EFFECT OF THERMAL ENVIRONMENT ON VIBRATION ANALYSIS OF ISOTROPIC MICRO PLATE WITH INCLINED CRACK BASED ON MODIFIED COUPLE STRESS THEORY

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An analytical model for vibration analysis of partially cracked thin rectangular isotropic micro plate as affected by crack orientation under thermal environment is presented. The analytical model is derived based on the Kirchhoff's classical plate theory and the modified couple stress theory. The arbitrarily oriented crack is in the form of continuous line and is located at the center of the rectangular plate. Line spring model is used to incorporate the effect of crack in the form of moments and in-plane forces. Also, the moment's and in-plane forces arising due to temperature is introduced in the analytical model. A single internal material length scale parameter is used to capture the size effect. The solution for fundamental frequencies of the cracked plate is obtained by Galerkin's method by neglecting the geometric non linearity. Results for the fundamental frequencies as affected by the temperature, internal material length scale parameter and crack orientations are presented for simply supported boundary condition.

Keywords: vibration, crack, micro-plate, thermal

1. Introduction

Thin plates are extensively used as various engineering structures and find its application in various fields such as civil, mechanical and aerospace. Vibration analysis of these plates is an important design parameter for such structures. Discontinuities and flaws in the form of holes, voids and cracks may affect the dynamic stability of these pates which may lead to severe performance threats. Rice and Levy [1] developed the line spring model to obtain the stress intensity factor of plate containing a line crack. They also showed the variation of tensile and bending compliances with crack depth. Israr et al. [2] extended the line spring model developed by Rice and Levy [1] and proposed an analytical model for vibration analysis of partially cracked isotropic plate. Ismail et al. [3] adopted a similar approach and developed an analytical model for vibration analysis of isotropic plate with arbitrarily oriented crack. Joshi et al. [4], further used line spring model to analytically evaluate the vibration characteristics of isotropic plate with centrally located two perpendicular cracks. Their model included the results for surface as well as internal crack. Further extending their work [5][6] they presented results for cracked orthotropic and FGM plate. The authors also presented the effect of thermal environment on vibration characteristics of cracked plates [7][8]. Based on modified line spring model Bose et al. [9] developed an analytical model for vibration analysis of cracked isotropic plate with various crack location and orientation.

Experiments show that the vibration characteristics of plate are affected by microstructure. Many theories have been proposed to capture the size effect of the plates. One such theory is modified couple stress theory (MCST) which is developed by Yang et al. [10]. In their model the couple stress tensor was symmetric and only one internal material length scale parameter was considered to capture the effect of microstructure. Yin et al. [11] used modified couple stress theory and developed an analytical model for vibration analysis of micro plate and concluded that the frequency is affected by the length scale. Recently Gupta et al. [12], developed an analytical model based on classical plate theory in conjunction with modified couple stress theory for vibration analysis of cracked isotropic and FGM micro plate. Further, Gupta et al. [13] also presented the effect of fiber orientation on fundamental frequency of cracked orthotropic plate with various crack location. The literature shows that the vibration characteristics of cracked plate depend on crack length, aspect ratio, crack location, crack depth and boundary conditions. To the best of the author's knowledge, literature lacks in the results for vibration analysis of isotropic micro plate with arbitrarily oriented crack in thermal environment.

2. Problem Description

The present work references the analytical model proposed by Ismail et al. [3] and Joshi et al. [7] for partially cracked isotropic plate with inclined crack [3] and for partially cracked isotropic plate under thermal environment. The present model adopts similar approach and applies it for the case of a rectangular partially cracked isotropic plate with inclined crack under thermal environment. The present model also includes the effect of microstructure on the vibration characteristics of plate. Thus the present work addresses the following;

i) Vibration analysis of isotropic plate with arbitrarily oriented crack.
ii) The effect of single internal material length scale parameter (based on the modified couple stress theory).
iii) The effect of thermal environment on vibration characteristics of cracked micro plate.

The configuration of the thin isotropic plate with arbitrarily oriented crack is shown in Fig.1. L_1 and L_2 are the plate lengths along x and y axis respectively. 'h' is the thickness of the plate along z axis. Crack length is represented by '2a' and the crack orientation is represented by angle β. Line spring model is used to incorporate the effect of crack in the form of moment and membrane forces.

Employing the equilibrium principle based on the classical plate theory, the equation of motion of the cracked isotropic plate is obtained. The effect of microstructure is incorporated by using the single internal material length scale parameter. Also, the effect of thermal environment is incorporated by moment's and in-plane forces arising due to temperature. The solution for fundamental frequencies of the cracked plate is obtained by Galerkin's method by neglecting the geometric non linearity. Results for the fundamental frequencies as affected by plate aspect ratio, crack length, crack orientation and internal material length scale parameter are presented for simply supported two boundary condition (SSSS).

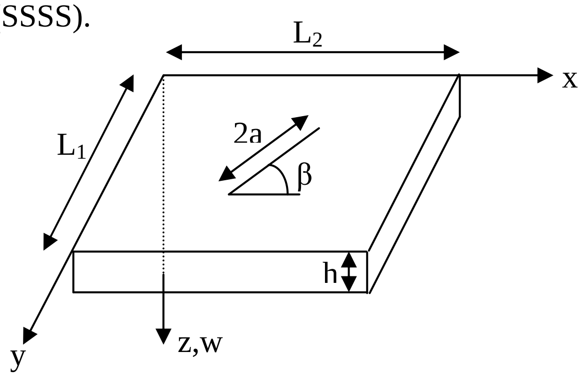

Figure 1. Isotropic plate containing a single crack located at centre of the plate

3. Governing Equation

The equation of motion governing vibrations of isotropic rectangular micro plate with arbitrarily oriented crack located at the centre of the plate is derived following the equilibrium principle. It is assumed that the thickness of the plate is small in comparison with its other dimensions. The stress normal to the mid plane and the effect of rotary inertia and shear deformation are neglected. The equation of motion for an intact rectangular isotropic micro plate in the absence of thermal environment based on modified couple stress theory can be stated as [11]

$$(D + D^l)\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h\frac{\partial^2 w}{\partial t^2} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + P_z \quad (1)$$

Where P_z is the load, w is the transverse deflection, ρ is the density, h is the thickness of the plate and $N_x, N_y, N_{xy} = N_{yx}$ are the in-plane forces per unit length. D is the flexural rigidity defined as $D = \frac{Eh^3}{12\,(1-\vartheta^2)}$ and $D^l = \frac{El^2 h}{2(1+\vartheta)}$ is the addition flexural rigidity due to size effect.

The equation of motion of an isotropic plate with arbitrarily oriented crack in the absence of thermal environment is given by Ismail et al. [3]

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h\frac{\partial^2 w}{\partial t^2} + N_{\not{y}}\frac{\partial^2 w}{\partial y^2} + N_{\not{x}}\frac{\partial^2 w}{\partial x^2} + 2N_{\not{x}\not{y}}\frac{\partial^2 w}{\partial x \partial y} + 2\frac{\partial^2 M_{\not{x}\not{y}}}{\partial x \partial y} + \frac{\partial^2 M_{\not{y}}}{\partial y^2} + P_z \quad (2)$$

Where $N_{\not{y}}$, $M_{\not{y}}$, $N_{\not{x}}$, $M_{\not{x}}$, $N_{\not{x}\not{y}}$ and $M_{\not{x}\not{y}}$ are the crack terms.

The equation of motion of cracked isotropic plate under thermal environment can be found in the work of Joshi et al. [7]. Apart from moment and in-plane forces arising due to crack they also included the in-plane moments and forces due to thermal environment.

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h\frac{\partial^2 w}{\partial t^2} - M_T\frac{\partial^2 w}{\partial x^2} - M_T\frac{\partial^2 w}{\partial y^2} - N_{Ty}\frac{\partial^2 w}{\partial y^2} - N_{Tx}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 M_{\not{y}}}{\partial y^2} - N_{\not{y}}\frac{\partial^2 w}{\partial y^2} + P_z \quad (3)$$

A similar approach has been adopted following the Eq. (1)-(3) to obtain the new governing equation for vibration analysis of isotropic micro plate with arbitrarily oriented crack in the presence of thermal environment. The new governing equation is derived for uniform heating of plate and can be stated as

$$(D + D^l)\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h \frac{\partial^2 w}{\partial t^2} - M_T \frac{\partial^2 w}{\partial x^2} - M_T \frac{\partial^2 w}{\partial y^2} - N_{Ty}\frac{\partial^2 w}{\partial y^2} - N_{Tx}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 M_{\bar{y}}}{\partial y^2} + 2\frac{\partial^2 M_{\bar{xy}}}{\partial x \partial y} - N_{\bar{y}}\frac{\partial^2 w}{\partial y^2} + P_z \quad (4)$$

Where

$$\left.\begin{aligned} N_{\bar{y}} &= \frac{2a}{(6\alpha_{bt}+\alpha_{tt})(1-\vartheta^2)h+2a} N_{Ty} \\ M_{\bar{xy}} &= \frac{a(sin2\beta)}{3\left(\frac{c_{bt}}{6}+c_{bb}\right)(1+\vartheta)h+2a} M_y \\ M_{\bar{y}} &= \frac{a(1+cos2\beta)}{3\left(\frac{\alpha_{bt}}{6}+\alpha_{bb}\right)(3+\vartheta)(1-\vartheta)h+2a} M_y \\ N_{Tx} &= N_{Ty} = \frac{E\alpha h T_c}{1-\nu} \text{ where } T_c \text{ is the temperature.} \end{aligned}\right\} \quad (5)$$

Thermal moment M_T is neglected in stiffness due to the uniform heating of the plate. The shear force N_{Txy} is also neglected as temperature does not affect the shear component [7]. It is assumed that $N_{\bar{y}}$ is not affected with the crack orientation. $\alpha_{bt}, \alpha_{bb}, \alpha_{tt}, c_{bt}, c_{bb}$ and c_{tt} are the crack compliance coefficients which are easily available in various literature [2][7]. The solution of the governing equation is obtained by defining the characteristic modal functions depending on the boundary conditions of the plate. Applying Galerkin's method, the solution for the transverse deflection is

$$w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} X_m Y_n \emptyset_{mn}(t) \quad (6)$$

where X_m and Y_n are the characteristic modal functions satisfying the boundary conditions. $A_{mn}$is arbitrary amplitude and $\emptyset_{mn}(t)$ is time dependent modal coordinate. On solving, the final equation for mass and stiffness can be written as

$$M_{mn} = \rho h \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \int_0^{L_1}\int_0^{L_2} X_m^2 Y_n^2 \, dxdy \quad (7)$$

$$K_{mn} = (D + D^l)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}\int_0^{L_1}\int_0^{L_2}\Big\{ (X_m{}^{iv}Y_n + 2X_m{}^{ii}Y_n{}^{ii} + Y_n{}^{iv}X_m) - \frac{a(1+cos2\beta)(\vartheta X_m{}^{ii}Y_n{}^{ii} + Y_n{}^{iv}X_m)}{3\left(\frac{\alpha_{bt}}{6}+\alpha_{bb}\right)(3+\vartheta)(1-\vartheta)h+2a} - \frac{2asin2\beta(\vartheta X_m{}^{iii}Y_n{}^{i} + Y_n{}^{iii}X_m{}^{i})}{3\left(\frac{\alpha_{bt}}{6}+\alpha_{bb}\right)(1+\vartheta)h+2a} + \frac{E\alpha h T_c}{1-\nu}A_{mn}\int_0^{L_1}\int_0^{L_2}\{X_m^{ii}Y_n^2X_m + Y_n^{ii}X_m^2Y_n + \frac{2aY_n^{ii}X_m^2Y_n}{(6\alpha_{tb}+\alpha_{tt})(1-\nu^2)h+2a} + \frac{2aX_m^{ii}Y_n^2X_m}{(6\alpha_{tb}+\alpha_{tt})(1-\nu^2)h+2a}\}\Big\} X_m Y_n dxdy \quad (8)$$

From Eq. (7) and Eq. (8) the natural frequency can be calculated as $\omega_{mn}{}^2 = K_{mn}/M_{mn}$.

4. Results and Discussion

The results are presented for a square isotropic plate made up of Aluminium with E=70.3 GPa, Poisson's ratio ν = 0.33 and density ρ = 2660kg/m^3. The dimension of the square isotropic plate is taken as L_1/h=100 and crack length 'a' is considered as 0.01m for all the cases. Table 1 shows the critical buckling temperature Tbcr for the cracked square isotropic plate. The effect of internal material length scale parameter l on critical buckling temperature is also presented. It is seen that the critical buckling temperature increases with the increase in internal material length scale parameter which is due to the inclusion of additional flexural rigidity by modified couple stress theory.

Table 1. Critical buckling temperature in ºC for cracked square plate (a=0.01m) of side 1m for different crack orientations and simply supported boundary condition (SSSS).

Crack Orientation	Critical Buckling Temperature Tbcr			
	CPT	MCST		
	l=0	l=0.0005	l=0.001	l=0.003
0	4.674*	4.721	4.862	6.365
20	4.708	4.755	4.897	6.410
40	4.794	4.842	4.986	6.529
60	4.892	4.940	5.088	6.661
80	4.956	5.006	5.154	6.748

*Ref [7]

The uniform rise in temperature is expressed as non dimensional temperature T*=Tc/Tbcr where Tc is the rise in temperature. Table 2 shows the variation of T* with crack orientation for square isotropic plate with crack length a=0.01m. It is seen that frequency increases with the increase in crack orientation which is in similar fashion as obtained by Ismail et al. [3] in the absence of thermal environment which is due to the fact that only uni-axial loading is considered for analysis. It is also seen in Table 2 that fundamental frequency reduces as the temperature increases towards critical buckling temperature which is indeed true and similar pattern was obtained by Joshi et al. [7] for their work on crack parallel to either axis in presence of thermal environment.

Table 2. Fundamental frequency for cracked square plate (a=0.01m) of side 1m for different crack orientations and simply supported boundary condition (SSSS)

T*	Fundamental Frequency (rad/s)				
	Crack Orientation (degrees)				
	0	20	40	60	80
0	301.1128	302.2046	304.9516	308.0456	310.0484
0.1	285.663	286.6986	289.3046	292.2391	294.1386
0.2	269.3285	270.3047	272.7616	275.5274	277.3175
0.3	251.937	252.85	255.1481	257.7343	259.408
0.4	233.2525	234.0975	236.225	238.6181	240.1666
0.5	212.9346	213.7057	215.6478	217.8307	219.243
0.6	190.4616	191.1508	192.8877	194.838	196.0994
0.7	164.9547	165.551	167.055	168.7408	169.8307
0.8	134.7016	135.1874	136.4151	137.7863	138.6721
0.9	95.2836	95.625	96.4923	97.451	98.0685

Fig. 2 shows the variation of fundamental frequency with T* for different internal material length scale parameter thereby depicting the importance of size effect in the case of micro plate. It can be seen that fundamental frequency of micro plate highly varies with the internal material length scale parameter in the presence of thermal environment. It can be seen that the difference in frequency is more for lower temperatures and as we move towards the critical buckling temperature this difference gets narrower.

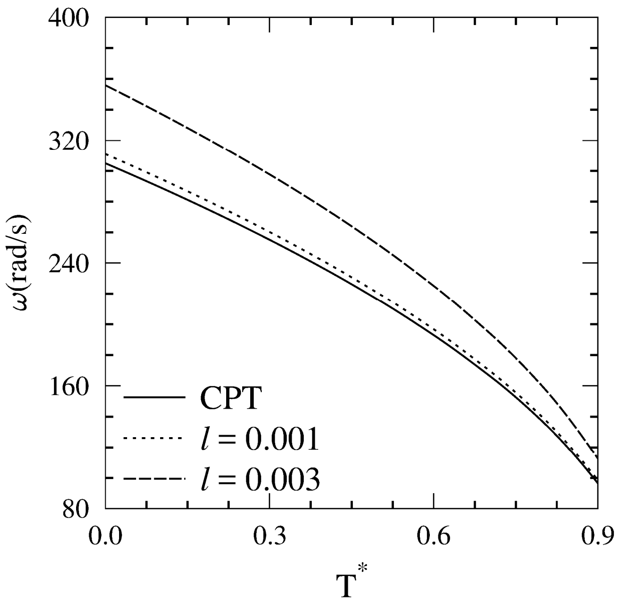

Figure 2. Variation of fundamental frequency with T* for cracked isotropic plate with a=0.01m and β=20º for various internal material length scale parameter

5. Conclusion

An analytical model for vibration analysis of partially cracked isotropic micro plate under thermal environment is proposed. The approach is based on the classical plate theory in conjunction with the modified couple stress theory. The arbitrarily oriented crack is located at the centre of the plate. An internal material length scale parameter is considered to capture the size effect of micro plate. It is concluded that the fundamental frequencies are affected by the crack length, crack orientation, temperature and boundary condition. Result shows that the critical buckling temperature increases with the increase in internal material length scale parameter and also increases with the increase in crack orientation. Also, the fundamental frequency decreases with the increase in temperature and increases with the increases in internal material length scale parameter. To the best of the authors' knowledge, this is the first attempt to analyze the effect of crack orientation on isotropic micro plate under thermal environment. The present work may be extended to some higher order shear deformation theories with multiple lines or curved cracks.

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