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ADVANCES IN THE MODELLING OF DISPERSION AND DIFFUSION OF ENTROPY WAVES IN GAS-TURBINE COMBUSTORS

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Entropy waves are an important source of indirect combustion noise and potentially contribute to the generation of thermoacoustic instabilities in gas-turbine combustors. Entropy fluctuations generated by unsteady combustion are known to disperse and diffuse as they travel towards the combustor exit. However, in low-order acoustic network codes, typically used for the analysis of gas-turbine combustors, such effects are often neglected and the transport of entropy perturbations is modelled in a simple way, with the entropy waves either advected without attenuation or completely diffused. Recent studies in simple flows have shown that both the dispersion due to a non-uniform mean velocity profile and the turbulent mixing can give important contributions to the attenuation of entropy waves. In gas-turbine combustors the effect of turbulent mixing can be even more pronounced due to the presence of swirling flows and recirculation regions, which may have a strong influence on the decay of the entropy perturbations. In this work, recent advances in the modelling of diffusion and dispersion of entropy waves are discussed with a focus on applications to gas-turbine combustors. A Large-Eddy Simulation of entropy advection in a swirling flow is presented to highlight the importance of both shear dispersion and turbulent mixing in the attenuation of entropy waves and the necessity of including both phenomena in low-order network models for a reliable evaluation of the transport of entropy perturbations in real combustors. Recently developed models for the dispersion of entropy waves, to be used in the context of low-order network codes, are also analysed and a revised model is proposed to include the effects of turbulent mixing, which is usually neglected in most of the available models.

Keywords: entropy waves, dispersion, diffusion, low-order models, gas turbines.

1. Introduction

In order to reduce the environmental impact of combustion systems, and in particular to reduce NO_x emissions, gas turbine manufacturers are developing technologies based on the use of lean flames. However, lean combustion systems are more prone to combustion instabilities [1] and generally lead to an increased level of combustion noise because of the higher unsteadiness of the reacting process [2]. Combustion instabilities, due to coupling mechanisms between unsteady combustion and acoustic waves propagating in the chamber, can lead to additional noise, vibrations and, in the case of very large pressure fluctuations, to flame quenching, flashback and damages of the combustor components [1]. For this reason, combustion instabilities are becoming one of the main issues in the development of lean burn technologies. In addition, also combustion noise is a very important aspect to be considered in the design of aero-engines due to its impact on the overall noise emissions produced by an aircraft [3]. The total noise radiated is usually distinguished into 'direct' and 'indirect' noise [4, 5]. Pressure perturbations directly generated by the unsteady heat release rate are referred to

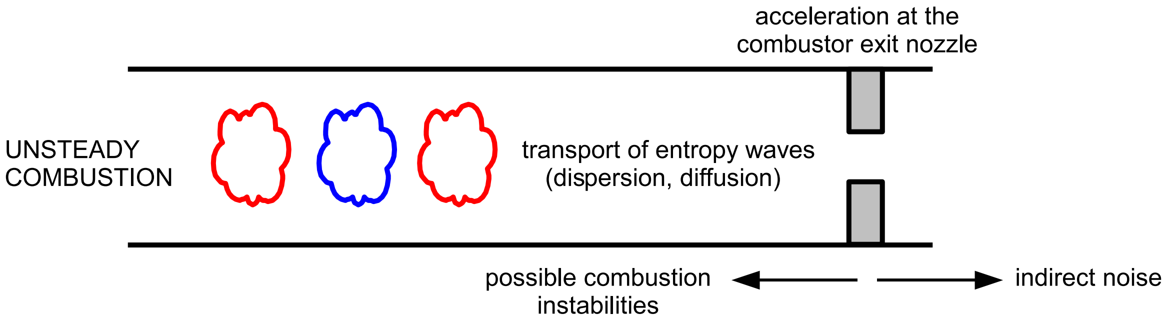


Figure 1: Simplified representation of entropy advection in a gas turbine combustor.

as direct noise whereas the indirect noise is an additional contribution due to the acceleration of a fluid with a non-uniform distribution of entropy [6, 7, 8] or vorticity and its level can prevail over the direct component [2]. Furthermore, as recently pointed out by Magri et al. [9], also inhomogeneities in the composition could give non-negligible contributions to the indirect noise. It is therefore crucial for the development of lean burn technologies to investigate such phenomena and develop proper tools to predict the behaviour of combustion systems.

In this context, entropy waves are receiving more and more attention due to their impact on both combustion noise and thermoacoustic instabilities [2, 10]. Entropy waves, sometimes described as hot and cold spots, are usually generated at the flame location by unsteady heat release rate and then advected by the flow towards the combustor exit where they are accelerated by the nozzle. As described in the early work by Marble and Candel [4], acceleration of entropy waves leads to the generation of pressure waves that propagate both upstream and downstream, contributing to the level of combustion noise. Furthermore, the perturbations propagating upstream can interact with the flame, possibly leading to combustion instabilities. This scenario is schematically described in Fig. 1. In some configurations, significant contributions to the entropy fluctuations may also come from the interaction with cold flows, as discussed in Ref. [11].

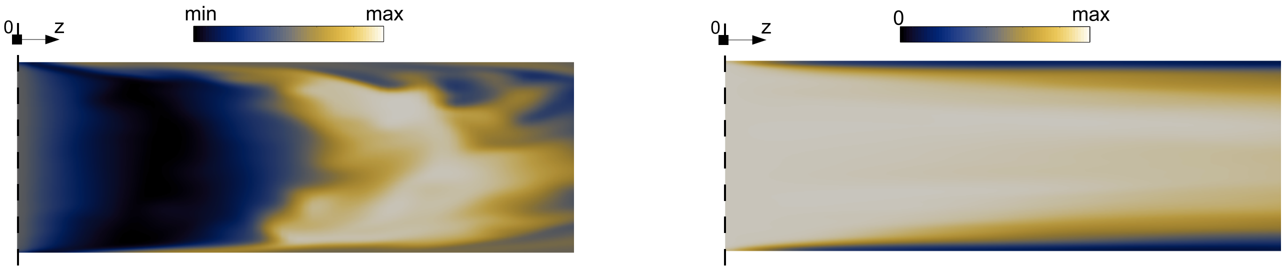
In order to predict the effect of entropy waves on both thermoacoustic instabilities and combustion noise, all the phenomena involving entropy wave generation, transport and interaction with the combustor exit nozzle should be properly addressed. In this paper the focus will be on the dispersion and diffusion of entropy waves. Recent advances in the understanding and modelling of entropy wave diffusion and dispersion are summarized with a focus on real engine combustors. Numerical simulations of entropy advection in swirling flows are presented to analyze the attenuation of entropy waves in a flow field much closer to the one typically found in real engines. Furthermore, a simplified low-order model able to include turbulent mixing effects on the dispersion and diffusion of entropy waves is introduced. Concluding remarks and recommendations for future research close the paper.

2. Entropy wave dispersion and diffusion

Entropy fluctuations generated in the combustion chamber are known to disperse and diffuse as they travel towards the combustor exit. In the following, some of the recent investigations on entropy dispersion and diffusion in simple configurations are first discussed. Then, the specific case of entropy advection in real engines is considered together with low-order modelling of the entropy transfer function.

2.1 Experiments and numerical simulations in simple geometries

One of the first attempts to analyse the dispersion and diffusion of transported scalars in a flow field is represented by the early work by Taylor [12, 13]. Both cases of laminar and turbulent flow were investigated, highlighting the role of diffusion and turbulent mixing. However, considering the



(a) Instantaneous entropy field

(b) *rms* of the entropy field

Figure 2: Convection of entropy waves in a straight duct [16] ($\mathrm{Re_d}$=5400, frequency of the entropy perturbation f=200 Hz).

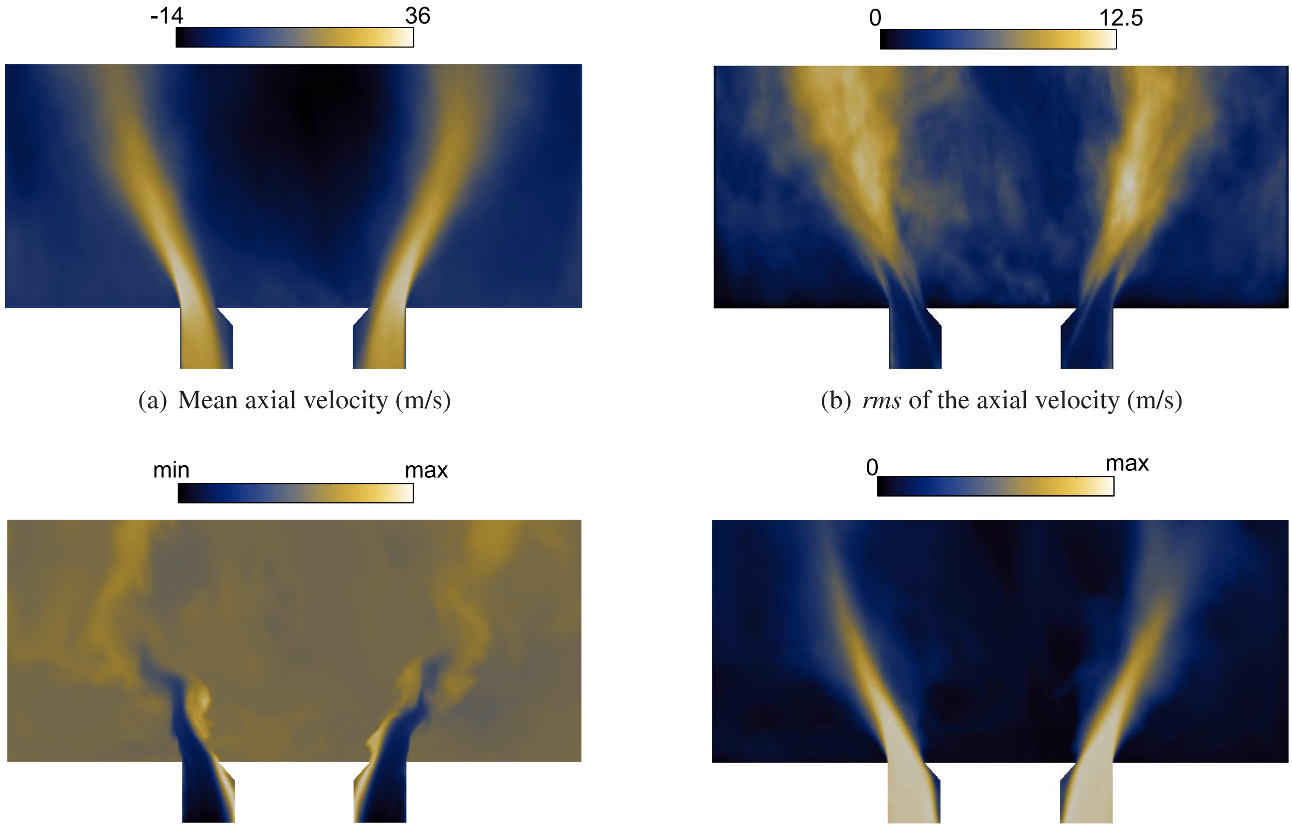
specific case of entropy waves, only very recently the phenomena characterizing the advection of the entropy spots have been investigated. Recent attempts in analysing the entropy wave diffusion and dispersion in a more comprehensive way are represented by the theoretical work by Sattelmayer [14], where the dispersion due to a non-uniform mean velocity profile was studied, and the investigations by Morgans et al. [15] and Giusti et al. [16] where the convection of entropy spots was analysed through numerical simulations and entropy transfer functions to be used in low-order models were proposed. Morgans et al. [15] performed Direct Numerical Simulations (DNS) of the advection of entropy perturbations in a channel flow between two parallel plates. This study highlights the fundamental role of entropy dispersion in the decay of entropy waves pointing out that entropy spots can reach the exit nozzle of the combustor with a significant amplitude. Giusti et al. [16] investigated the dispersion and diffusion of entropy waves in a pipe flow using both experiments and numerical simulations based on the Large-Eddy Simulation (LES) approach. Simulations of entropy convection in fully-developed pipe flows showed that the decay of entropy fluctuations is affected by both the shear dispersion due to a non-uniform mean velocity profile, and turbulent mixing and diffusion. The effect of turbulent mixing and diffusion was found to increase with increasing axial distance and with decreasing wavelength of the entropy fluctuations.

Some of the results obtained by Giusti et al. [16] in a turbulent pipe flow simulation are shown in Fig. 2 (for details regarding the numerical setup the reader is addressed to Ref. [16]). Uniform entropy perturbations are imposed at the inlet of the pipe and then transported by the turbulent flow. The standard deviation (*rms*) of the entropy field indicates that the entropy perturbation decays on moving towards the outlet and that the attenuation also varies along the radius of the duct. Fluctuations along the axis of the duct survive longer whereas close to the walls the perturbations decay faster. Because of the smaller convection velocity, entropy perturbations near the wall are characterized by a smaller wavelength, making the turbulent mixing and diffusion more effective [10, 2], therefore justifying the behaviour observed in the simulations.

Results obtained in numerical simulations can also be exploited to compute entropy transfer functions to be used in low-order network codes. For example, considering entropy fluctuations averaged over the cross section, the relation between the entropy perturbation at a given axial location and the one imposed at the inlet can be found [15, 16, 17]. It was found [16] that in a fully developed pipe flow the magnitude of the entropy transfer function scales well with a Helmholtz number defined as $He = fz/U_b$, where z is the axial distance along the duct, U_b is the bulk velocity of the flow and f is the frequency of the perturbation (see also the results shown in Fig. 4).

2.2 Dispersion and diffusion of entropy waves in real engines

The flow field in real combustors is usually characterized by complex flow structures that can be far from the simple channel and duct flows considered in the fundamental studies described so



(a) Mean axial velocity (m/s)

(b) *rms* of the axial velocity (m/s)

(c) Instantaneous entropy field

(d) *rms* of the entropy field

Figure 3: Convection of entropy waves by a recirculating swirling flow (frequency of the entropy fluctuation f=200 Hz).

far. Recirculation regions induced by swirling flows are usually present to stabilize the combustion process and the cross section of the combustor changes along the axial direction. The residence time of the entropy perturbation strongly depends on the region of the combustors where the fluctuation is generated and the flow path followed by the perturbation. For instance, the residence time of perturbations trapped in the recirculation region is expected to be very high and the strong velocity fluctuations can further increase their attenuation. Therefore, further investigations in configuration much closer to real engines are necessary to better understand the dispersion and diffusion of entropy waves in realistic cases.

Figure 3 shows results from a LES simulation of the transport of entropy waves in a flow field closer to the one typically found in real combustors, with the presence of swirling flows and recirculation regions. Following the approach described in Ref. [16], a uniform entropy perturbation (peak value equal to 5% of the mean value) at a given frequency was introduced upstream of the bluff-body and then transported in the main chamber by a swirling flow. The mean axial velocity shows the typical flow structure of swirl-stabilised flames with the presence of a recirculation zone in the region above the bluff-body. The strong swirling flow leads to high fluctuations of the velocity field, as highlighted by the *rms* of the axial velocity. The entropy fluctuations transported from the inlet are partly entrained by the central and side recirculation zones. It is also evident from the *rms* of the entropy field that the entropy fluctuations decay faster as they approach the shear layer and the region characterized by high turbulent fluctuations. It is important to point out that results also depend on the frequency of the perturbation and a different attenuation of the entropy perturbation might be found by changing the frequency of the entropy wave. However, this example allows us to point out that the increased level of turbulence and the recirculation regions induced by swirling flows can give an important contribution to the attenuation of the entropy fluctuations, especially in the case of small wavelength of the entropy wave compared to the length scales associated with mixing.

A more realistic simulation of the advection of entropy fluctuations in gas-turbine combustors was recently conducted by Xia et al. [17] using LES. A Gaussian entropy perturbation was introduced at the flame location and then transported as a passive scalar. Both the case of advection due to the mean flow only and advection with velocity fluctuations were considered. The results show that the presence of velocity fluctuations, and therefore turbulent mixing, further contributes to the smearing of the entropy perturbation leading to a decrease in the magnitude of the entropy transfer function, evaluated from the flame to the outlet of the combustor, compared to the case where only the transport by the mean flow was considered [17]. This consideration is also important for the use and development of low-order models for the entropy advection. Models that consider only the dispersion due to the mean velocity profile may be inaccurate when highly unsteady flows are present, as also recently pointed out by Mahmoudi et al. [18].

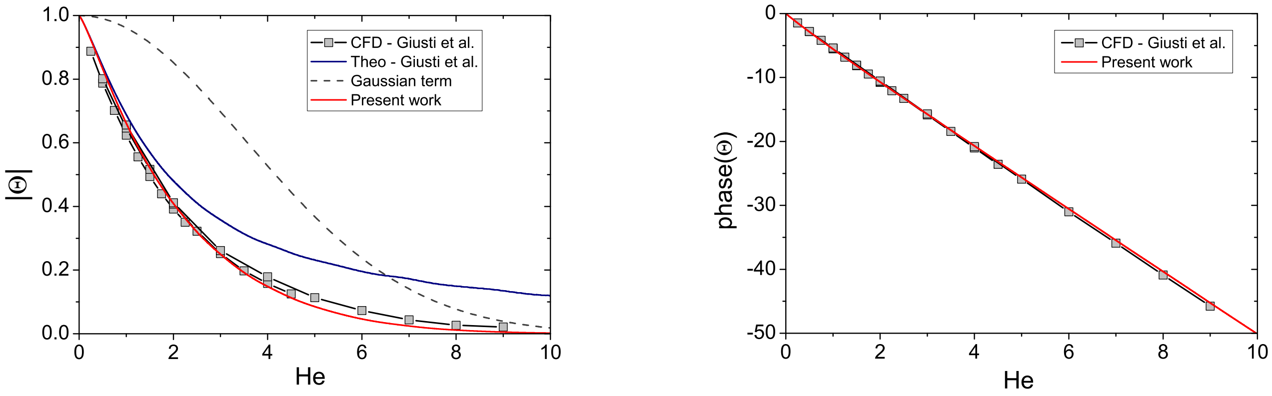
The characterization of entropy dispersion and diffusion in a configuration representative of real engines was also the focus of the experimental work by Wassmer et al. [19]. The attenuation of entropy perturbations was investigated in a model combustor characterized by a realistic swirling flow. It is interesting to note that also in this study the entropy transfer function was found to scale well with the Helmholtz number previously introduced, supporting the findings by Giusti et al. [16] for the fully developed pipe flow. However, this does not imply that low-order models developed for simplified flows are also applicable in the case of real combustors and further investigations are necessary to assess their reliability.

2.3 Low-order models

Low-order acoustic network codes are widely used as preliminary design tools for the thermoacoustic analysis and noise prediction of real systems. However, the convection of entropy waves is usually modelled in a simple way with the entropy waves either advected without attenuation or completely diffused [20]. The first attempt to include scalar dispersion in low-order network codes is represented by the theoretical work by Sattelmayer [14] where a model based on the differential time delay of perturbations advected by a non-uniform mean velocity profile was presented. Based on their DNS simulations, Morgans et al. [15] developed a Gaussian model for the entropy dispersion able to give a good agreement with the magnitude of the entropy transfer function evaluated from the numerical simulation. It should be noted that, in principle, this model can also consider the effect of turbulent mixing and diffusion if the model parameters are evaluated using the entropy profile predicted by the numerical simulation (as long as such effects are included in the computation). A theoretical model based on the advection of entropy perturbations in fully developed pipe flows was also proposed by Giusti et al. [16]. This model considers only the effect of the differential time delay due to a non-uniform mean velocity profile and the resulting entropy transfer function, Θ, can be formulated as follows:

$$\Theta = 2\int_0^1 \widetilde{r}\widetilde{u}_z(\widetilde{r})\exp[-2i\pi He/\widetilde{u}_z(\widetilde{r})]\mathrm{d}\widetilde{r} \qquad (1)$$

where $\widetilde{r}$ is the non-dimensional radius and $\widetilde{u}_z(\widetilde{r})$ is the non-dimensional mean axial velocity inside the duct (for more details see Ref. [16]). According to the model of Eq. 1, the entropy dispersion due to the mean velocity profile only depends on two parameters, the Helmholtz number and the non-dimensional velocity profile. The former gives a measure of the distance travelled by the wave relative to a representative wavelength of the entropy perturbation (or the residence time compared to the period of the perturbation), whereas the latter is related to the dispersion of the time delay over the cross section. Furthermore, it is important to point out that this model only requires the mean velocity profile as an input which can obtained for example through RANS simulations of the flow. A good agreement between this model and the entropy transfer function predicted by numerical simulations was found (see also Fig. 4). In particular, the phase of the entropy transfer function



(a) Magnitude (b) Unwrapped phase (rad)

Figure 4: Comparison between the entropy transfer function of Eq. 2 and numerical data from Ref. [16]

predicted by the model was in excellent agreement with numerical simulations in the entire range of He number, suggesting that the time delay of entropy perturbation averaged over the cross section is mainly influenced by the convection due to the mean velocity profile. As far as the magnitude of the entropy transfer function is concerned, the agreement was good for low values of He, whereas for high values of He the magnitude of the entropy transfer function predicted by the theoretical model was higher than the value from the numerical simulations. As already pointed out, the theoretical model considers only the dispersion of entropy waves due to a non-uniform mean velocity profile and therefore this deviation was attributed to the effect of turbulent mixing and diffusion which might have an important effect at high He (i.e. long distances travelled by the wave or short wavelengths of the entropy perturbation).

Because of the complex features of the flow field in gas-turbine combustors, low-order models derived for simplified flows might not be fully representative of the entropy advection in real engines and a proper calibration for every specific case might be necessary. Considering the dispersion (differential time delay) due to a non-uniform mean velocity profile as the key contribution for the attenuation of entropy waves, Mahmoudi et al. [18] proposed a model for entropy dispersion, suitable for gas turbine applications, able to capture the complex flow features of a real combustor. The model is based on the advection of entropy perturbations along mean streamlines (extracted for example from a numerical simulation of the combustor). The capability of the model to capture the entropy attenuation was indirectly assessed through comparisons with noise measurements at the exit of the engine. The better prediction of noise level at low and high frequencies, compared to simulations without entropy dispersion, indicates the importance of including entropy attenuation in low-order models of real engines [18]. However, the over-prediction of the noise level at intermediate frequencies also points out that much more effort is required in the modelling of entropy transport in both the combustor and turbine nozzles, and an aspect to be considered is the modelling of the effect of turbulent mixing in the combustion chamber [18].

As previously mentioned, an attempt to describe the effect of turbulence on the dispersion of passive scalars in pipe flows was already made in the early work by Taylor [13]. It was shown that a small amount of a passive scalar (concentration of a species), rapidly injected in a turbulent pipe flow, is transported by the flow field in such a way that the profile of the concentration at long distance from the injection location is very close to a Gaussian profile as long as the Reynolds number of the flow is sufficiently high. At short distances and for low Reynolds numbers the shape of the concentration profile is not symmetric and the Gaussian-like shape exhibits a longer tail on one side (as also observed in recent DNS simulations [15]). The deviation at smaller Reynolds numbers was attributed to the effect of the laminar sublayer [13]. It should be noted that the resulting shape observed at a given location of the duct is the result of both the dispersion due to a non-uniform mean velocity profile and the turbulent mixing. In the work by Morgans et al. [15], the non-symmetric shape at the exit of the channel was approximated with a Gaussian profile to build a Gaussian model for the entropy dispersion. On the contrary, in the model proposed by Giusti et al. [16] the contribution of turbulent mixing was neglected and a good prediction of both the phase and the magnitude of the entropy transfer function was found in the case of negligible turbulent mixing.

On the basis of the previous observations and considering the case of a turbulent pipe flow, a semi-empirical model for the entropy transfer function that includes the effect of both dispersion and turbulent mixing can be obtained by combining the model by Giusti et al. [16] with a Gaussian model [15], where the former is used to describe the dispersion due to a non-uniform mean velocity profile and the latter to model the turbulent mixing contribution. The resulting entropy transfer function can be formulated as follows:

$$\Theta = 2\mathrm{exp}[-(He/K)^2] \int_0^1 \widetilde{r}\widetilde{u}_z(\widetilde{r})\mathrm{exp}[-2i\pi He/\widetilde{u}_z(\widetilde{r})]\mathrm{d}\widetilde{r} \tag{2}$$

where K is the parameter describing the strength of the turbulent mixing and generally decreases with the increase of the turbulence effects. Figure 4 compares the entropy transfer function predicted by this model (a constant value K=5.0 has been used) with data from LES simulations [16]. The model shows a promising capability of capturing both the magnitude and the phase of the entropy transfer function in the entire range of Helmholtz numbers. However, it is important to point out that a proper calibration of the parameter K might be necessary depending on the characteristics of the flow. Extension of this model to configurations closer to real engines will be the focus of future work.

3. Concluding remarks

Some of the recent advances in the modelling of entropy dispersion and diffusion have been reviewed and discussed with a focus on the application to gas-turbine combustors. As highlighted through LES simulations of entropy advection in a configuration much closer to real engines, the presence of swirling flows and recirculation regions that typically characterize real combustors, makes both the dispersion due to a non-uniform mean velocity profile and phenomena associated to turbulent mixing important aspects to be considered for a proper evaluation of the attenuation of entropy waves. A simplified semi-empirical model able to include both dispersion and turbulent mixing in low-order acoustic network codes was proposed. Assessment against numerical simulations of entropy advection in fully-developed pipe flow shows that the model is able to properly predict both the magnitude and the phase of the entropy transfer function. The flow field in a combustion chamber is quite far from the simplified configurations investigated in fundamental studies and models developed for simple flows might not give reliable predictions in engines without a proper calibration. Therefore, future work should focus on the investigation of entropy attenuation in configurations much closer to engine applications and on the development of low-order models able to give a good representation of the dispersion and diffusion of entropy waves in real engines.

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