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DEFINITION OF STATIONARITY BASED ON MONITORING THE UNCERTAINTY AT REAL MEASUREMENT CONDITIONS

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In the statistical standard literature the stationarity of a time dependent process generally is defined by the invariance in time of the distribution of the variable like a sound pressure level fluctuating in time. However in reality there cannot exist constant distribution respectively characteristics in time in the strict mathematical sense because the time intervals of observation only can be finite due to practical reasons. Hence on every distribution and characteristics based on it a certain, but evaluable uncertainty is imposed. For monitoring these uncertainties the online-measurement technique, i. e. primarily appropriate software, is already available, also for customers. According to this state of the art the following expanded definition of the stationarity is proposed: Stationarity during a quality controlled measurement process becomes established, when the *upper* confidence limit of the interesting specific characteristic has *no positive* slope in time and correspondingly the *lower* confidence limit of the specific characteristic *drifts upwards or is constant* and, as a third condition, the interesting specific characteristic has adjusted itself to a constant position in time. From this a systematic criteria scheme is established and in examples applied on different in- and outdoor situations of sound impact.

Keywords: Stochastic processes, stationarity, uncertainty, criteria.

1. Introduction

A stochastic process basically is called stationary if its statistical properties are invariant to a shift of the origin [1]. This means that the processes **x**(t) and **x**(t+c) have the same statistics for any value of parameter c. From this it follows for the cumulative distribution $0 \le F(x;t) \le 1$ of the single "states" of – for example – a time series of instant values, that F(x; t) = F(x; t+c) for any c. Hence, if the process is stationary, the cumulative distribution is independent of time, i. e.

$$F(x;\, t) = F(x). \tag{1}$$

In consequence of Eq. (1) also the following Eq. (2),

$$\frac{\partial F(x;t)}{\partial t} = 0 \tag{2}$$

is valid and further, by definition

$$\frac{\partial F(x;t)}{\partial x} := f(x)\,, \tag{3}$$

where f(x) denotes the probability density of the instantaneous states of the stochastic process. If Eq. (1) is valid then the process is strict-sense, respectively strongly, stationary.

In a wide sense a stationary process is defined if only its expectation E{**x**(t)}, i. e. its mean, is invariant in time [1]. Principally not only the average but also a certain partition of the distribution can be stationary in the sense, that for a certain parameter value x_0 the distribution $F(x_0;t)$ = const. in time, whereas $\frac{\partial F(x;t)}{\partial t} \neq 0$ occurs for $x \neq x_0$. This is possible in real sound measurements, what can be demonstrated by appropriate examples. Furthermore there exist several extensions of the concept "stationarity" which are not relevant in this context and therefore are not mentioned here.

For stochastic processes, fluctuating by their nature, Eqs. (1) and (2) evidently imply that time intervals to be infinite. But in reality this is never the case. A stationary process must show variations of its elements. Otherwise it would be trivial and there would be no stochastic performance. Thus for real fluctuating processes, provided that the in practice always limited observation time is at least several times the nonzero autokorrelation time, "stationarity" exists, but only and inevitable within nonzero variations of distribution parameters.

The variation in time of sound coming from one or – at outdoor situations - simultaneously from more sources can generally be regarded as „stochastic processes". Such processes, more precise the (cumulative) distribution F(x; t) of their instant values can be described by characteristics. Those are the quantiles (percentiles) and the mean. If a fixed quantile, say $1\text{-}q = F(x; t)$, where $q \leq 1$ is set then, if stationarity is absent, x is varying in time. It evidently only is constant in time, if Eq. (1) respectively Eq.(2) is fulfilled.

Within this context there are at least two important aspects to be accounted for especially in noise control matters: For a qualified outdoor noise control stationary boundary conditions concerning the sound impact situation are indispensable. This applies primarily to authorization processes of industrial plants and similar facilities. This of course also applies to measurements for the assessment of a sound situation. On the other hand as a rule in practice it is a tacit understanding that the precondition "stationary", whatever it is, is met.

A further aspect is that comparative measurements only can be meaningful, if the observed situations to be compared each obviously are stationary at their specific levels, relevant for the current assessment. Otherwise it is not clear which status within the instationary process is to be applied for the comparison process.

2. Assessment of stationarity of time limited acoustic signals

2.1 Method

As is well known the kinds of parameters respectively characteristics to describe the distribution F(x) of the instant sound pressure levels (SPL) are the N percent exceedance levels $L_{N\%}$, resp. q := N/100 (for q see below), with N preferably 1; 5; 10; 50; 70; 90; 95 and 99. In addition the equivalent continuous sound pressure level L_{eq} is calculated encompassing the complete distribution. In real measurement situations the mutual relative distant $L_{N\%}$ characteristics in the percent scale can be quite different determined by accordingly different sound sources acting on the immission site. This in consequence can impose accordingly different time scales until they become stationary as defined.

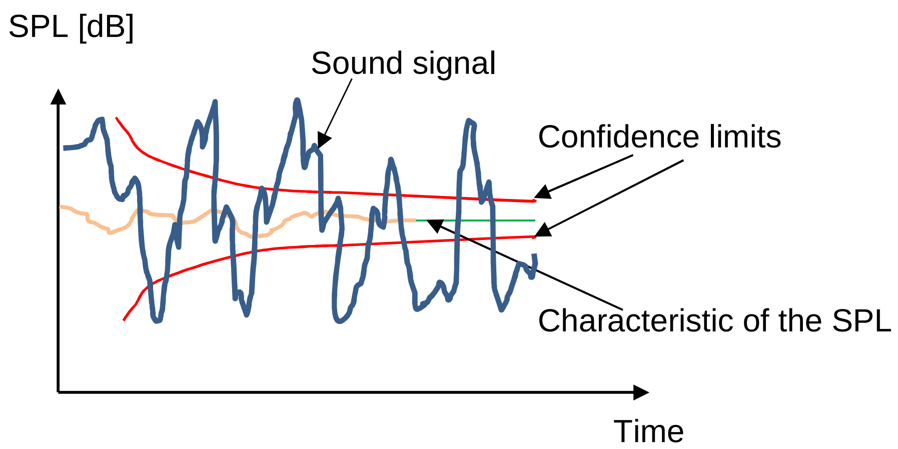

Figure 1: The elements of the method for testing on stationarity,
including the uncertainty due to the random fluctuations of sound pressure level (SPL).

However, the common state of the art is to record and present on screen the kinds of characteristics mentioned above. By this method merely can be ascertained in zero order whether the interesting specific characteristic is approximately stationary or not at the end of the observation time interval. But by this way it cannot be judged in addition whether the characteristic has reached a stationary state within its uncertainty due the stochastic nature of the process or not.

To evaluate also the intrinsic, i. e. inevitable uncertainty of sound measurements over finite time intervals and make it visible, the Company also named on the top of this paper developed an appropriate software in close cooperation with the author, who worked out the theoretical foundations [2][3][4]. This software provides signal processing in online operation followed by optional offline data evaluation in high performance, including also all kinds of characteristics mentioned above.

The confidence interval [5] is one of the measures available to test the degree of achieved stationarity. For the exceedance level half confidence interval, i. e. the level distance V_L of the upper and of the lower confidence limit L_{upper} and L_{lower} from the measured $L_{N\%}$ value itself, respectively the level positions of L_{upper} and L_{lower} can be evaluated according to [2] by

$$V_L := L_{upper} - L_{N\%} = L_{N\%} - L_{lower} = t_{n-1;1-\alpha/2} \cdot \left|\frac{dL}{dq}\right| \sqrt{\frac{\bar{f}}{T}\left(q_w^2 \cdot s_u^2 + q_u^2 \cdot s_w^2\right)} \ . \qquad (4)$$

In Equation (4) are $t_{n-1;1-\alpha/2}$ Student's coverage factor for degree n-1 of freedom and for coverage probability 1-α [5], in the measurement software with α = 0,2. The inverse slope of the cumulative probability distribution function of sound pressure level is denoted by dL/dq and T stands for the measurement time interval. The crossing down and crossing up time intervals of the continuous sound pressure level signal with respect to the exceedance level $L_{N\%}$ are denoted by u_i and w_i. Their associated standard deviations are written as s_u and s_w. The mean frequency of the n occurring stochastic periods u_i+w_i within T is by common definition $n/T := \bar{f}$. By further definition are $q_w \equiv q$, where q denotes the excess fraction, already mentioned above and $q_u+q_w \equiv 1$. Only the excess parameter q is preset as constant in time.

Equation (4) is valid if the crossing up and down time intervals are statistically independent as within the same kind of intervals as in cross correlation between crossing up and down. This is confirmed by observations available until now at least for outdoor sound impacts [6].

Within the observation of a sound impact situation by measurement based on the kind of characteristics mentioned above, there is need of an initial time interval for a configuration of minimum meaningful statistics, i. e. edition of confidence intervals. This is indicated by a gap between the origin of the time scale and the first occurrence of the confidence limits. This is due to the partition variance, which is only meaningful within the percent-space $0 \leq N \leq 100$. This leads to the minimum crossing number n_{min}, until the confidence limits appear which can be estimated by

$$n_{min} = t^2_{n-1;1-\alpha/2}\,(N/100)^2\,(v_u^2 + v_w^2) \qquad (5)$$

with $N \leq 50$ [3]. This is implemented in the software mentioned above. By v_u is denoted the ratio of s_u and the mean of the crossing down intervals. Accordingly v_w is defined. Usually the minimum number of crossings is in the order of magnitude from 5 to 10 at outdoor sound impact measurements. Thus by additional taking into account the existent measurement uncertainty a stationary state cannot be confirmed from the very beginning of an observation.

2.2 Graded criteria for stationarity

For the judgement whether over the measurement time interval a stationary situation of sound impact could be revealed or not, in principle can be symbolized by the following Figures 2 and 3.

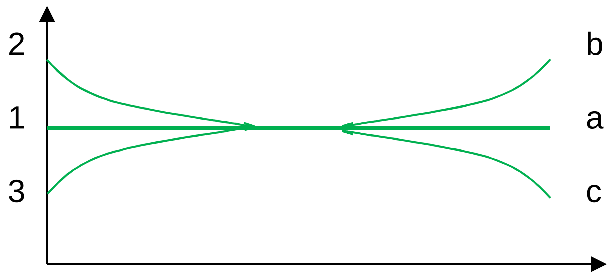

Figure 2: Possible graphs of the characteristic (in green) in dB over time (schematic).

In Figure 2 each end (character) can combine with each initial phase (number). However, for the branches b and c it is irrelevant with respect to a check on stationarity, which configuration out of 1, 2 or 3 combines with one of them because b and c anyway are to be ruled out. For the confidence limits respectively the confidence interval, three kinds of situations at the end of measurement are relevant for the ascertainment of stationarity or not. This shows Fig. 3.

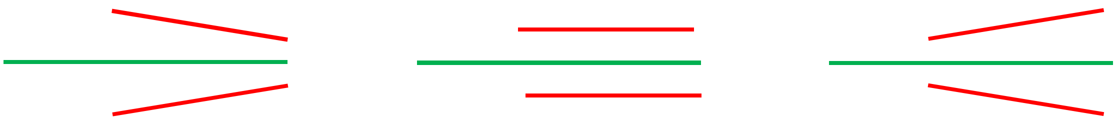

Figure 3: Possible configurations of the confidence limits (in red)
in the final phase of a measurements (schematic).

Table 1: Classification in steps 1.1 to 2.2 of stationary SPL-fluctuations over time

Characteristic at the end of the measurement → / Conf. interval at the end of the measurement ↓	Is constant in time		Drifts
	Final value is retrospective **never** outside of the whole confi-dence track	Final value is retrospective **tem-porary** outside of the whole confi-dence track	
Convergent	**1.1**	**2.1**	none
Constant	**1.2**	**2.2**	none
Divergent	none	none	none

Additional conditions: 1) At the end of a measurement the confidence interval may not exceed a preset width, preferably ± 1 dB or ± 0,5 dB.
2) If in case 1.2 or 2.2 the confidence interval is very narrow, say ≤ ± 0,2 dB, then these ratings can be regarded equivalent with 1.1 and 2.1 respectively.

From the graphs Fig. 2 and Fig 3 classes of stationary fluctuating sound pressure levels over time can be derived. This is shown in Table 1. It also contains the set of disqualifying criteria.

The characteristics and their confidence limits, respectively confidence intervals, are evaluated continuously in online operation. It is to be emphasized that they are representative for the *complete* time interval of measurement. This on the other hand means also that according to Table 1 stationarity can only confirmed (or not) for the specific measurement interval available.

3. Examples

3.1 Emission

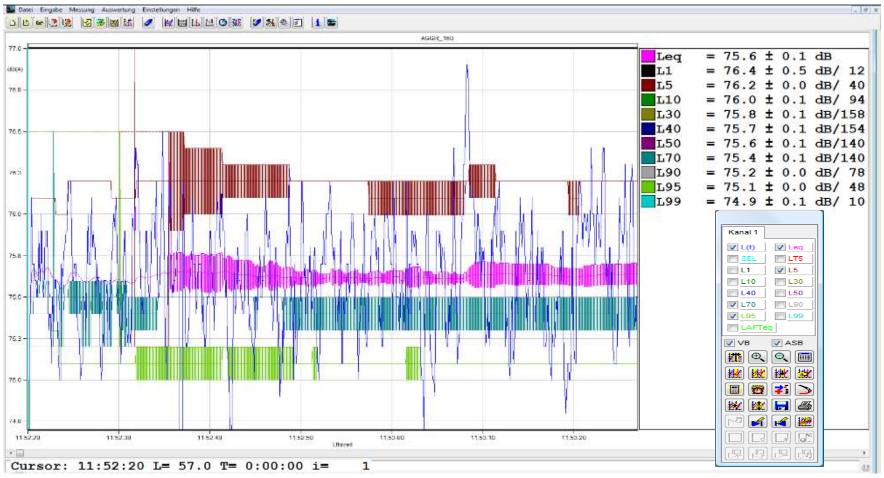

Figure 4: Machinery. Pump station. Distance 2 m. Duration of measurement: 1 min, 10 sec. Range: L_1-L_{99} = 1,5 dB(AF). The edited characteristics L_{95} and L_5 meet criterion 1.1., L_{70} and L_{eq} meet criterion 1.2. The sound emission is completely stationary, as to be expected.

3.2 Speech

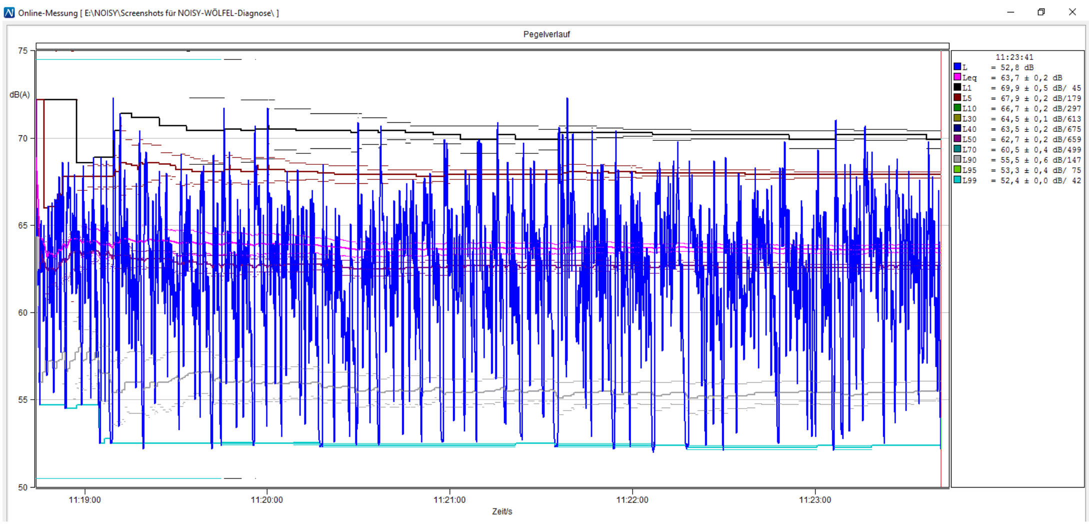

Figure 5: Speech. Lecture on the radio. Duration of measurement: 5 minutes. The edited characteristics L_{70}, L_{eq}, L_5 and L_1 meet criterion 1.2. L_{50} and L_{10} fulfill criterion 2.2. The situation is stationary with respect to all edited characteristics, i. e. stationary at all.

3.3 Impulsive events

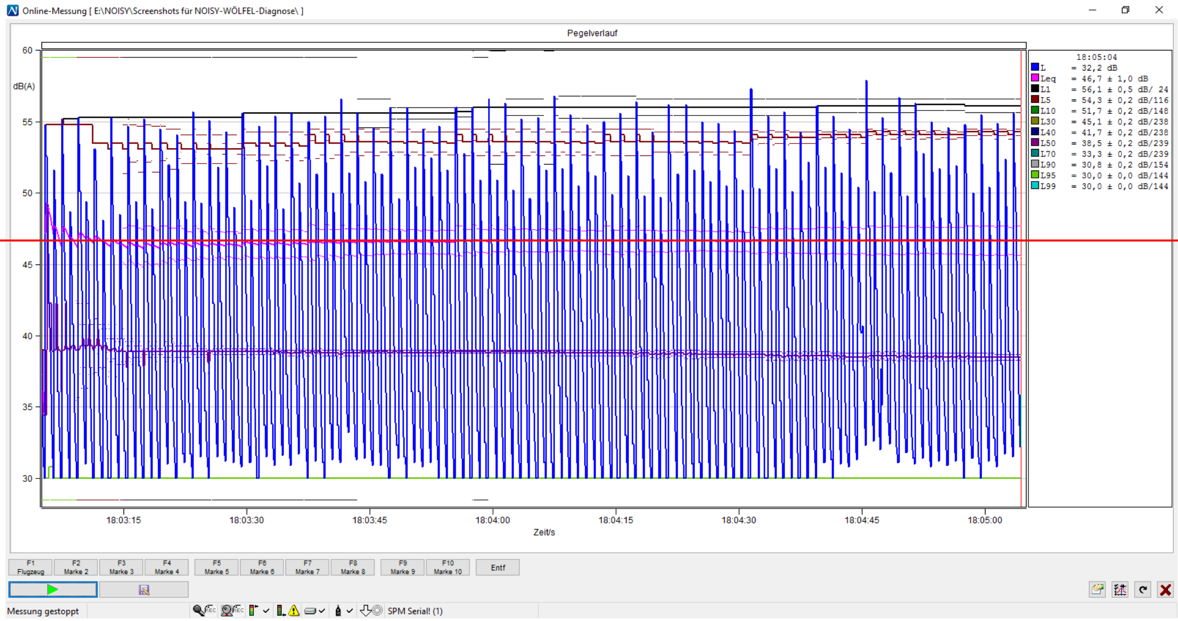

Figure 6: Clock ticking 1 Hz – as a periodical sound for 2 minutes. L_{eq} and L_5 are in accordance with criterion 1.1, whereas L_1 meets 1.2. and L_{50} fullfills only criterion 2.2.
Superposed at the final level of L_{eq}: Red horizontal checkline. The situation is stationary at all.

3.4 Urban sound environment

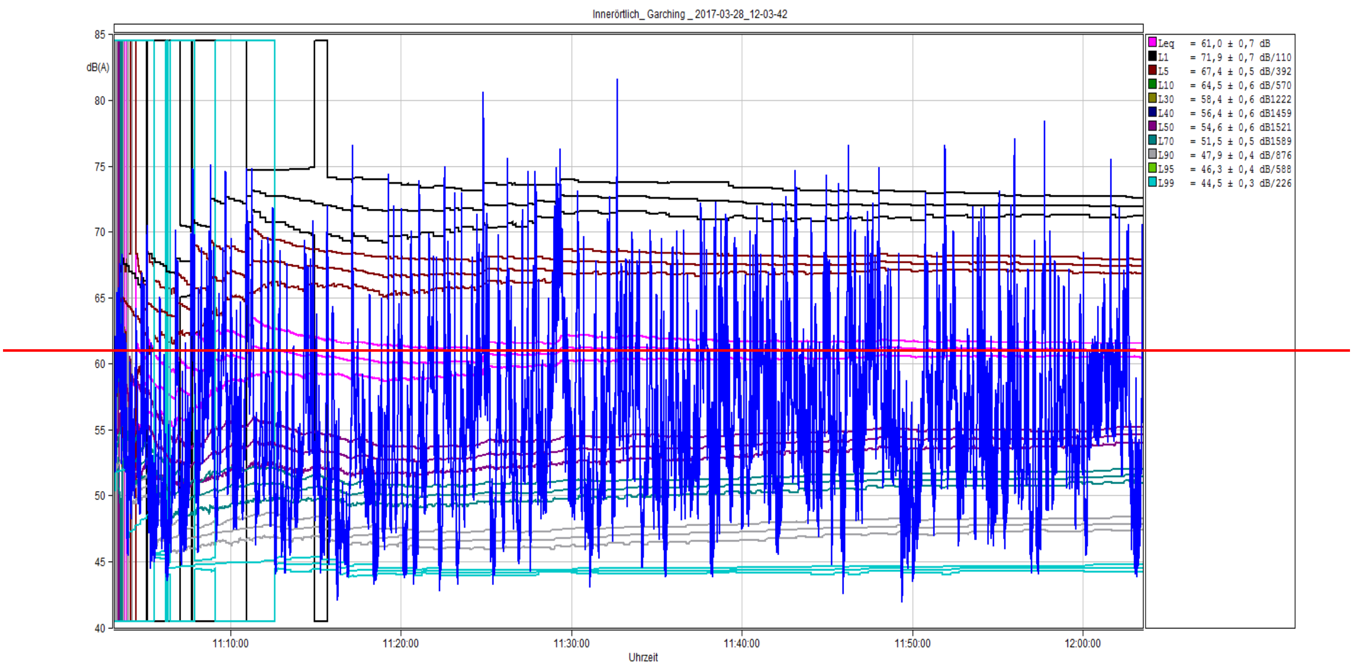

Figure 8: Urban sound situation, dominated by nearby city traffic. Duration of measurement 1 hour. Range: L_1-L_{99} = 27,4 dB(AF). The edited higher characteristics L_{eq}, L_5 and L_1 meet criterion 1.1. Thus they are highly stationary. L_{90}, L_{70} and L_{50} drift upwards: Not stationary. L_{99} is quite stationary, only slightly divergent at the end. Relatively low uncertainty: No confidence interval exceeds ± 0,7 dB(A).
Superposed at the final level of L_{eq}: Red horizontal checkline.

3.5 **Power plant (500 MW)** with traffic noise as background

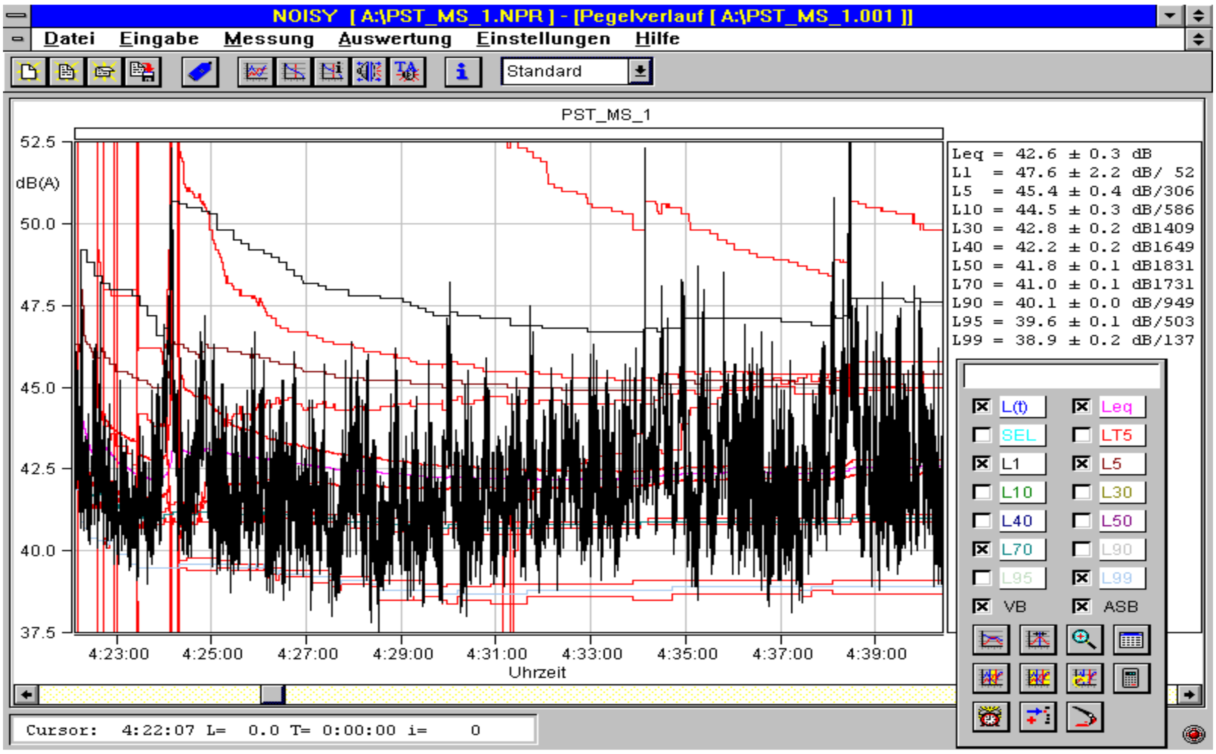

Figure 9a: Power plant. Distance 200 m. Duration of measurement 18 minutes.
Range: L_1-L_{99} = 8,7 dB(A).

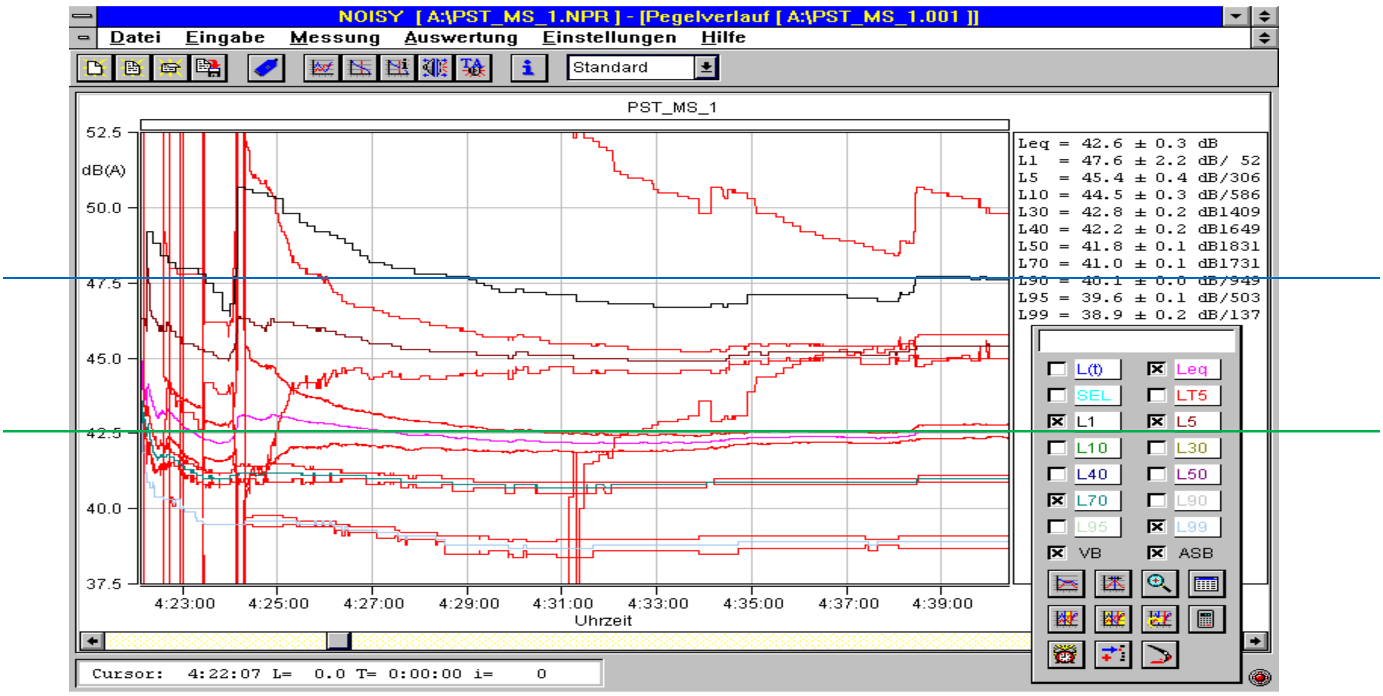

Figure 9b: Power plant, the same as in Fig. 9a. Here only the characteristics over time are edited. L_{70}, L_{eq} and L5 meet criterion 1.2. L_{99} satisfies 2.2. The uncertainty of L_1 is too high although for it criterion 1.1 is met formally. Result: The situation is sufficiently stationary for significant assessment based on L_{eq}. Superposed at the final level of L_{eq} green at final L_1 blue horizontal checkline.

3.6 Aviation

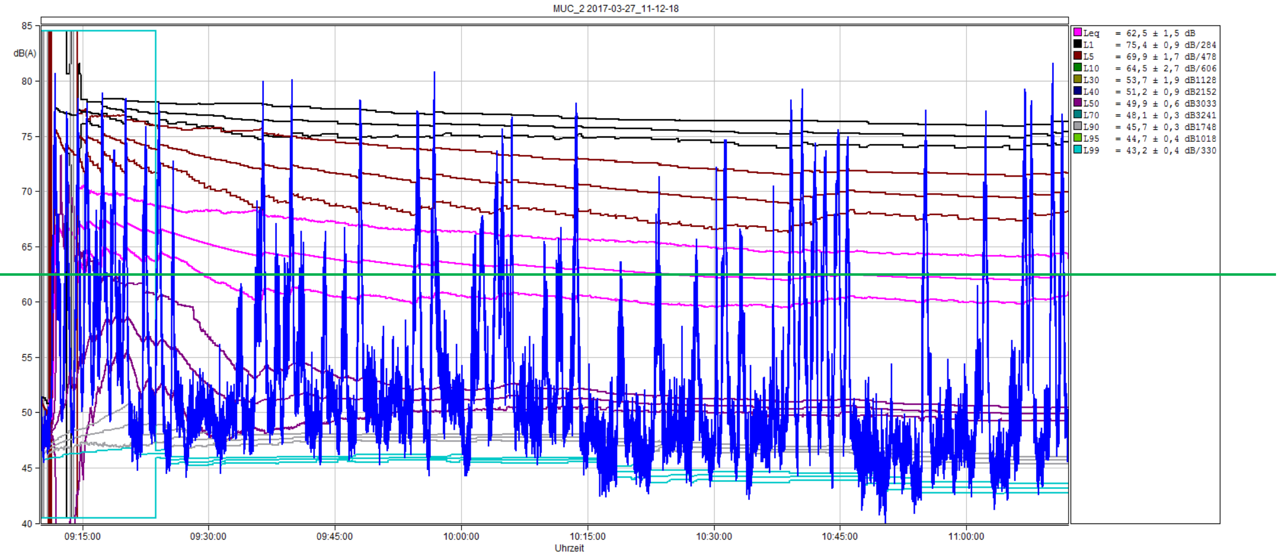

Figure 10: Aircraft noise in the immediate vicinity of a civil airport measured direct under exclusively departure flights. Duration of measurement 2 hours. Range L_1-L_{99} = 32 dB(AF). The edited higher characteristics L_{eq}, L_5 and L_1 meet criterion 1.1 partially justified by the observed drift down of the background noise levels L_{50}, L_{90} and L_{99}. Hence these only just can meet criterion 2.2.

4. Conclusion

The usual definition of stationary processes, that means independence of system parameters from time, have been extended by additionally taking into account that on real observations of stochastic processes, in the rule also of fluctuating sound pressure level in time, an unavoidable but evaluable uncertainty is imposed.

Based on a software, which besides other numerous features also allows to measure the uncertainty of the process parameters L_{eq} and $L_{N\%}$ in online operation a convenient tool for sophisticated assessment of stationarity is established. Its application within measurement performance is demonstrated by different sound examples, testing whether and to which extent an observed sound situation is stationary not only in the traditional sense, but also within the confidence limits of characteristics assigned to the specific process of interest.

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