; i.e. when there are no discrete connection points. This means that the method is not necessarily restricted to separable sub-systems (such as a typical source and receiver) and can in fact be applied to determine the point impedance or mobility at any point and in any degree of freedom on an arbitrary virtual boundary separating two sets of measurement points as shown in Figure **1**.

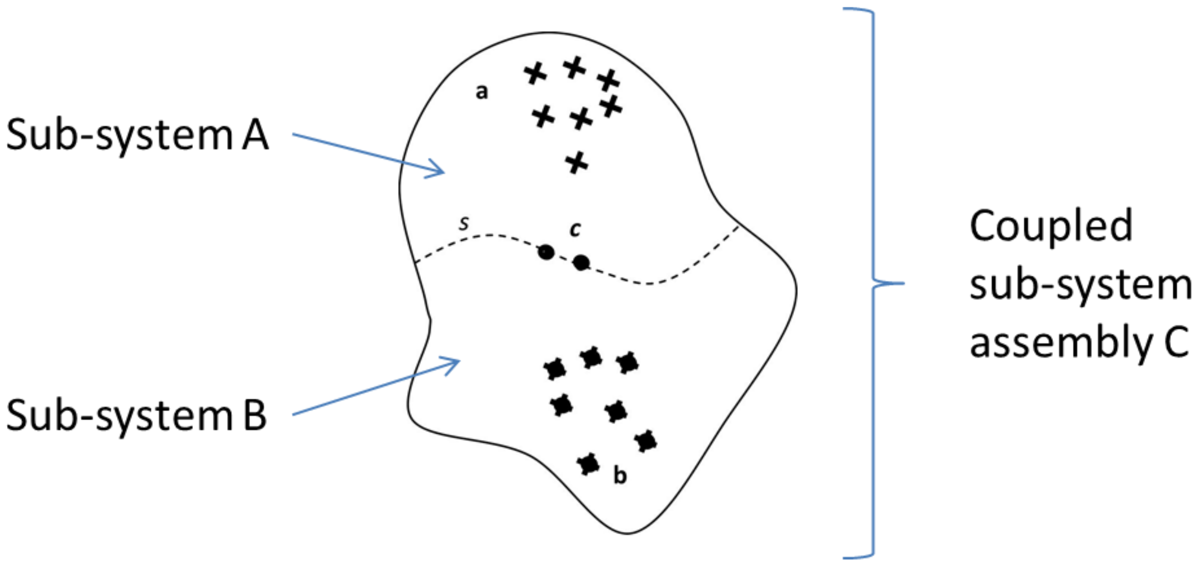

Figure 1: Continuous structure with an arbitrary interface passing through reconstruction points (c), showing excitation points, (a) and excitation-response points (b).

Figure **1** shows a virtual interface or surface (s) on which there are two points (c) where the required mobilities or impedances lie. These points are referred to as being passive meaning only response measurements at these positions is required; represented in the figure by a black dot (•). Either side of the interface are two further sets of points (a) and (b) which must both be excited (×) to complete the round trip using the following relationship [5],

$$\mathbf{Y}_{C,cc} \approx \mathbf{Y}_{C,ca}\mathbf{Y}_{C,ba}^{-1}\mathbf{Y}_{C,cb}^{T} \tag{1}$$

where **Y** is mobility and the uppercase subscript denotes the assembly C which is a combination of two arbitrary sub-systems A and B which need not be separable. The lower case subscripts then denote the response and excitation positions in that order so for example $\mathbf{Y}_{C,ca}$ is a matrix of transfer mobilites where responses are measured on the interface and excitations are made at points (a). $\mathbf{Y}_{C,cc}$ is therefore a square symmetric matrix containing the point and transfer mobilities at and between the points on the interface (c) and can be of any size providing the number of points n employed at (a), (b) and (c) obey the following, $n_a \geq n_b \geq n_c$. As mentioned previously the theory is exact for discreet interfaces so equation (1) is therefore a more general but approximate form of that reported in [4]. Furthermore, equation (1) is just one of many potential forms the round trip can take since the theory relates to Green's functions in general [5] not just those of the type vibration velocity due to force. The purpose of this paper is therefore to present and explore some other potential applications of the round trip theory employing frequency response functions obtained by airborne excitation with a calibrated volume velocity source.

2. "Round Trip" theory: airborne excitation

Until now the round trip theory has only been implemented on solid structures excited by point forces. However, by following the same derivation steps as in [4,5] an alternative formulation for the structural mobility $\mathbf{Y}_{cc}$ can be derived using an airborne rather than point force excitation,

$$\mathbf{Y}_{C,cc} \approx \mathbf{H}_{C,ca}\mathbf{X}_{C,ba}^{-1}\mathbf{H}_{C,cb}^{T} \tag{2}$$

Where $\mathbf{X}$ is acoustic impedance (p/Q - the sound pressure due to a unit volume velocity excitation) and $\mathbf{H}$ is a vibro-acoustic frequency response function (v/Q – the vibration velocity due to a unit volume velocity excitation).

Equation (2) is of interest because it could potentially be used to obtain mobilities at locations that are difficult to access providing one has a sound source powerful enough to produce a measurable velocity level at the interface (c). The experimental setup for such a test would require one or more accelerometers at the interface (c) and a number of microphones at positions (b) which is greater than or equal to the number of sensors at (c). To complete the round trip, excitations with a volume velocity source would be required at points (a) and (b).

Although there are likely many instances where this relationship could be employed one particularly interesting example is in-situ transfer path analysis [6]. For example, a vehicle interior could be instrumented with multiple microphones (positions b) and the feet of a piece of vibrating equipment with accelerometers (interface c). Volume velocity excitations at points (b) and (a) might then provide all the data required to determined $\mathbf{Y}_{C,cc}$ which could then be used, together with operational velocities on the equipment's feet, to determine its blocked force using,

$$\mathbf{F}_{bl} = \mathbf{Y}_{C,cc}^{-1}\mathbf{v}_{C,c} \tag{3}$$

where, $\mathbf{F}_{bl}$ is the blocked force and $\mathbf{v}_{C,c}$ is the operational velocity measured on the feet of the vibration source [1]. Multiplying these blocked forces with $\mathbf{H}_{C,cb}^{T}$ (already measured for equation 2) would then provide the structure borne path contributions to the interior noise, $\mathbf{p}_{C,b}$, which can be shown to be equal to $\mathbf{X}_{C,ba}^{-1}\mathbf{H}_{C,cb}^{T}\mathbf{v}_{C,c}$ by combining equations (2) and (3); which is an interesting aside.

Such a TPA method could potentially be appealing from a practical point of view because force excitations at the source receiver interface would be avoided and because a vehicle interior, for example, would anyway be instrumented with several microphones for such tests. Achieving a good signal to noise ratio on the interface sensors is likely to be challenging however, especially for heavy structures, and although it was shown to be feasible to employ the round trip method for continuous 2D line interfaces [5] a 3D system where the interface is a plane has not previously been investigated. As an interim point it is interesting therefore to first consider a slightly less challenging application of the round trip theory which involves a planar interface but not the difficulties associated with the airborne excitation of solid structures: the remote measurement of acoustic impedance.

Following the same derivation steps as in [4,5] it can be shown that,

$$\mathbf{X}_{C,cc} \approx \mathbf{X}_{C,ca}\mathbf{X}_{C,ba}^{-1}\mathbf{X}_{C,cb}^{T} \tag{4}$$

where $\mathbf{X}$ is the acoustic impedance p/Q. In order to invoke this form of the round trip relationship a set of microphones would be employed at a set of positions (b) and on an arbitrary interface (c) which separates points (b) from a further set of points (a). Points (a) and (b) would be excited with a volume velocity source to complete the round trip and obtain a matrix of acoustic impedances between the points on the interface (c). Besides its usefulness here the relationship could potentially also be of use for the measurement of impedances in duct systems where it is not possible to mount a volume velocity source due to space restrictions for example.

It is important to stress at this point that equations (2) and (4) are approximate and have not as yet been validated to the same extent as equation (1) but the surprising level of accuracy achieved when employing equation (1) to obtain point mobilities on an arbitrary continuous interface suggests that the relationships could be useful in practise. In the following sections equation (4) will be investigated further using experimental data and a numerical room model but before doing so we shall first conduct a basic validation of equation (2), the main point of interest, using a simple analytical model of the system shown in Figure 2 below.

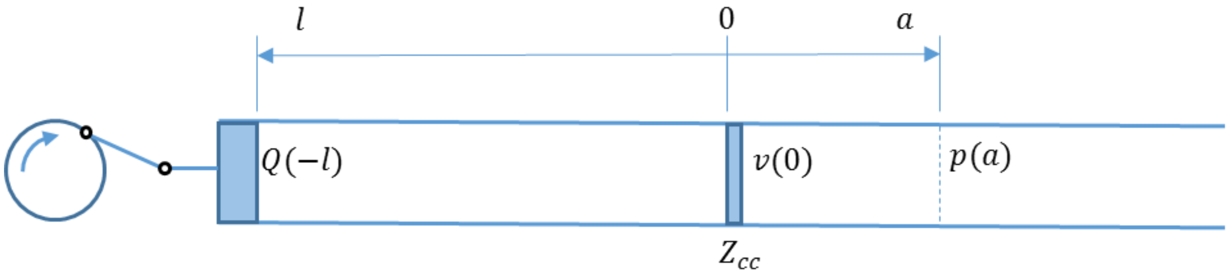

Figure 2: An infinite duct driven with a volume velocity Q at –*l* and blocked with a solid plate with acoustic impedance Z_{cc} at 0 (the interface c). Further downstream in the duct there is a resulting sound pressure p(a).

For the duct system shown in Figure 2 the following relationships can be defined which relate to the terms of the round trip relationship in equation (2),

$$\frac{v(0)}{Q(-l)} = \frac{1}{S}\left\{\frac{\rho c}{iZ_{cc}\sin kl + \rho c \cos kl}\right\} \tag{5}$$

$$\frac{p(a)}{Q(-l)} = \frac{\rho c}{S}\left\{\frac{\rho c}{iZ_{cc}\sin kl + \rho c \cos kl}\right\} e^{-ika} \tag{6}$$

$$\frac{p(a)}{f(0)} = \frac{1}{S}\frac{\rho c}{Z_{cc}} e^{-ika} \tag{7}$$

Thus, noting that pressure/force is equivalent to vibration velocity/volume velocity by reciprocity, i.e. $H = p/F = v/Q$, we may write,

$$Y_{cc} = \left[\frac{1}{S}\left\{\frac{\rho c}{iZ_{cc}\sin kl + \rho c \cos kl}\right\}\right]\left[\frac{\rho c}{S}\left\{\frac{\rho c}{iZ_{cc}\sin kl + \rho c \cos kl}\right\} e^{-ika}\right]^{-1} \frac{1}{S}\frac{\rho c}{Z_{cc}} e^{-ika} = (SZ_{cc})^{-1} \tag{8}$$

which demonstrates that equation (2) is valid, albeit for this trivial case, and is therefore worthy of further investigation. For completeness, the terms in equation (8) are those used most commonly in literature, i.e. c is the speed of sound, ρ is the density of air, k is the wavenumber, L is the distance between piston and impedance change and a is the distance between the impedance change and sound pressure.

It is worthwhile reiterating at this point that equations (1), (2) and (4) are just three of the many possible forms of the round trip theory that can be formulated and others, for example, that include electrical impedance could also be of use in other areas or fields such as electro-acoustics and beyond. The purpose of this paper however is only to explore applications of the round trip theory involving acoustic excitation to determine mechanical or acoustic impedance and no other forms of the round trip theory shall be considered here.

3. Experimental Validation

A simple experiment was conducted to demonstrate the application of equation (4) and obtain by measurement the acoustic transfer impedance between two positions in an anechoic environment using excitation at remote positions only. Two microphones were employed at the interface (c) and a further two microphones at (b). Excitations using a volume velocity source were carried out at six positions (a) and at the two (b) microphone positions beyond the interface which was arbitrarily defined. Equation (4) was then used to estimate the point and transfer impedances between the two interface microphones so that the transfer impedance between these microphones could be compared to that which was also measured directly by exciting at one of the two interface positions. The result is shown in Figure **3**.

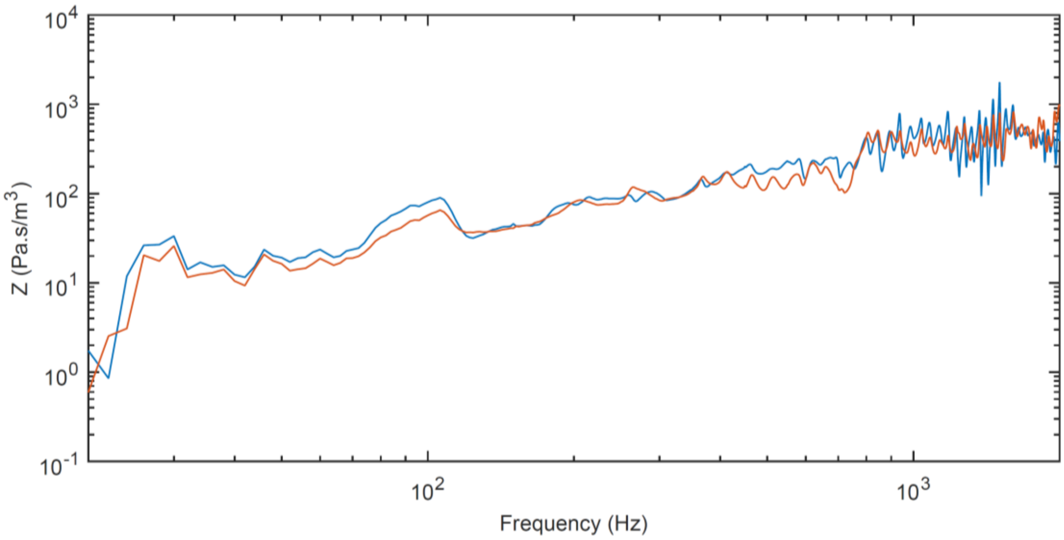

Figure 3: Directly measured transfer impedance (blue line) compared to that estimated using the round trip theory (red line) in the form of equation (4).

Although imperfect the reconstructed Green's function obtained by employing equation (4) is in fairly good agreement with the one that was directly measured, especially when considering the dynamic range of the input data which is shown graphically in Figure 4. This implies that the experimental test case is not entirely trivial and that the relationship may hold for more resonant systems.

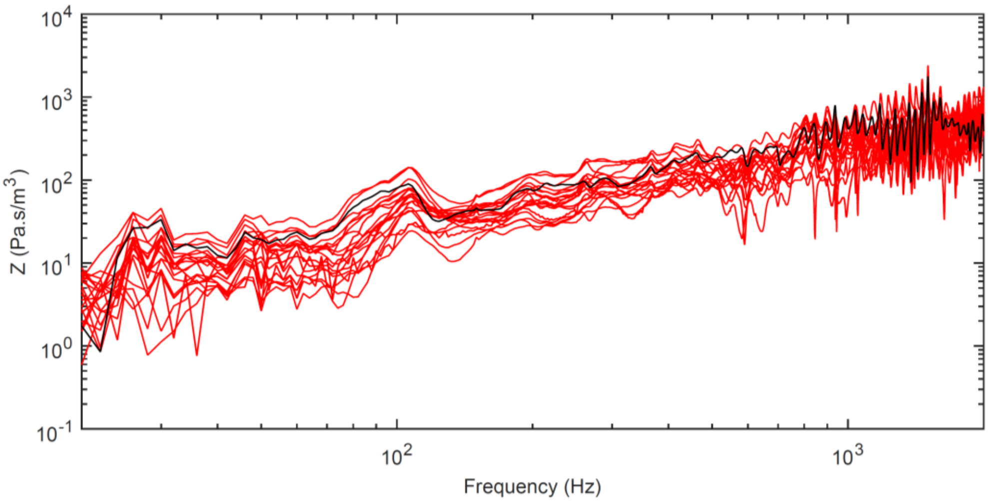

Figure 4: Directly measured transfer impedance (black line) compared to all other FRF's used in in its reconstruction by equation 4 (red line)

4. Room Model Validation

Figure 4 suggests that acoustic transfer impedance can be estimated well from equation (4) in an anechoic environment. In practice however, one will typically encounter a system that is to some degree resonant and it is useful therefore to also demonstrate how well the method performs in a space with a stronger modal behaviour. To this end we also include here a simulation of the method using data generated from a basic modal room model [6].

The room dimensions, chosen arbitrarily, were x=8m, y=5m, z=3m and the room was split into two parts by a virtual plane set at x=3m; its orientation need not be defined providing it does not intersect the (a) or (b) positions in a way that invalidates the round trip. A total of 8 points were included at (a), five at (b) and two on the interface (c). Figure 5 shows the transfer impedance magnitude and phase between the two points on the interface obtained directly from the room model and indirectly using relationship (4) with the required input data generated using the same room model. No regularisation was used to obtain the result shown in Figure 5 and the result has not been optimised in terms of the ideal number of (a) and (b) positions so it could likely be improved on. Nevertheless the result is anyway fair considering the complexity of the system and because equation (4) is known to be approximate when the interface is not composed of discrete points.

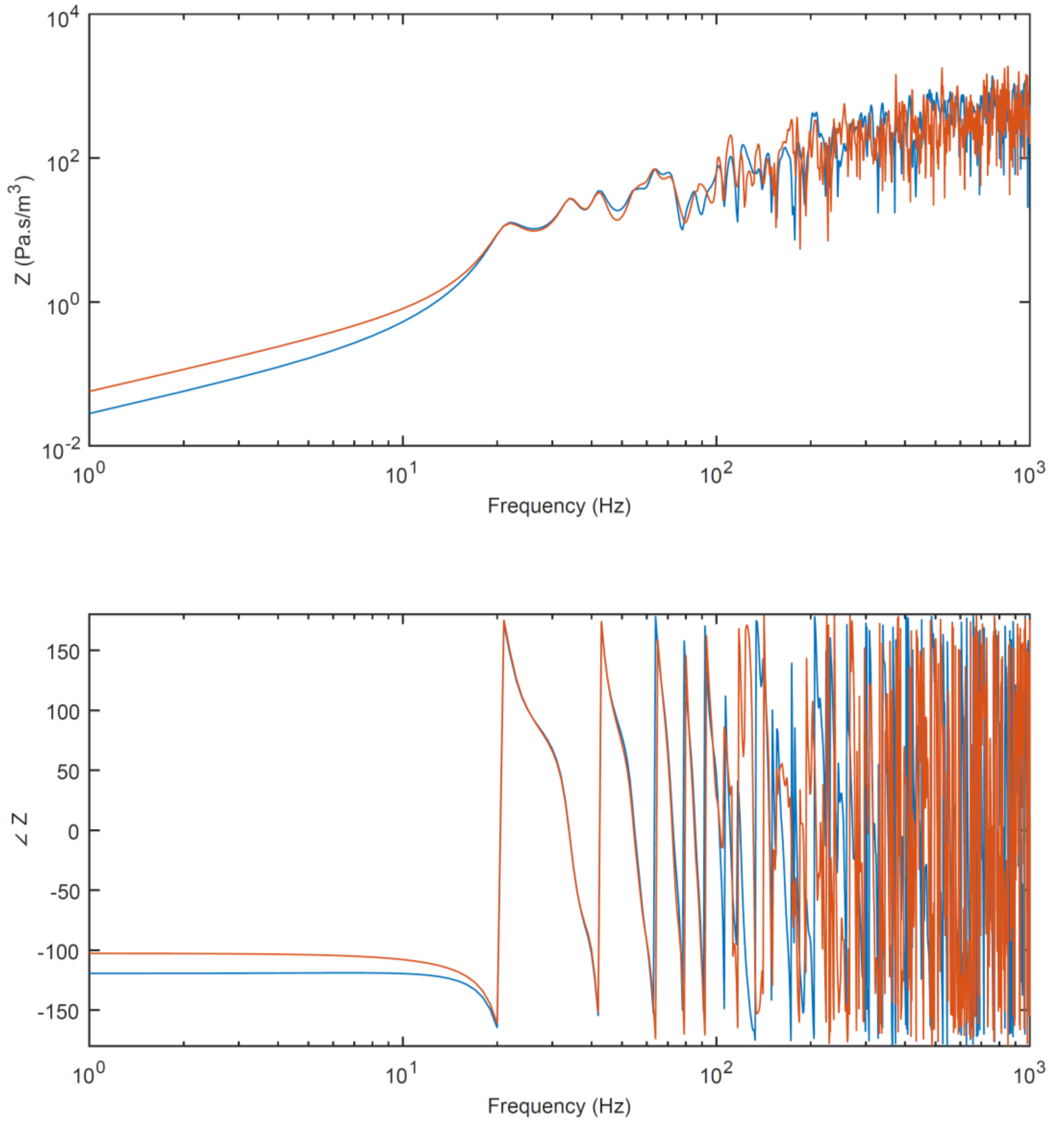

Figure 5: Transfer impedance from model (blue line) compared to the transfer impedance estimated using modelled FRF's and the round trip theory (red line) in the form of equation (4). Upper plot magnitude and lower plot phase in degrees.

5. Concluding Remarks

A method to determine point impedance or mobility in air or on solid structures by remote excitation via the "Round trip" theory has been outlined. The method is practically appealing because it avoids the usual requirement to excite at the points of interest which is advantageous when these points or degrees of freedom are difficult to access. The round trip method has previously been implemented successfully as reported in [4,5] but only using point force excitations of a solid applied with an instrumented hammer. The method described here differs in that the excitations are made with a calibrated volume velocity source.

It is shown in the paper that the point impedance of a physical structure or the impedance of a fluid can be obtained by remote acoustic excitation and an experimental validation is provided for the latter in an anechoic environment. A further validation using a modal room model is also provided and in both cases reasonable agreement was obtained considering that the relationship is known to be approximate for continuous interfaces. Further study is required to improve this agreement and this may be achieved by optimising the number and/or the positions of the remote measurement positions employed and/or through regularisation. The results presented are part of an ongoing study and further results will be presented at ICSV24.

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