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AN EXAMINATION OF THE 'OVERLAPPING RESONANCES' THEORY OF ACOUSTIC TRANSMISSION THROUGH AN ELASTIC PLATE

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INTRODUCTION

For an infinite, elastic plate immersed in a fluid and insonified by a plane wave, the exact expression for the transmission coefficient, T, is of a complicated form and yields little direct insight into the physical reasons for the highly varied transmittivity curves which are met. Some insight has been gained by consideration of modes. The incident radiation can excite Lamb modes in the plate, and for steel in water Fig. 1 illustrates dispersion curves for the first few antisymmetric and symmetric modes. The ordinate is presented as modal phase velocity c normalized to the shear velocity c_s, and it is also shown as a scale of incidence angle θ_r. The abscissa is the frequency thickness product fd, or alternatively the normalized plate thickness d/λ_s, where λ_s is the shear wavelength. Passage along a vertical line in Fig. 1 corresponds to varying the angle of incidence for fixed normalized plate thickness, and at an intersection with a modal branch, i.e. at a "coincidence" angle, the mode is excited and the transmittivity is mostly total. Similarly, passage along a horizontal line on Fig. 1 corresponds to a fixed angle of incidence and fixed absolute plate thickness and varying frequency, and it is again at intersections with modal branches that total transmission may be obtained.

A recent theory by Fiorito, Madigosky and Überall (FMU) [1] has extended the insight into the effect of modes on the transmittivity. By truncated Taylor series expansion of T about intersections with modal branches, with either sin θ_r or fd as the variable, FMU obtain an approximate expression for the transmission coefficient, T_{FMU}, in the form of a sum of two series, one for antisymmetric and one for symmetric modes. This can be visualised as the sum of contributions from all Lamb modes encountered by passage along either a vertical or a horizontal line on a dispersion diagram such as Fig. 1. Each term of either series represents a resonance curve which is accurate in the vicinity of its resonance position but becomes increasingly less accurate with departure from resonance. There is a phase difference of π between corresponding points on resonance curves of "antisymmetric type" and "symmetric type", and the 3 dB width of each curve is a function of each mode and varies along the modal locus. In each of several instances, with fd as the variable in some cases and θ_r in others, FMU show a comparison of the exact transmittivity magnitude curve with the corresponding set of resonance curves. Near the resonance peaks the agreement is very good. Although some curves are presented in Ref. 1 showing that there can be overlap of resonance curves at significant levels, FMU do not present comparisons of T and T_{FMU} in the overlap regions. However they do explain the formation of transmission nulls by such overlap, and Madigosky and Fiorito (MF) set out to illustrate this in Ref. 2. But MF do not take account of the true nature of the phase variations of each resonance contribution, and the schematic illustrations in their Figs. 5 and 8 contain significant errors. This lack of quantitative testing of the overlapping resonances concept away from the vicinity of resonance points and the potential usefulness of the concept in explaining some hitherto unexplained effects have led the author to the present study.

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THE RESONANCE CURVES

The resonance curve of FMU theory portraying the contribution T_m to T_{FMU} of the mth antisymmetric mode is shown in Fig. 2, the value of the phase, in degrees, being marked at a number of points. For a symmetric mode the curve is the same, but the phase values are increased by 180°. The abscissa h_m represents the displacement from the resonance position in units of the half width g_m between 3 dB points. To calculate T_{FMU} the basic information needed is the position of each resonance peak (on a scale of sin θ_f or of fd) and the associated value of g_m.

TRANSMISSION NULL FORMATION DUE TO TWO AND TO THREE LAMB MODES

For steel in water, resonance curves have been constructed for the two relevant modes of a 0.45 λ_s thick plate (Fig. 3(a)). The resultant $|T_{FMU}|$ is compared in Fig. 3(b) with the exact transmittivity curve, $|T|$. The approximate curve has indeed the general characteristics of the exact curve and includes a transmission null. The latter is slightly displaced from that in $|T|$, and this is largely caused by the inaccuracy of the individual resonance curves away from their peaks. A comparison of the phases of T_{FMU} and T in Fig. 3(c) also provides convincing support for the correctness of the overlapping resonances concept, apart from near grazing incidence. The FMU theory assumes that $\rho_f c_f \cos\theta_d \ll \rho_p c_d \cos\theta_f$, where ρ_f and ρ_p are densities of fluid and plate, and θ_d is the refracted angle for dilational waves in the plate; thus the theory does not apply near grazing incidence, for there $\cos\theta_f \approx 0$. A similar construction for a steel plate in water of thickness 0.6 λ_s, for which the A_0, S_0 and A_1 modes are relevant, while yielding good agreement near the resonance peaks, fails to reproduce the two transmission nulls exhibited by the exact curve of T.

TRANSMISSION BEHAVIOUR AT CLOSELY SPACED MODES

For plate thicknesses greater than about λ_s the coincidence angles for the A_0 and S_0 modes approach close together and the transmittivity in the region of the two coincidence angles is no longer unity but drops to small values with increase of d/λ_s. The overlapping resonances concept accounts for this behaviour, as is illustrated in Fig. 4 for a steel plate in water when d/λ_s = 2.5. Fig. 4(a) shows the resonance curves for the A_0 and S_0 modes, while Figs. 4(b) and (c) compare the magnitude and phase, respectively, of T_{FMU} and T. In the calculation of T_{FMU}, the contribution of modes other than A_0, S_0 and A_1 have been neglected. The general type of behaviour yielded by the approximation is in good accord with that given by the exact solution.

In the limit of large d/λ_s, when the plate becomes a half space, the resonance widths of modes other than of zero order become vanishingly small and these modes make no contribution to the transmittivity. The A_0 and S_0 resonance curves then superpose and exactly cancel each other out so that their resultant is zero at all incidence angles.

SYMMETRICAL AND ANTISYMMETRICAL BOUNDARY BEHAVIOUR

Calculations of the antisymmetrical and symmetrical boundary normal displacements for 0.55 λ_s [3] and 2.5 λ_s [4] thick steel plates in water show that, apart from the A_0 and S_0 modes for the thicker plate, the normal boundary motion at the coincidence angles, although predominantly antisymmetrical or symmetrical, as

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appropriate, also contains a small proportion of the opposite type of motion. At the coincidence angles of the A_0 and S_0 modes of the 2.5 λ_s thick plate almost equal proportions of both types of motion are present. These observations are in accord with what would be expected from the overlapping resonances concept [4], the behaviour for the zero order modes of the thick plate being explained by the near-overlaying of the resonance curves illustrated in Fig. 4(a). This near-superposition of the A_0 and S_0 resonance curves corresponds to there being virtually no motion of the far plate boundary, and this must persist with increase of plate thickness right to the half space limit.

ON THE CONCEPT OF $(fd)_{critical}$

The overlapping resonances concept implies that some fractional excitation of a Lamb mode can occur at other angles than the coincidence angle. Thus the concept that below $(fd)_{crit}$, (where the flexural wave velocity is subsonic), the A_0 mode does not contribute to the transmittivity has to be revised. Magnitude and phase transmittivity curves for steel plates in water for d/λ_s values as low as 0.001, (corresponding approximately to 1/80 $(fd)_{crit}$), have shown evidence of interference effects due to the presence of the A_0 mode. This is so despite the fact that at subsonic velocities the A_0 mode becomes highly damped.

INCOMPLETENESS OF FMU SOLUTION

When $c < c_s$, a horizontal line on a diagram such as Fig. 1 encounters only one modal branch. As the FMU treatment depends on Taylor series expansions about resonance points, no solution is yielded for the antisymmetric or symmetric modes, respectively, when no resonance of the given type is encountered. So, beyond the second critical angle the FMU solution is only partial. This probably accounts for the FMU conclusion that at the half space limit only one of the zero order modes survives, whereas it is concluded here that both survive.

CONCLUSIONS

The overlapping resonances concept is generally consonant with transmittivity behaviour yielded by exact theory and fits in with behaviour both at the high and at the low limits of plate thickness. Limitations which have been demonstrated are that the FMU theory does not apply near grazing incidence and that at fixed incidence angles beyond the second critical angle it yields only a partial solution.

REFERENCES

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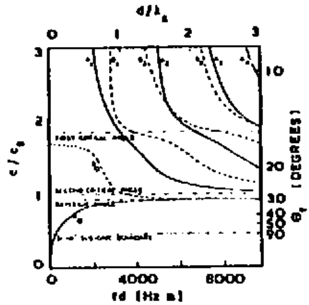

Fig. 1. Dispersion curves for steel plate in water.

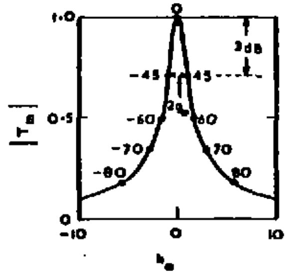

Fig. 2. Transmittivity resonance curve for an antisymmetric mode.

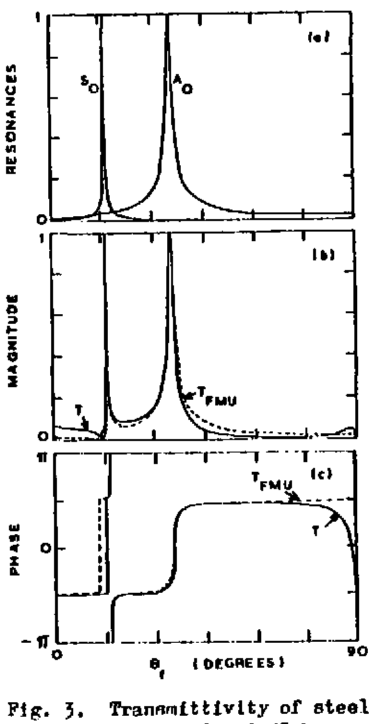

Fig. 3. Transmittivity of steel plate in water. d = 0.45 λ_s.

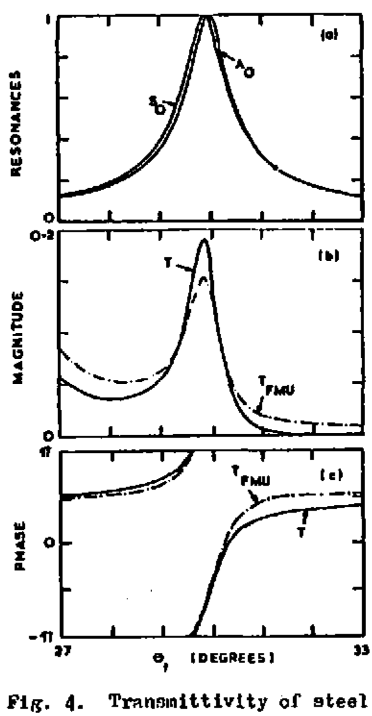

Fig. 4. Transmittivity of steel plate in water. d = 2.5 λ_s.