MEASUREMENT ACCURACY OF QUANTILES DETERMINED FOR CONTINUOUS RANDOM SIGNALS AND APPLICATION TO SELECTIVE ASSESSMENT OF ENVIRONMENTAL NOISE

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1. INTRODUCTION

It is well known that noise control in many cases is to be performed by aid of measurements and that in practice noise occurs in the form of continuous sound signals fluctuating randomly. For this reason it is useful to take into account the uncertainties, which are inevitably inherent to any evaluation index describing those signals. Additionally this may lead to a better understanding of the structural features of the stochastic process depicted by the observed signal. Further possible consequences are also adequate applications.
In the following treatment we will concentrate on the accuracy of the quantile (100q% level), a noise evaluation index frequently used. In a preceding paper the main basic features of this topic already have been reported [1]. But there the calculation procedure for the bracket confidential interval of the quantile was restricted to the case that, for simplicity, a local linearization of the cumulative probability distribution function is justified. In the following the procedure is extended to the general case of a cumulative distribution function having an arbitrary curvature.

2. SIGNAL PROCESSING FOR THE EVALUATION OF THE VARIANCE

As has been shown in [1], the quantile confidence interval is accessible through the variance of the partition which is imposed to the observed signal's cumulative distribution by a fixed reference level. This variance can be estimated by

$$\mathrm{Var}\, q = \frac{\hat{\nu}}{T} \left(\hat{q}_u^2 s_w^2 + \hat{q}_w^2 s_u^2\right) \tag{1a}$$

$$= \frac{1}{T} \frac{\hat{q}_u^2 \hat{q}_w^2}{\hat{\nu}} \left(v_u^2 + v_w^2\right) = \frac{1}{n} \hat{q}_u^2 \hat{q}_w^2 \left(v_u^2 + v_w^2\right) \tag{1b, c}$$

(see also [1]). In eq. (1) denote: w, u: Indices for crossing-up and crossing-down respectively; T: Measurement time interval; n: Observed number of independent crossings-up (or crossings-down); $\hat{\nu}$: Observed mean of the crossing-up (or crossing-down) frequency; $\hat{q}$: Observed partition parameter, i. e. estimate of q; s: Standard deviation; $\hat{v}_u = s_u/\overline{u}$: Coefficient of variation; $\overline{u}$: Mean of the single continuous undershoot time intervals; denotions for w are corresponding to u. Eq. (1) is valid if there are, at least approximate stable conditions i. e. $v_u, v_w \lesssim 1$ and $n \gtrsim 5$ [1]. Then, as is well known, the

Central Limit Theorem [2] holds approximately and a bracket confidence interval of the partition q can be calculated by

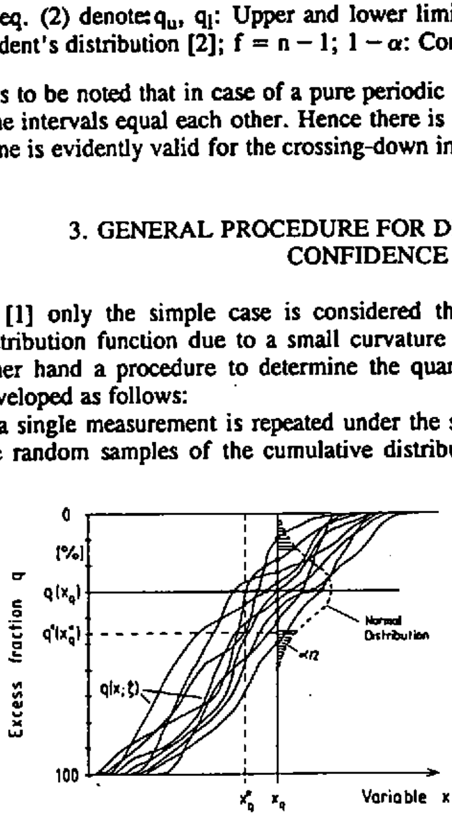
$$q_u - \hat{q} = \hat{q} - q_l = t_{f;\,1-\alpha/2}\ [\mathrm{Var}\,\dot{q}]^{1/2}. \qquad (2)$$

In eq. (2) denote: q_u, q_l: Upper and lower limit of the confidence interval; t: Quantile of student's distribution [2]; $f = n - 1$; $1 - \alpha$: Confidence coefficient [2].

It is to be noted that in case of a pure periodic signal the single crossing-up (= overshoot) time intervals equal each other. Hence there is no variance, i. e. $s_W = 0$, as expected. The same is evidently valid for the crossing-down interval.

3. GENERAL PROCEDURE FOR DETERMINING THE QUANTILE CONFIDENCE INTERVAL

In [1] only the simple case is considered that a local linearization of the cumulative distribution function due to a small curvature within the spread of q is justified. On the other hand a procedure to determine the quantile confidence interval in general can be developed as follows:
If a single measurement is repeated under the same stationary conditions, the ensemble of the random samples of the cumulative distribution arranges as depicted schematically in



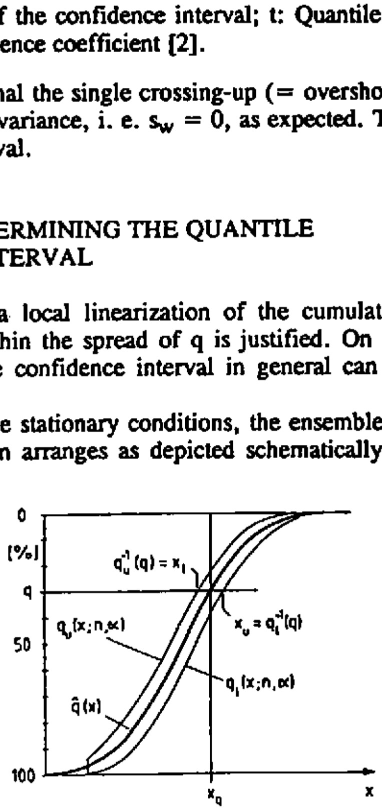


Fig. 1. Definitions and relations used to evaluate the confidence interval in the general case of arbitrary curvature of the cumulative probability distribution function (schematically).
In a) denote: x_q: Fixed reference level; $q(x, \zeta)$: Samples of the cumulative distribution function; ζ: Stochastic ensemble parameter; $q(x_q)$: Expectation value associated to x_q.
In b) denote: $\hat{q}(x)$: Estimate of the (expected) cumulative distribution; $q^{-1}(q)$: Inverse function of $\hat{q}(x)$, i. e. the function x(q). Further explanations see text.

Fig. 1a). The points of intersection on the vertical axis $x = x_q$ are approximately normally distributed. The intersections along the horizontal line $q = q(x_q)$ indicate the quantile samples.
Since the slope of the cumulative distribution is always positive, it is evident that the number of intersections of the field of sample curves, for instance with the line $q = q(x_q) = \text{const.}$ for $x < x_q$ is the same as with the line $x = x_q = \text{const.}$ for $q > q(x_q)$. Thus the sample fraction below an arbitrary value x_q^* on the $q(x_q)$-line is the same as the fraction above $q(x_q)$ along the line $x = x_q^*$.
The analogue is valid for the opposite quadrant. Therefrom one can derive that once the limits of a (symmetric) bracket confidence interval for q, dependent on x, are established, the locations of the confidence limits of an arbitrary quantile also are determined.
This procedure is shown in Fig. 1b): As a function of x the estimator $\hat{q}(x)$ of the partition and its upper and lower confidence limits $q_u(x; n, \alpha)$ and $q_l(x; n, \alpha)$, dependent also on the parameters n and α, are depicted in a diagram or stored in a computer. It is to be emphasized that this is already achievable by running the measurement over the time interval T only once and then applying the procedures described in the second chapter (provided that there is enough data storage capacity). Then, for a qiven q, the measured quantile itself follows in the common way from the inverse of $\hat{q}(x)$. The upper and lower confidence interval limits result in an analogous manner from the inverse functions (see Fig. 1b)):

$$x_u (q; n, \alpha) = q_l^{-1}(q; n, \alpha) \tag{3a}$$

$$x_l (q; n, \alpha) = q_u^{-1} (q; n, \alpha). \tag{3b}$$

From Fig. 1b) it is evident that if the slopes do not change considerably within the interval $x_l \lesssim x \lesssim x_u$, the linearization as already used in eq. (8) in [1] is convenient.

By aid of the procedure underlying eq. (3) the statistical stability as mentioned in [3] with regard to sound level quantities can be expressed and judged quantitatively. Also the question for the lowest meaningful $L_{100q\%}$ background level (see [4]) can principally be answered quantitatively as well. A more directly applicable tool to check for statistical stability, at least within the measurement time interval, already has been established by the minimum measurement time criterion, derived and presented by eqs. (6) and (7) in [1].

4. A VARIANCE RELATION BETWEEN THE QUANTILES AND THE MEAN OF A SIGNAL

From Fig. 1a) easily can be inferred that

$$\overline{x}(\zeta) = \int_0^1 x(q; \zeta)\, dq \tag{4}$$

is a sample of the mean observed signal amplitude. Let $\Delta x(q; \zeta)$ denote the deviation of the single cumulative distribution function from the corresponding expectation function $E\{x(q; \zeta)\} = \overline{x}(q)$. Then the mean of $(\Delta x(q; \zeta))^2$ taken over the stochastic variable ζ is the quantile variance. Application of the Cauchy-Schwarz inequality leads to the relation

$$[\mathrm{Var}\,\overline{x}]^{1/2} \leq \int_0^1 [\mathrm{Var}\,x(q)]^{1/2}\,dq \qquad (5)$$

for the variance of the time average $\overline{x}$ of the signal, taken over T. The consequence of eq. (5) is that if the quantiles measured turn out to be statistically stable over nearly the total admitted q-variable space $S = \{q: 0 \leq q \leq 1\}$ and this at a given acceptable high confidence level (see eq. (7) in [1]), then the mean of the signal taken over the measurement interval also can be regarded statistically meaningful. This, together with the experimental results in [1] and in the following, confirms the statement in [5] that the adoption of (traffic noise) measurement time intervals about 15 - 20 minutes „usually provides good results".

5. APPLICATION TO THE SEPARATION OF DIFFERENT SOUND INTENSITY CONTRIBUTIONS

5.1 Basic steps of the procedure

The method presented above can be applied for instance to separate with a verifyable accuracy the constant sound level of an interesting source from a simultaneously present randomly fluctuating environmental noise with a comparable or even higher level. Similar residual noise correction problems have already been treated by other authors using other signal processing theories like in [6] and [7]. It is presumed in our case for simplicity that the sound source to be assessed can be switched off. This interesting steady noise signal can be calculated by an appropriate difference based on the additivity property of physical sound intensity. The difference of the states with and without the interesting steady source is performed by starting from approximately the one sound level, i. e. quantile, which shows the smallest variance of sound intensity. This level is evidently very near to the lower edge of the distribution function. The chosen partition parameter value q for the quantile is to be maintained strictly during the whole evaluation.
As is known from statistics the resolution limit of the procedure is determined by the variance of the sound intensity difference of the quantiles. Here this variance is about twice the intensity variance of the environmental random noise due to the practically nonfluctuating source to be separated acoustically. Taking this into account, then in terms of the symmetric confidence interval V_I on the sound intensity antilog scale of the reference quantile, the sound pressure level which indicates the resolution limit is determined by

$$L_{resol} \simeq 10\,\lg \frac{V_I}{\sqrt{2}} \quad \mathrm{dB}. \qquad (6)$$

ACCURACY OF QUANTILES FOR RANDOM SIGNALS

The condition underlying eq. (6) is, that to achieve still a significant separation, the lower confidence interval limit of the quantile sound intensity difference is to be above zero.

5.2 Simulation experiment

To test whether the method described above works also in practice, a simulation experiment was performed. As the randomly fluctuating environmental sound signal the noise emitted from a nearby highway was taken. To this signal some different constant and known sound levels have been superimposed separately in the laboratory. Then these levels were regarded to be unknown and reevaluated by the procedure described above. The characteristic data for that are presented in Fig. 2 and Table 1.
The total recording time interval over 26 minutes was subdivided into three parts denoted by I1, II and I2 which have the ratios 1 : 2 : 1 in duration. During interval II the additional source levels constant in time were superimposed successively. For comparison with II the intervals I1 and I2 stayed unaltered to serve as the switched off state.

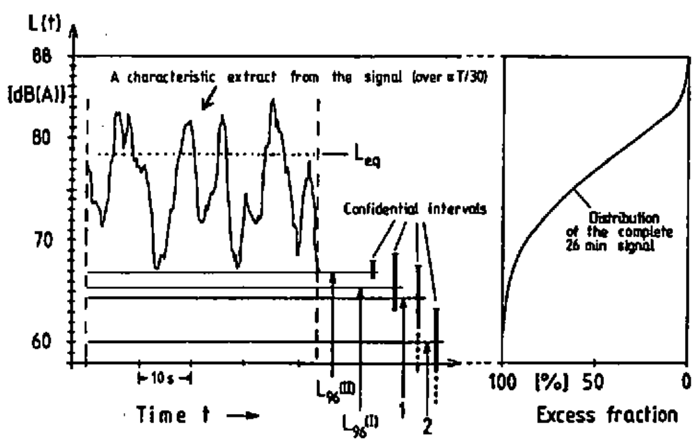


Fig. 2. Simulation experiment. Environmental-signal properties and resolution limits for the constant signal source levels to be separated.

Confidential coefficient: $1 - \alpha = 0{,}8$.

Reference level: L_{96}; for time interval I: $L_{96} = 65{,}5$ dB(A).

Level 1: Resolution limit at 64,5 dB(A) if signals in intervals I and II are taken as occurred i. e. slightly non-stationary.

Level 2: Resolution limit at 60 dB(A) if the signal would have been permanently stationary.

$L_{eq} = 78{,}5$ dB(A).

The relatively great difference between the confidence intervals for I and II (see Fig. 2) is due to an observable weak drift which started in I2. This demonstrates that the system, as far as described by the eqs. (1) - (3), reacts quite sensitive on the fact, that within the total interval I the signal observed cannot be regarded as stationary in the strict sense. This was also clearly to be seen by the level recorder output. On the other hand within intervall II no deviations from stationarity could be observed. The confidence limits evaluated for $L_{96}(II) = 66{,}9$ dB(A) are in positions +1,0 and −0,7 dB(A) to this percentile.
The results of the simulation experiment are arranged in Table 1. The main result is that the confidence intervals of the levels evaluated from experiment in all cases include the true level of the single steady source. The realistic resolution level 1 at 64,5 dB(A) represents approximately the L_{98} of the 26 minutes total measurement time for I + II.

ΔL_{pos} dB(A)	ΔL_{meas} dB(A)	d_+ dB(A)	d_- dB(A)
−1,3	+2,0	+4,3	−3,0
+1,7	+1,0	+2,6	−1,4
+6,7	+0,7	+0,2	−0,3

Table 1. Results of the simulation experiment for separation of a constant source level.

ΔL_{pos} = Positions of the „true" single source levels, imposed successively during time interval II, with respect to the reference level $L_{96} = 65{,}5$ dB(A) for time interval I.
ΔL_{meas} = Difference between evaluated level and true level
d_+ = Distance of the upper confidence limit of the evaluated level to the true level.
d_- = Distance of the lower conf. lim. respectively.

What resolution in comparison with this can be achieved by a sound level separation on the basis of an L_{eq}-difference?
One example of noise from nearby intraurban traffic, i. e. similar to the simulation experiment demonstrated here, is reported in [1]. Fig. 2a) in [1] and the application of unequality (5) let suppose a realistic confidence interval for the L_{eq} of about 2 dB. Applying eq. (6) this leads to an L_{eq}-resolution level which is roughly 4 dB lower than the reference sound signal. In case of the the signal shown above in Fig. 2 this yields a resolution level of 74,5 dB(A). So the method demonstrated here makes possible a resolution improvement of about 10 dB in this example.

On the basis of the considerations made above the resolution limit, when applying the quantile, also can be estimated for other examples of real noise signals. In [1], Fig. 2a) the quantile having minimum sound intensity variance is L_{96} = 49 dB(A), with a sound level confidence interval V = 2 dB(A). This is about the same as for L_{eq} = 67 dB(A). Thus in this example the new method improves the resolution by more than 15 dB.
For the second example in [1] Fig. 2b) ambient noise in a rural environment during night, V = 0,3 dB(A) is observed for the optimal quantile L_{75} = 30 dB(A). This implies a resolution limit of L = 17 dB(A). L_{eq} is about 32 dB(A). The inequality (5) yields an L_{eq}-confidence interval not greater than about 2 dB(A). This gives a resolution limit for L_{eq} of about 28 dB(A). Thus the resolution improvement is at least 10 dB(A) for this example.

The work reported here and in the preceding paper [1] is in an early stage and so has been primarily focused on its basic methodological aspects. Accordingly there remain some more sophisticated aspects, to be discussed, for instance the explicite proof of stochastic independence and its detailed preconditions or the influence of periodicity superimposed to the pure stochastic signal component.

6. SUMMARY

A method is developed to evaluate the confidence interval of quantiles (percentiles) determined for the cumulative distribution function of a measured continuous stochastic signal. There are no restrictions to be imposed on the distribution function. On the basis of the quantile variance determined over the whole cumulative probability distribution an upper limit for the variance of L_{eq} of the observed random signal can be calculated. The possible application of the procedure in the practice of environmental protection is demonstrated by a short term separation of a sound source having a constant level from additionally present randomly fluctuating environmental noise with comparable or even higher levels. In comparision with the L_{eq} as a tool for separation of the noise contributions from different sources the application of the presented method makes possible a resolution improvement between about 10 dB and slightly more than 15 dB, the actual value depending on the individual case.

7. ACKNOWLEDGEMENT

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