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A METHOD OF DETECTING AN INFORMATION OF LINEAR AND NON-LINEAR CORRELATIONS FOR TIME SERIES OF ACTUAL RANDOM NOISE AND VIBRATION

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INTRODUCTION

As is well-known, an ordinary linear correlation is very often studied as the first order approximation of fundamental information reflecting the time correlation character for actual random noise and vibration waves. In the next stage of study, several types of non-linear correlation functions must be additionally taken into consideration as the higher order approximation of correlation. On the other hand, the fluctuation data of random noise and vibration is very often sampled in a digitalized level form at discrete time intervals (e.g., the road traffic noise measured by a digital type sound level meter). Under the above situation, the use of digital computer is essential in a case of extracting statistical information (like mean, covariance, higher order moments, correlation function, (100-X) percentile Lx, etc.) due to various types of statistical evaluation.

From the above point of view, in our previous study [1], a general expression of probability distribution with quantized level form was derived in the unified expression form of orthonormal series expansion with parameters reflecting hierarchically linear and non-linear correlation properties. Therefore, through some statistical treatment based on this general expressions, it can be principally expected to detect arbitrary types of information on linear and/or non-linear correlations being latent in the time series of random noise and vibration.

In this paper, first, as a specialized case of the above general type expansion expression on probability distribution, we introduce an explicit expression on bivariate joint probability function in a concrete form of expansion series taking the product of two well-known binomial distributions as the first expansion term. Obviously this expression is suitable to deal with an arbitrary random noise and vibration of discrete level type. Next, the general type of linear and/or non-linear correlation functions can be explicitly estimated through the lower order statistic of conditional average derived from the above expansion series type expression. Moreover, from the viewpoint of practical use, a simplified method of detecting these linear and non-linear correlation functions is approximately derived. Finally, the validity and the usefulness of present theory are confirmed by applying to experimentally observed road traffic noise data in our big city.

THEORETICAL CONSIDERATION

First consider a level fluctuation $X(t)$ of general noise or vibration waves and denote the instantaneous values at two different time point t_1 and t_2 by X_1 and X_2 respectively. Furhtermore, assume that X_i ($i=1, 2$) are measured in a digitalized level form with level difference interval h_i, and that X_{s_i} ($s_i=M_i, M_i+1, \cdots, N_i$) denotes the s-th quantized level value of X_i. Then, the joint probability function $P(X_1,X_2)$ of the above discrete level type random variables X_1 and X_2 with level difference interval h_i can be expressed in a general form of orthonormal expansion series by taking the product of two binomial distributions $B_i(X_i; N_i, p_i, h_i)$ ($i=1, 2$) with newly introduced level difference h_i as the first expansion term as follows [1]:

$$P(X_1, X_2)=\prod_{i=1}^{2} B_i(X_i; N_i, p_i, h_i)\{1+\rho \frac{\sigma_{x_1}\sigma_{x_2}}{\mu_{x_1}\mu_{x_2}} BP_1(X_1)BP_1(X_2)$$

$$+\sum_{n=3}^{\infty}\sum_{n_1+n_2=n} A(n_1, n_2)BP_{n_1}(X_1)BP_{n_2}(X_2)\}, \qquad (1)$$

with

$$B_i(X_i; N_i, p_i, h_i)=\frac{(N_i/h_i)!}{(X_i/h_i)!((N_i-X_i)/h_i)!} p_i^{\frac{X_i}{h_i}} (1-p_i)^{\frac{N_i-X_i}{h_i}}$$

$$(X_i=0, h_i, 2h_i, \cdots, N_i),$$

and

$$BP_{n_i}(X_i)=\frac{1}{h_i^{n_i}} \sum_{j=0}^{n_i} \binom{n_i}{j} (-1)^{n_i-j} \left(\frac{p_i}{1-p_i}\right)^{n_i-j} (N_i-X_i)^{(n_i-j)} X_i^{(j)},$$

$$A(n_1, n_2)=\left\langle \prod_{i=1}^{2} \frac{(1-p_i)^{n_i}}{N_{oi}^{(n_i)} n_i! p_i^{n_i}} BP_{n_i}(X_i) \right\rangle, \quad N_{oi}=N_i/h_i,$$

$$\mu_{xi}=N_i p_i, \quad \sigma_{xi}^2=h_i(1-p_i)N_i p_i, \quad \rho=\langle (X_1-\mu_{x_1})(X_2-\mu_{x_2})\rangle/\sigma_{x_1}\sigma_{x_2}.$$

Based on the definition of conditional probability and by using orthogonal condition of polynomial $BP_n(X)$, the conditional probability function of X_2 conditioned by X_1 can be derived from Eq. (1) as follows:

$$P(X_2|X_1)= B_2(X_2; N_2, p_2, h_2)\{1+\rho \frac{\sigma_{x_1}\sigma_{x_2}}{\mu_{x_1}\mu_{x_2}} BP_1(X_1)BP_1(X_2)$$

$$+\sum_{n=3}^{\infty}\sum_{n_1+n_2=n} A(n_1,\ n_2)\prod_{i=1}^{2} BP_{n_i}(X_i)\}/\{1+\sum_{n_1=3}^{\infty} A(n_1,\ 0)BP_{n_1}(X_1)\}. \quad (2)$$

Next, after letting X_2 express in a linear combination form of orthogonal polynomial: $X_2=C_0BP_0(X_2)+C_1BP_1(X_2)$ with $C_0=N_2p_2=\mu_{x_2}$, $C_1=h_2(1-p_2)$, the conditional average of X_2 conditioned by X_1 can be derived as follows:

$$<X_2|X_1> \triangleq \sum_{X_2=0}^{N_2} X_2P(X_2|X_1)$$

$$=\{\mu_{x_2}+\rho\frac{\sigma_{x_1}\sigma_{x_2}}{\mu_{x_1}h_1(1-p_1)}(X_1-\mu_1)+\mu_{x_2}\sum_{n_1=3}^{\infty} A(n_1,\ 0)BP_{n_1}(X_1)$$

$$+\mu_{x_2}\sum_{n_1=2}^{\infty} A(n_1,\ 1)BP_{n_1}(X_1)\}/\{1+\sum_{n_1=3}^{\infty} A(n_1,\ 0)BP_{n_1}(X_1)\}, \quad (3)$$

by use of orthogonal relation of polynomials $BP_n(X)$.

As an information on the usual linear correlation ρ is reflected in the right side of Eq. (3), we can easily extract this information as follows:

$$\rho=\frac{<X_2|X_1>-\mu_x}{X_1-\mu_x}\{1+\sum_{n_1=3}^{\infty} A(n_1,\ 0)BP_{n_1}(X_1)\}$$

$$-\frac{\mu_x}{X_1-\mu_x}\sum_{n_1=2}^{\infty} A(n_1,\ 1)BP_{n_1}(X_1), \quad (4)$$

where the stationary properties: $\mu_{x_i}=\mu_x$, $\sigma^2_{x_i}=\sigma^2_x$, $N_i=N$, $p_i=p$ $(i=1,2)$ for random noise or vibration wave are introduced. In the same way, by taking several degree of approximation into consideration in Eq. (3), several types of simplified method for detecting an information of linear and/or non-linear correlations can be principally derived. For example, among several kinds of correlation properties between X_1 and X_2, if a linear correlation property is dominantly latent in the phenomena, the relation $A(n_1,\ 1)\simeq 0$ can be hold and Eq. (4) can be expressed only by a fairly simplified formula. Furthermore, in a typical case when the binomial distribution in the first expansion term can well express the total shape of one-variate distribution, the expansion coefficients $A(n_1,\ 0)$ $(n_1\geq 3)$ show nearly zero value. Hereupon, the following property must be noticed: Even a binomial distribution $B_i(X_i;\ N_i,\ p_i,\ h_i)$ in the first expansion term is very useful to express approximately the dominant part of whole distribution form of discrete level type as compared with other probability expression with one parameter (like Poisson type distribution), since it has two parameters N_i and p_i and so can originally reflect two measures on center and spread of distribution form. Thus, a more simplified method of detecting a linear correlation can be reduced as a approximation of first order:

$$\rho\simeq(<X_2|X_1>-\mu_x)/(X_1-\mu_x). \quad (5)$$

In addition to the above linear correlation, an information on non-linear correlation $\rho_{1n}(=\langle (X_1-\mu_{X_1})(X_2-\mu_{X_2})^n\rangle/\sigma_{X_1}\sigma_{X_2}^n)$ $(n\geq 2)$ reflected in expansion coefficients $A(1, n_2)$ can be detected, first by regarding ρ and ρ_{1n} as unknown parameters, and then by solving the simultaneous equations in the same form as Eq. (4) with different values of X_1, after adopting several values of the conditional level X_1. In this case, it must be noticed that the non-linear correlation of higher order can be obtained from the conditional moment data of lower order (such as mean and variance) supported only by small number of data due to conditioned level X_1.

EXPERIMENTAL CONSIDERATION

For purpose of confirming experimentally the legitimacy of proposed method, we have applied the present theory to road traffic noise data observed in our big city. Road traffic noise data are measured at every one second by use of sound level meter of digital type. The estimated result for a linear correlation coefficient ρ by use of Eq. (5) is shown in Fig. 1. In spite of fairly simplified method, the result agree well with the experimental value evaluated directly by the original definition formula of linear correlation. Other experimental results by use of Eqs. (4), (5) and the estimated result of non-linear correlation function have also shown a good agreement with the present theoretical method.

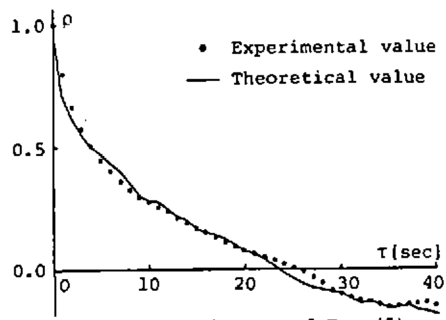

Fig. 1 Estimates by use of Eq. (5).

CONCLUSION

In this paper, first, a new detection method of linear and non-linear correlations being latent in a time series of random noise and vibration has been theoertically proposed based on a fairly small number of experimental data conditioned by a specific level of random wave, especially in a suitable form for the actually observed data with discrete level type. Next, the theoretical method has been applied to the actual road traffic noise data measured by use of the sound level meter of digital type. Thus, the validity and the usefulness of the proposed method has been confirmed experimentally from various points of view.

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REFERENCE

[1] M. Ohta and K. Hatakeyama, Preprints of 12th JAACE Symposium on Stochastic Systems, 145-148 (1980).