APPROXIMATE SCALING LAWS OF ACOUSTIC ARRAY INTERACTION

A. FREEDMAN
65 Mount Pleasant Avenue, Weymouth, Dorset DT3 5JF, U.K.

ABSTRACT

For a fixed set of parameters, computing the performance of an array of N transducers when taking acoustic coupling into account usually involves the solution of a real matrix equation of order 2N. If N is large, the computation for just one set of parameters can be a considerable task. In array interaction studies many parameters are variable, and the computational effort to cover even a limited range of variables can be formidable indeed. One approach to reducing the magnitude of effort required is to search for scaling laws, even of an approximate nature, which will enable conlusions drawn from a study of one array to be applied to a family of arrays.

THE SCALING LAWS

This paper, which is based on Ref 1, describes the derivation of approximate scaling laws for planar arrays of piezo-electric elements vibrating either within an infinite, plane, rigid baffle or with no baffle present; the transducer faces are deemed to be rigid and circular, with diameter less than $\lambda/2$ and, for convenience, the medium is assumed to be water. All transducers of an array are presumed identical. The several scaling laws which are deduced can be applied in combination, providing the restrictions applicable to each individual law are complied with.

Over a limited frequency band, a piezo-electric transducer may be represented by the equivalent circuit of Fig 1, where upper and lower case subscripts denote electrical and mechanical units, respectively. Instead of specifying the equivalent circuit components directly, we specify the following quantities: f_o, mechanical resonant frequency of an isolated transducer in water; a/λ_o, piston radius in wavelengths at f_o; ρ, density of water; c, velocity of sound in water; η_{ma}, mechano-acoustic efficiency of an isolated transducer in water; Q_w, motional Q factor of an isolated transducer in water; k, coupling coefficient; B, electrical to mechanical transform coefficient; tan δ, dielectric loss factor. This method of specification, coupled with the use of simple approximations (Ref 2) for the mutual radiation impedance, and with certain restrictions on optional external components L_E and R_S, enables the impedances of all circuit components of Fig 1 to be expressed in terms of the self radiation resistance and reactance at f_o, $R_{r(s)o}$ and $X_{r(s)o}$, of the normalised frequency, f/f_o, and of quantities which are constant for a specified element. For given boundary conditions and fixed a/λ_o, $R_{r(s)o}$ and $X_{r(s)o}$ are each equal to a constant times f_o^{-2}. Hence one may deduce the dependence factors of Table 1 for the parameters E, B and f_o (which we shall call scaling law 1).

With constant voltage drive and no external series impedance, the force across the input to the mechanical part of the equivalent circuit is unaffected by the values of k and of tan δ; hence the acoustic performance is then unaffected by these two parameters. (Scaling law 2.)

Applying the previously mentioned relations and using approximations for the radiation impedance of small, circular pistons, further scaling laws can be deduced; these relate performance curves (over a limited bandwidth) of elements with different specifications when these curves are presented on a generalised frequency scale of $2Q_w\Delta$, where Δ is the fractional bandwidth, $(f-f_o)/f$.

For single transducers with specifications differing solely in the value of Q_w, any parameter depending only on the mechanical impedance $Z_{m\,r}$ (see Fig 1) will yield, when plotted on the generalised frequency scale, curves (approximately) independent of Q_w. (Scaling law 3a.) When there is a constant voltage source and no series impedance in the electrical feed, the acoustic quantities (e.g. piston velocity, force on piston, radiated power) are such parameters. Although law 3a does not hold, in general, for the electrical quantities (such as input impedance and power), it does hold for such quantities if the elements have negligible dielectric loss, are parallel-tuned to f_o and have equal values of $Q_w(k^2/(1-k^2))^{1/2}$. (Scaling law 3b.) Computational checks for some small arrays indicate that laws 3a and b hold not only for isolated elements but for elements in arrays. An example of law 3a is shown in Fig 2 for two baffled and uniformly energised arrays whose specifications only differ in that they have Q_w values of 6 and 12, respectively. There are 3 unique element positions, namely corner, mid-side and centre; thus 3 separate full line curves are shown on each graph. In addition, a broken line curve shows the behaviour of an isolated element. The severe interaction effects follow very similar curves for the two arrays.

The following relations apply to corresponding elements of arrays with different a/λ_o, but otherwise identical specification. Piston velocity, radiated power, input current and power are proportional to $1/R_{r(s)o}$, whereas input impedance and mutual and total radiation impedance are proportional to $R_{r(s)o}$. (Scaling law 4.) An example is shown in Fig 3, where, on the right hand graph of each pair, the expected highest point of the array curves as scaled from the left hand graph is indicated by a short, horizontal line.

Similarly, an array in an infinite, rigid baffle and an unbaffled array with otherwise identical specifications will scale approximately in the ratio or inverse ratio of the respective values of $R_{r(s)o}$. (Scaling law 5.) Fig 4 shows an example.

By combination of scaling laws 4 and 5 one can deduce that two similar arrays, one baffled and one unbaffled, yet both with elements having (nearly) the same equivalent circuits, will yield approximately the same performance if a/λ_o for the baffled array is 1/6th less than for the unbaffled one.

References

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in the random medium problem. In the limit $\Phi >> 1$ Rayleigh statistics obtain and

$$\langle I^n \rangle = n! \quad , \tag{5}$$

corresponding to the saturated regime for wave propagation in a random medium. Actually, it can be shown that a sufficient condition for eqn. (5) to hold, at least for the lower orders, is $\Phi\Lambda > 2$ and $\Phi > 1$.

Correlation effects for the scattered field received at spatially and temporally separated points have also been considered. Let $p_1 = p(\underset{\sim}{r}_o, \underset{\sim}{r}_1, t)$ and $p_2 = p(\underset{\sim}{r}_o, \underset{\sim}{r}_1 + \Delta\underset{\sim}{x}, t + \tau)$ be the complex envelopes of the received field, where $\Delta\underset{\sim}{x}$ and τ are the spatial and temporal separations of the observations. Then, by a development parallel to that for a single receiver, the joint distributions for the scattered wave envelopes can be derived. The general results are rather complicated and will not be reproduced here; however for the case $\Phi\Lambda > 2$, $\Phi > 1$, there is obtained

$$\langle I_1^n \, I_2^n \rangle = (n!)^2 \; {}_2F_1\left[-n, -n; 1; \exp(-2\Phi^2(1 - \Psi))\right] \quad , \tag{6}$$

where $\Psi = \Psi(\underset{\sim}{\xi}, \tau)$, and $\underset{\sim}{\xi}$ can be simply related to $\Delta\underset{\sim}{x}$ [8].

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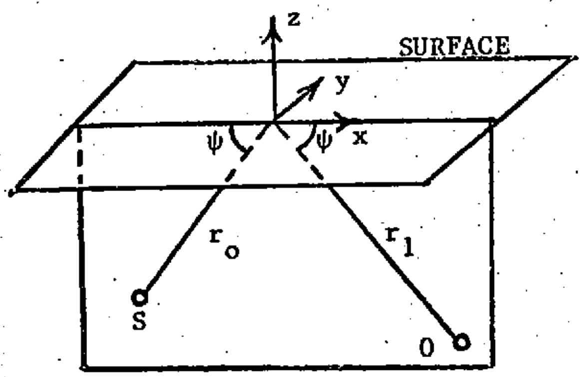

Fig. 1. Surface scattering geometry