INSTITUTE OF ACOUSTICS
UNDERWATER ACOUSTICS GROUP

PROCEEDINGS OF THE
CONFERENCE

RECENT DEVELOPMENTS IN UNDERWATER ACOUSTICS

HELD AT THE ADMIRALTY UNDERWATER
WEAPONS ESTABLISHMENT, PORTLAND

31st MARCH AND 1st APRIL 1976

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The Underwater Acoustics Group of the Institute of Acoustics was duely constituted in July 1975. The membership already exceeds 50.

The Portland meeting on 'Recent Developments in Underwater Acoustics' is the first general meeting organized by the Group. The quality of the papers submitted, as well as their numbers, justifies the holding of such meetings from time to time.

As the Chairman of the Steering Committee of the Group, I would like to record our thanks to the Director of the A.U.W.E. for kindly hosting this meeting, and to the members of the Organizing Committee for their hard work in putting it together.

H. O. Berktay

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THE USE OF LINEAR SYSTEM THEORY IN ACOUSTIC RADIATION AND SCATTERING ANALYSIS

By A. Freedman

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INTRODUCTION

Some years ago I put forward theories relating to scattering and to radiation of pulsed acoustic or electromagnetic waves. Those theories were based upon certain physical approximations (of the physical optics type) and were also limited to input signals of small fractional bandwidth. In essence, the scattered radiation at a fieldpoint was shown to be made up, in general, of a set of "Image Pulses" whose envelopes were approximate replicas of the envelope of the input signal [1,2,3]; similarly, at a point in the field of a radiator, the radiation was shown to be made up of a set of so-called "Replica Pulses" [1,4,5,6].

More recently Wiekhorst[7] brought out a very generalised theory, applicable to most linear physical processes, and giving a picture of system outputs with no bandwidth limitations on it. Wiekhorst explained that my Image Pulse concept of echo formation corresponded to a particular case of this linear system theory, and I gave a very brief description of Wiekhorst's interpretation of acoustic echo formation in Ref. 8. Since then I have considerably developed the formalism of the above linear system approach and have explicitly derived the solution in the small fractional bandwidth situation. This has enabled formal relationships to be established between the Image and Replica Pulse theories and the linear system theory. It has also enabled, on the one hand, the former band-limited results of the Image and Replica Pulse theories to be presented free of bandwidth limitations, and, on the other, it has enabled one to see the limits of useful applicability of the previous band-limited theories.

I shall summarise the salient features of the linear system theory, show its relations to my earlier Image Pulse and Replica Pulse theories and present examples of results obtained using the non-restricted and the restricted bandwidth formulae.

THE LINEAR SYSTEM APPROACH

If a Dirac pulse, $\delta(t)$, where t denotes time, is applied to the input of an arbitrary linear system, the output is the so-called impulse response function, $h(t)$, of the system. For an arbitrary input signal, $s(t)$, the output, $e(t)$, is given by the convolution integral

$$e(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau \equiv s(t) * h(t). \qquad (1)$$

In general, the impulse response function may contain Dirac pulses (Fig. 1(a)), and we denote these by $a_{g,0}\,\delta(t-t_g)$. The times t_g, where $g = 1,2,\ldots,F$, refer to all times at which Dirac pulses occur in any order of derivative of $h(t)$, and not just to times at which pulses occur in the zero order derivative. Thus some or all of the amplitudes $a_{g,0}$ may be zero (Fig. 1(b)). For convenience we introduce the nomenclature $h(t) \equiv h_{-1}^{(0)}(t)$, and we denote the remainder of $h_{-1}^{(0)}(t)$ after removal of any Dirac pulses by $h_0^{(0)}(t)$ (Fig.1(c)). Thus

$$h_{-1}^{(0)}(t) = \sum_{g=1}^{F} a_{g,0}\,\delta(t-t_g) + h_0^{(0)}(t). \qquad (2)$$

The first derivative, with respect to time, of $h_0^{(0)}(t)$, namely $h_0^{(1)}(t)$, may contain Dirac pulses (Fig. 1(d)) which we denote by $a_{g,1}\,\delta(t-t_g)$. We also denote the remainder of $h_0^{(1)}(t)$ after removal of any such Dirac pulses (Fig. 1(e)) by $h_1^{(1)}(t)$ (Fig. 1(f)). So

$$h_0^{(1)}(t) = \sum_{g=1}^{F} a_{g,1}\,\delta(t-t_g) + h_1^{(1)}(t). \qquad (3)$$

We can continue this process of "stripping" Dirac pulses and differentiating the "stripped" remainder, and for the qth stripped derivative we shall have

$$h_{q-1}^{(q)}(t) = \sum_{g=1}^{F} a_{g,q}\,\delta(t-t_g) + h_q^{(q)}(t). \qquad (4)$$

Let the mth be the last differentiation of the impulse response function to yield Dirac pulses, i.e. repeated further differentiation of $h_m^{(m)}(t)$ yields no further such pulses.

Then it can be shown that the following equations apply for the output signal.

$$e(t) = e_A(t) + e_B(t), \qquad (5)$$

$$e_{A,g,n}(t) = a_{g,n-y}\, s^{(-[n-y])}(t-t_g), \qquad (6)$$

$$e_{A,g}(t) = \sum_{n=0}^{m} e_{A,g,n}(t), \qquad (7)$$

$$e_A(t) = \sum_{g=1}^{F} e_{A,g}(t), \qquad (8)$$

$$e_B(t) = s^{(-m)}(t) * h_m^{(m)}(t). \qquad (9)$$

y is an integer, which in the cases of acoustic radiation and scattering takes the value of unity. Expressions such as $s^{(-m)}(t)$ denote the mth integral of the function with respect to the argument.

The above equations show that the output of a linear system consists, in general, of two parts. The first is made up of components, $e_{A,g,n}(t)$, each of which is associated with a Dirac pulse, at time t_g, in the nth derivative of the impulse response function of the system; each such component is a scaled, and appropriately delayed, signal whose nth derivative is a replica of the input signal. The output, $e_{A,g}(t)$, due to all such components associated with a given t_g is the sum of a set of signals, each starting at the same time and each having an envelope of different shape, as each is associated with a different value of n. As all components of $e_{A,g}(t)$ start at the same time they are not physically resolvable from one another, and $e_{A,g}(t)$ forms a physical output entity. Its envelope shape will depend upon the relative distribution of the amplitudes, $a_{g,n}$, of its components. The various $e_{A,g}(t)$ start at different times, t_g, and, subject to appropriate pulse lengths and bandwidths, are in principle resolvable from one another. The second part of the output of a linear system consists of a single component which is the convolution of the mth integral of the input signal with the mth "stripped" derivative of the impulse response function; the shape of this second part is thus not simply related to that of the input signal, and its duration could be the sum of the durations of the input signal and of the impulse response function. Either, but not both, of the two parts may be zero. When the first part is zero, the second part degenerates to the convolution of the input signal and the impulse response. Almost all the cases I have investigated so far have either yielded outputs where $e_B(t)$ is zero or small, or, in a few instances, where $e_A(t)$ is zero.

As an illustration of the principles involved, assume that an input signal of the form $t \exp(-t^2)$ (Fig. 2a) is fed into a system whose impulse response function has the form $[4\delta(t-8) + 3H(t-8) - 3H(t-14)]$ (Fig. 2b), where $H(t)$ represents a Heaviside step function. Then it can be shown straightforwardly that $e_A(t) = e_{A,1,0}(t) + e_{A,1,1}(t) + e_{A,2,1}(t)$ and $e_B(t) = 0$. The three components of $e_A(t)$, i.e. of $e(t)$, are illustrated in Figs. 2c, d and e, and the total output signal is presented in Fig. 2f.

THE SMALL FRACTIONAL BANDWIDTH SOLUTION

A considerable simplification is brought about when the bandwidth of the input signal is appropriately restricted.

Making the assumption that $s(t)$ is band limited to within the limits $\omega_o-(\Delta\omega/2)$ and $\omega_o+(\Delta\omega/2)$, and that ω^{-n} varies little over this band, (i.e. that there is a sufficiently small fractional bandwidth), the following approximate formulae are derived. That the results are band limited is denoted by an upper tilde.

$$\tilde{e}(t) = \tilde{e}_A(t) + \tilde{e}_B(t), \tag{10}$$

$$\tilde{e}_{A,g,n}(t) \simeq a_{g,n-y}\,(i\omega_o)^{-(n-y)}\,s(t-t_g), \tag{11}$$

$$\tilde{e}_{A,g}(t) = \sum_{n=0}^{m} \tilde{e}_{A,g,n}(t), \tag{12}$$

$$\tilde{e}_A(t) = \sum_{g=1}^{F} \tilde{e}_{A,g}(t), \tag{13}$$

$$\tilde{e}_B(t) \simeq (i\omega_o)^{-m}\,s(t) * h_m^{(m)}(t). \tag{14}$$

Each component, $\tilde{e}_{A,g,n}(t)$, associated with a Dirac pulse, at time t_g, in the nth derivative of $h(t)$ is now a scaled, phase shifted and appropriately delayed replica of the input signal. Thus, the overall envelope shape of $\tilde{e}_{A,g}(t)$ is also a replica of the envelope of the input signal. $\tilde{e}_B(t)$ consists of a scaled convolution of the input signal with the mth stripped derivative of the impulse response function.

For an input signal consisting of an amplitude modulated pulse with half sine envelope and containing an integer number of cycles at carrier frequency ω_o, scaled output components $e_{A,g,n}(t)$ and $\tilde{e}_{A,g,n}(t)$ are compared in Fig. 3. They are plotted for input pulses containing 10, 5 and 3 cycles, respectively, for three values of n-y. It turns out that, in the case of backscattering of a plane wave or of farfield radiation, these values of n-y are associated with certain geometrical features of the scattering or radiating surfaces, and these are also indicated in Fig. 3. For $n-y = 0$, the approximate and exact curves are identical; for the other values of n-y the agreement is good even for pulses as short as three cycles. Thus, the so-called "small fractional bandwidth" approximation appears to be satisfactory for fractional bandwidths as large as unity.

THE EFFECT OF ASYMPTOTES IN $h^{(n)}(t)$

In some quite ordinary physical situations, at a given t_g, no Dirac pulses occur for any order of derivative of the impulse response function; instead asymptotes are encountered. For example, Fig. 4 shows the impulse response function relating the normal velocity of a circular piston to the resulting velocity potential at a nearfield point; the first two derivatives are also illustrated, and asymptotes are met from the first derivative onwards. Present information indicates that where the lowest derivative of $h(t)$ in which an asymptote occurs is the rth, an output contribution is produced which is intermediate between what would be produced by Dirac pulses occurring in the rth and (r+1)th derivatives.

THE EFFECT OF DIRAC PULSES IN $s^{(n)}(t)$

Where the input signal or any of its derivatives also contain Dirac pulses, further series of output components result. For brevity, these are not dealt with here.

THE IMAGE PULSE AND REPLICA PULSE THEORIES

In the Replica Pulse theory of acoustic radiation[1,4] the model assumes the input signal to be the normal velocity of the radiating surface and the output to be the resulting pressure at a field point. The model for the Image Pulse theory of echo formation[1,2] included transmitting and receiving transducers, but we drop these electro-acoustic links to avoid bandlimiting terms; the input signal in the echo formation model is now the incident field pressure normalised to unit distance in the reference direction from a directional source, while the output is the pressure of the scattered field back at the source. The impulse response function for the radiation model turns out to be $h(t) = (\rho c^2/2\pi)\, C_w^{(2)}(r)$ and that for the echo formation model to be $h(t) = (-c/2\pi)\, W_w^{(2)}(r)$. ρ is the density, c the sound velocity, $C_w(r)$ the "velocity-weighted, range-normalised" area of the radiating surface[4] within range r of the field point, and $W_w(r)$ the "directivity-weighted" solid angle subtended at the source[2] by those parts of the scattering surface within range r. Taking account of these values of h(t) (and of the removal of the electro-acoustic transducers) the solutions in Refs. 4 and 2 for pulsed inputs reduce identically to Eq. (13) for $\tilde{e}_A(t)$. In the Image Pulse and Replica Pulse treatments of Refs. 1, 2 and 4 a remainder term in a series expansion of the scattering integral and of the radiation integral was erroneously discarded. After recovering this remainder term in each case, substitution of the appropriate value of h(t) shows that, for each of the two models, the remainder term reduces identically to $\tilde{e}_B(t)$ as given by Eq. (14). This completes the demonstration that the Image Pulse and Replica Pulse theories are particular cases of the more general linear system approach.

We are now in a position to compare the approximate, band-limited results given by the Replica and Image Pulse theories with the results when there is no bandwidth approximation. For convenience, examples of radiated fields are presented, but similar results could be shown for backscattering. Three types of situation are illustrated. (i) Where for given t_g, Dirac pulses occur in only one order of differentiation of h(t), (ii) where, for given t_g, Dirac pulses occur in more than one order of differentiation of h(t), (iii) where no Dirac pulses occur at any time in any derivatives of h(t). Situations (i), (ii) and (iii) are illustrated in Figs. 5 to 7 by the fields of rectangular, conical and circular, uniformly vibrating radiators, respectively. For the latter example, the Replica Pulse approach was supplemented by an asymptotic expansion treatment[9] in order to get the small fractional bandwidth solution. These examples demonstrate that, for an input pulse containing even a very few cycles, the small fractional bandwidth approximation (used in the Image and Replica Pulse theories) gives results very close to the true waveforms.

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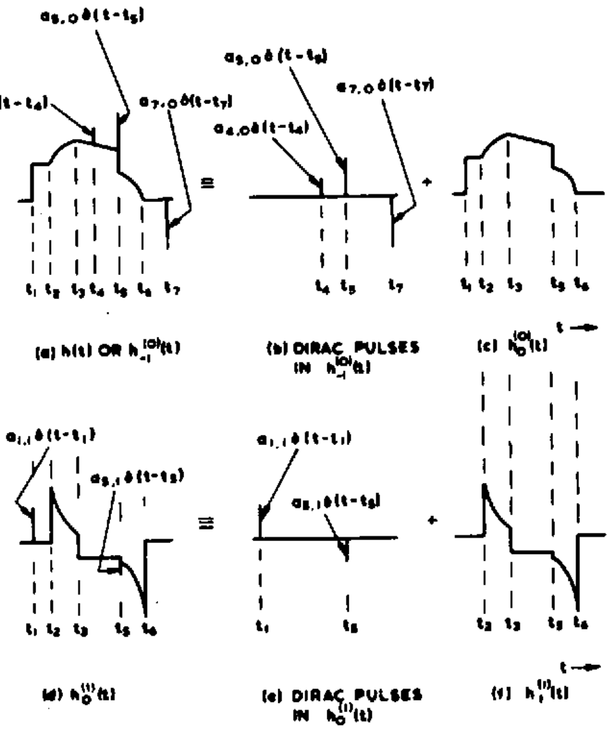

FIG. 1 SCHEMATIC ILLUSTRATION OF IMPULSE RESPONSE FUNCTION AND OF ITS DERIVATIVE AFTER REMOVAL OF ANY DIRAC PULSES

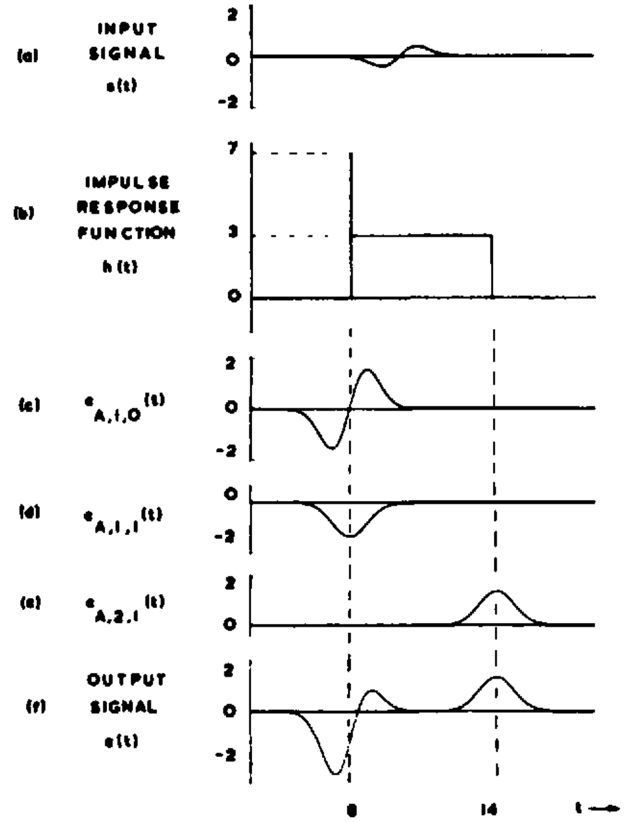

FIG. 2 EXAMPLE OF EXACT WAVEFORMS OF THE OUTPUT COMPONENTS AND OF THE TOTAL OUTPUT FOR WIDE BAND INPUT. (FRACTIONAL BANDWIDTH OF INPUT SIGNAL = 3)

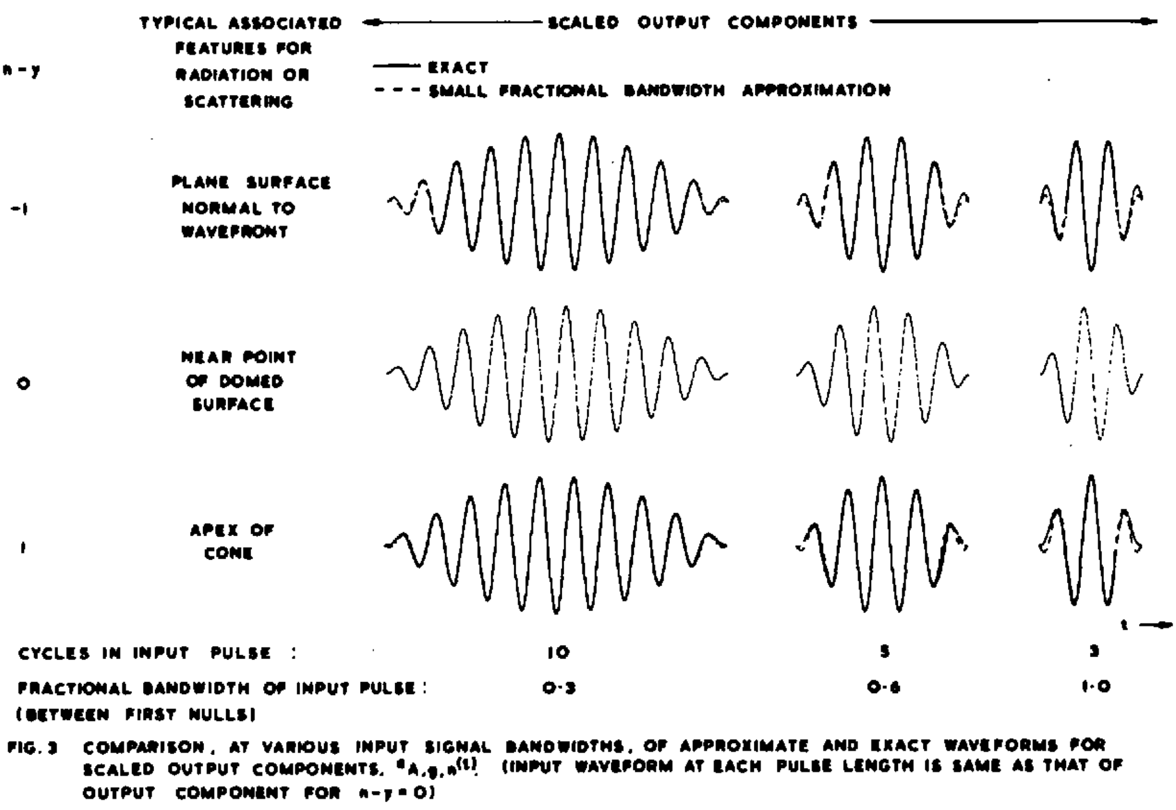

FIG. 3 COMPARISON, AT VARIOUS INPUT SIGNAL BANDWIDTHS, OF APPROXIMATE AND EXACT WAVEFORMS FOR SCALED OUTPUT COMPONENTS. $e_{A,g,n}(t)$ (INPUT WAVEFORM AT EACH PULSE LENGTH IS SAME AS THAT OF OUTPUT COMPONENT FOR $n-\gamma=0$)

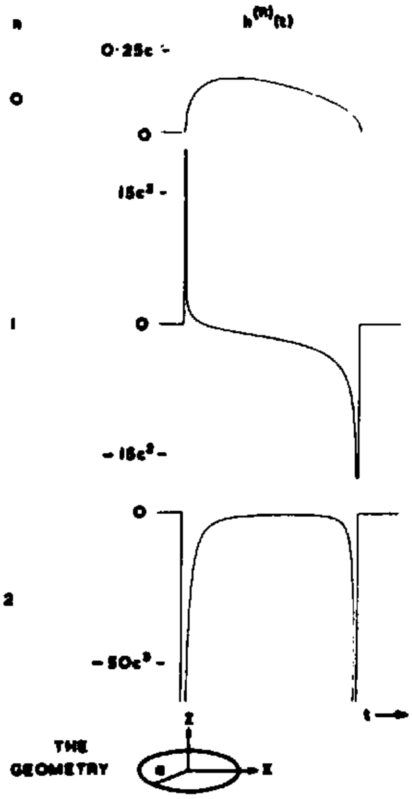

FIG. 4 IMPULSE RESPONSE FUNCTION OF CIRCULAR PISTON AND ITS DERIVATIVES, (X = 2a, Z = 5a)

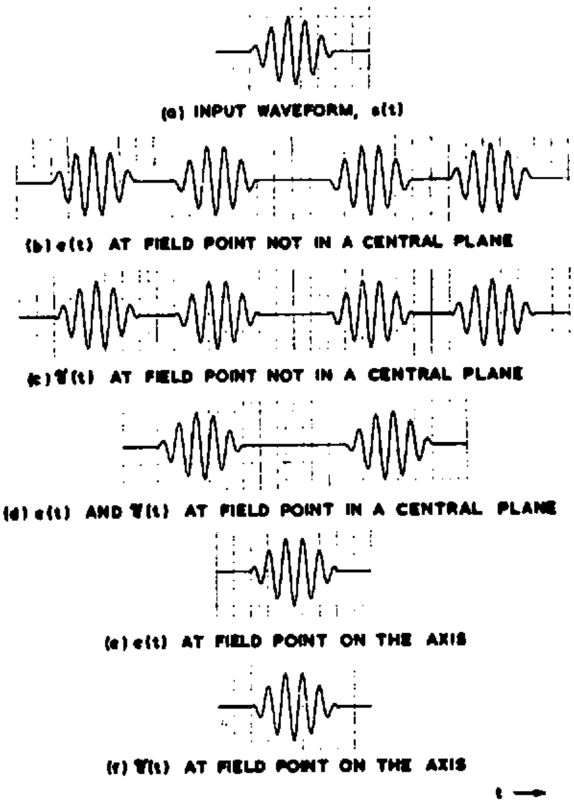

FIG. 5. PRESSURE IN FARFIELD OF RECTANGULAR PISTON

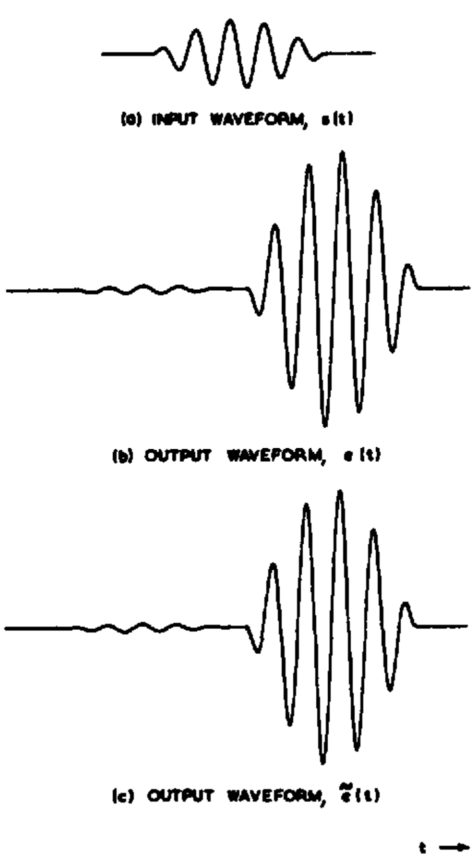

FIG. 6. AXIAL PRESSURE IN FARFIELD OF CONICAL RADIATOR OF HEIGHT $5 \cdot 5\lambda_0$

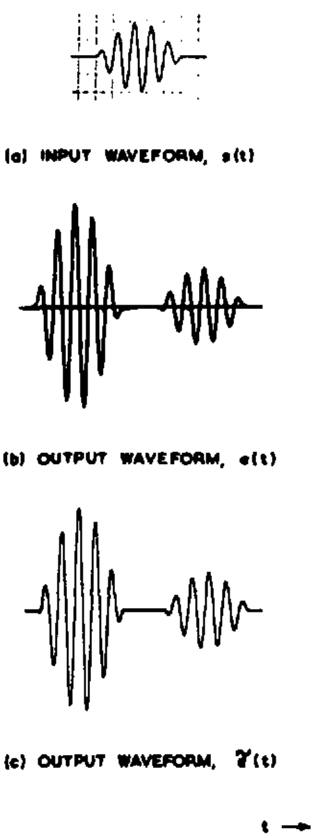

FIG. 7 PRESSURE IN NEARFIELD OF CIRCULAR PISTON ($a = 10\lambda_0$, $X = 2a$, $Z = 5a$)