NUMERICAL COMPARISON BETWEEN OBJECTIVE AND SUBJECTIVE DETECTION MODELS

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Traditionally, there have been two different ways of viewing the problem of acoustic detection. According to the conventional theory (reference 1), the problem reduces to one of testing the statistical hypothesis that the sample is merely noise against the alternate hypothesis that the sample is signal plus noise. Alternatively (reference 2), one may think of the problem of acoustic detection as one of evaluating, within a Bayes statistical framework, the degree of confidence in the hypothesis that the sample is made of signal plus noise.

It has been stated that the Bayesian observer extracts more information regarding the presence of the target from the sample measurement than does the conventional observer (reference 3). We wish to quantify this contention under the simplifying assumption, often used in present day processing technology (reference 3), that the sample, whether it be signal plus noise or noise alone, can be represented by a band-limited white, stationary and Gaussian random process.

In our model, the conventional observer uses the input voltage measurement to evaluate the energy contained within some previously chosen observation time-interval, thresholds that energy subject to a fixed probability of false alarm, and then repeats the procedure for observation intervals obtained by successively dropping the oldest measurement in the previous interval and adding the latest one. He records the random outcome of this succession of threshold operations in a vector whose components are either 0 or 1 according to whether the corresponding energy has fallen below or above the chosen threshold. Such a vector closely models a LOFAR gram. On the basis of his output, the conventional observer evaluates, using Bayes' rule, a posterior probability that the sample contains the signal.

The Bayesian observer, on the other hand, dispenses with the automatic thresholding procedure described above and evaluates his posterior probability that the sample contains the signal directly from the voltage measured at the output of the sonar array.

Using the concept of sample-path information introduced in 1951 by Woodward & Davies (reference 3), we relate the posterior probability to the amount of information extracted by the two observers regarding the presence of the target and find that on the average, the Bayesian observer does indeed obtain uniformly more information about the presence of the target from his measurement than does the conventional observer. This difference is as much as 25 percent of the value extracted by the conventional observer when the signal-to-noise ratio ~~is ω~~ = 0.5 and the observer's prior is $\lambda = 0.5$ (see figure 1).

Finally, we require both observers to make a final decision regarding the presence of the target by having them threshold their corresponding posterior probabilities against an appropriately chosen threshold, and compare their probabilities of having made the correct decision. Again, we find that the Bayesian observer uniformly outperforms the conventional one (see figure 2).

REFERENCES

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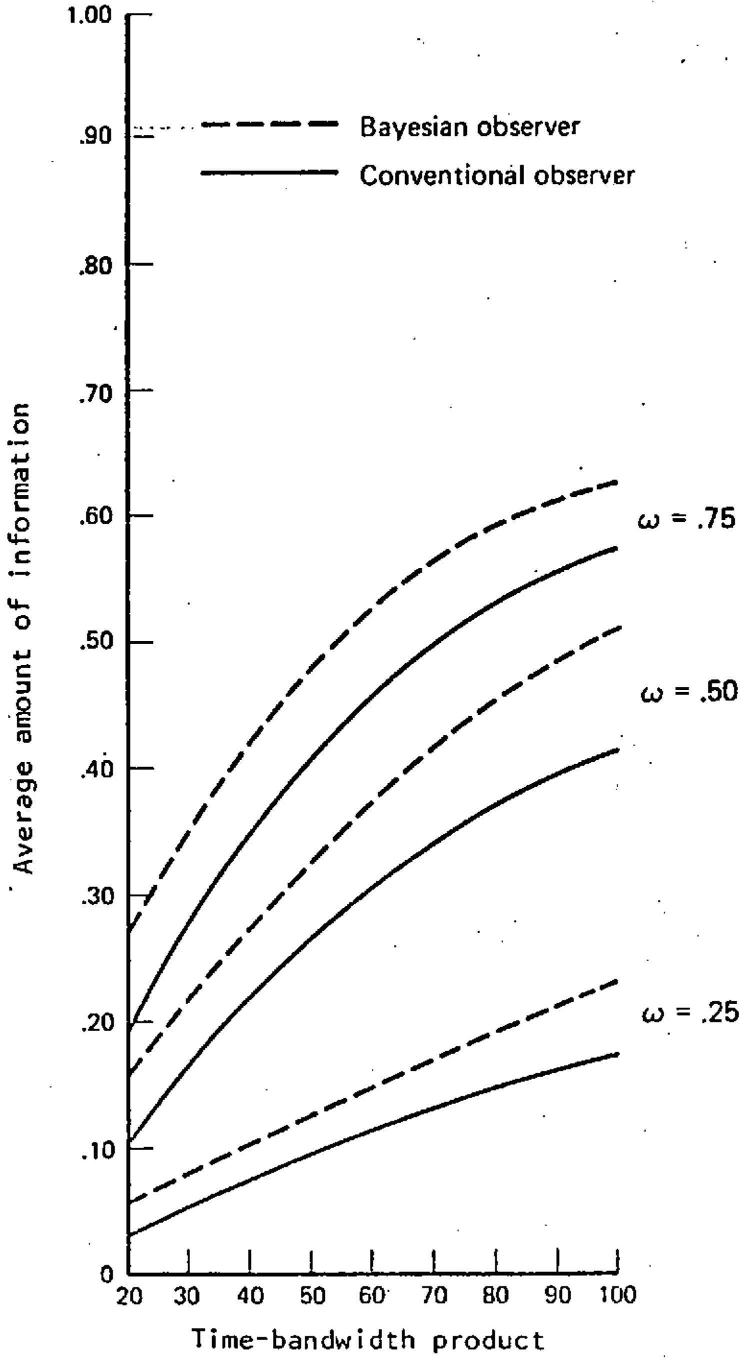

FIG. 1: AVERAGE INFORMATION EXTRACTED BY THE CONVENTIONAL AND BAYES OBSERVERS, λ = .50

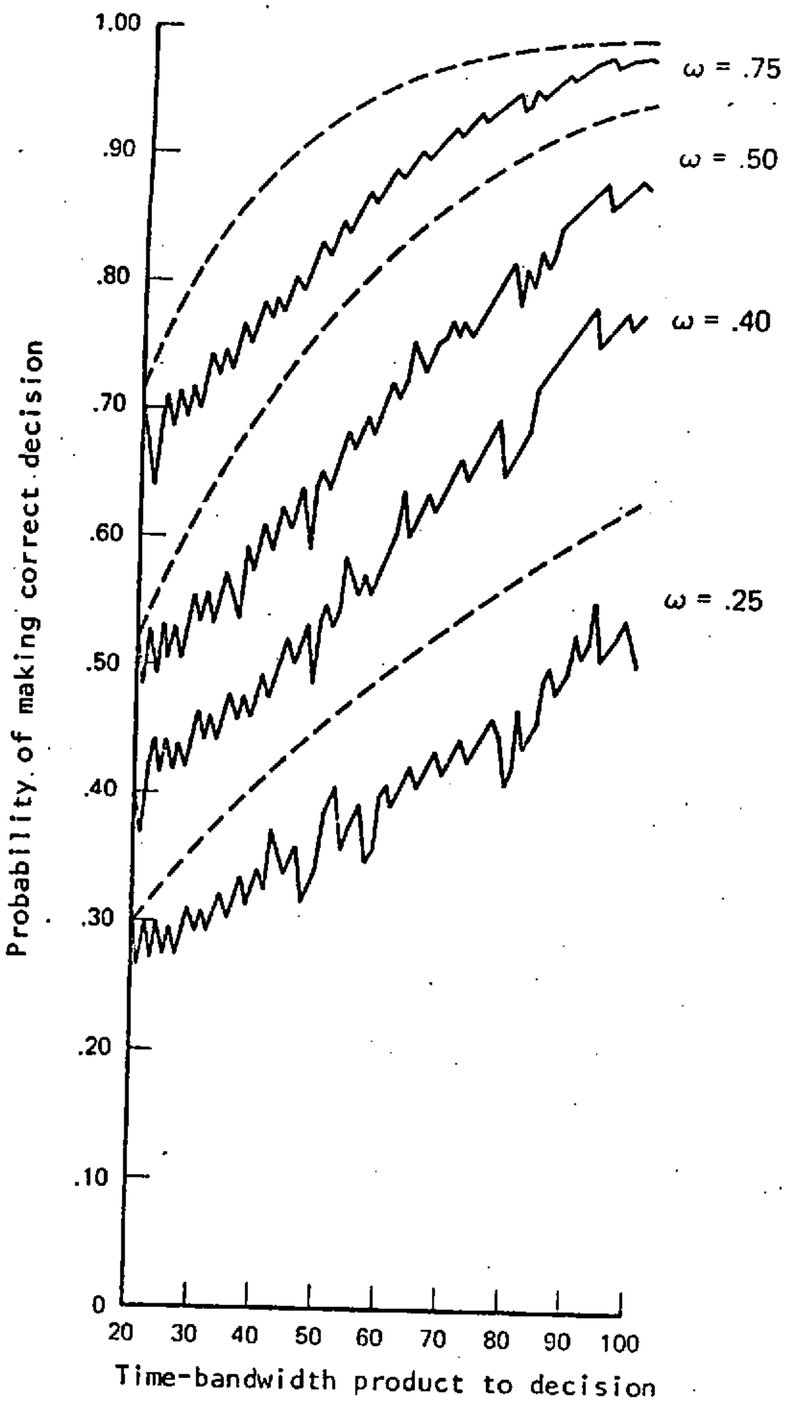

FIG. 2: DETECTION PROBABILITY OF THE CONVENTIONAL AND BAYES OBSERVER, P_{FA} = .10