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STRONG ANOMALOUS SCATTERING OF SOUND AT A LINEAR CHAIN OF PERFORATED CYLINDER

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A rigorous theory of diffraction for finite and infinite chain of perforated cylindrical shells shows that the transmission spectrum exhibits a sharp minimum, which is due to excitation of leaky wave with anomalous dispersion. This wave is a weakly-decaying symmetric eigenmode that propagates along the array and provides very effective 90º-redirection of sound. The antisymmetric counterpart of this eigenmode has normal dispersion but it turns out to be a deaf mode since it cannot be excited at normal incidence. However, at slightly oblique incidence both modes can be excited at different but close frequencies, thus leading to two minima. The symmetric mode redirects sound in the "wrong" direction, thus allowing splitting of incoming signal containing two harmonics into two beams propagating along the chain in the opposite directions. The results are presented for ideal and viscous fluid.

Keywords: Physical acoustics, phononic crystals, sound scattering, Acoustic transmission

1. Introduction

A periodic array of scatterers with internal structure (mass-in-mass units) may serve as a waveguide for mechanical energy, also exhibiting negative mass density [1]. A metamaterial behavior where both the effective mass and the elastic modulus are negative was predicted for a chain consisting of elastic mass-in-mass units with lateral resonances [2]. A dispersion curve of an array of acoustic Helmholtz resonators (soda cans) with two units per unit cell has a passing band within a polaritonic gap where the effective index of refraction becomes negative [3]. These and other examples demonstrate that acoustic interactions between weak scatterers may be quite strong giving rise to a set of variety metamaterial properties.

A perforated object usually serves as a weak scatterer if the radius of perforations *r* exceeds much the thickness of the viscous layer δ. Since a single perforated cylindrical shell is a weak scatterer without internal resonances, a periodic linear chain of perforated shells turns out to be almost transparent for sound at normal incidence [4]. However, anomalously low transmission was numerically predicted for normal incidence at the frequencies close to the position of Wood anomaly when the wavelength λ is close to the period a of the chain, $\lambda \approx a$. Here a microscopic mechanism of this low transmission is proposed and explained byresonant interaction of external plane wave with symmetric leaky eigenmode. With this purpose we develop a theory of scattering of external plane wave by a linear chain of perforated cylindrical shells of finite and infinite length in viscous fluid.

2. Modelling for an infinite periodic chain

Scattering of waves by an in infinite periodic array of cylinders is one of the classical problems of theory of diffraction. For conducting cylinders illuminated by electromagnetic wave the expansion of the scattered field was proposed in Ref. [5]. For sound wave the field distribution resulted from diffraction at periodic array of solid spheres in fluid was calculated in Ref. [6]. Similar problem for elastic waves scattered by a monolayer of elastic spheres was solved in Ref. [7]. In these studies the chain of scatterers was considered to be in finite, i.e. the Bloch theorem was applicable. This section describes our modelling for the case of a periodic infinite chain.

We apply a method of expansion over cylindrical waves that leads to an infinite set of linear equations for partial transmission and scattering amplitudes. This results in a transcendental equation whose numerical solution for the few lowest bands is presented in Fig. 1. Since perforated shells are weak scatterers, the dispersion curve (blue dots) is very close to the band structure obtained in the empty-lattice model, shown by the straight broken line (red). It is observed that away from points of degeneracy the dispersion is practically linear with speed equal to the speed of sound in the background fluid. However, at the Γ-point and at the edges of the Brillouin zone the repulsion of levels gives rise to doublets. Particularly, at the Γ-point $q = 0$ the effect of level repulsion leads to gap opening of 35 Hz and essentially nonlinear dispersion (see the left insert in Fig. 1). The eigenfunctions corresponding to the components of the doublet are either symmetric or antisymmetric functions of coordinates. At normal incidence only the symmetric mode can be excited, the antisymmetric one turns out to be a deaf mode [8]. In accordance with this symmetry, at normal incidence a sharp minimum in the transmission spectrum appears when the frequency of the external wave coincides with the frequency of the symmetric component, which turns out to be the lower level of the doublet. There are also some singularities in the dispersion of the upper band which are not resolved in Fig. 1. Away from the Γ-point the dispersion of the both bands is practically linear but it is normal for the upper band and anomalous for the lower one.

For real values of the wavevector q all the solutions $\omega(q) = 2\pi f(q)$ of the transcendental equation are complex. This means that the acoustic eigenmodes of an infinite chain of shells are leaky modes, even if the viscosity of the fluid is neglected. The imaginary part of frequency describes weak radiative decay, i.e. the acoustic eigenmodes are not strongly localized near the chain. However, since the decay is very slow the leaky eigenmodes are long-living excitations. The imaginary part is negative, $\mathrm{Im}\, f < 0$, in order for the time-dependent factor $e^{-i\omega t}$ to decay with time. The rate of radiative decay $|\mathrm{Im}\, f|$ is at least two orders of magnitude smaller than the frequency of sound $f \approx \mathrm{Re}\, f$ as it can be seen from the right insert to Fig. 1. For the frequencies near the Γ-point the rate of radiative decay is about 1-3 Hz; i.e., it practically does not contribute to the width of the resonance minima. Figure 2 shows the plots of the transmittance of an infinite chain for inviscid (thick line) and viscous air (thin line). The minimum of the width of resonance, $\Delta f = 30$ Hz, is observed for the ideal situation of inviscid air.

Excitation of the eigenmodes by an external plane wave is possible since the spectrum of eigenfrequencies is complex, i.e. all the eigenmodes of a linear chain are leaky modes radiating acoustic energy into the background fluid. Excited eigenmode transmits energy along the chain, i.e. the initial flux of energy is partially redirected by 90◦. Only symmetric modes can be excited by external plane wave at normal incidence. The upper level, being an antisymmetric mode, cannot be excited and for this geometry it is referred to as deaf mode [8]. The effect of redirection of acoustic energy was recently predicted not only for periodic systems of scatterers [4, 9] but also for a narrow fluid channel in a solid elastic plate [10].

At oblique incidence the symmetry of the incoming wave is broken and both eigenmodes (symmetric and antisymmetric) can be excited. Therefore, the transmission spectra exhibit two minima. Each minimum is observed at the frequency where the following matching condition for the wave vectors is satisfied:

$$k_y = (2\pi f/c_0)\sin\theta = q(f), \tag{1}$$

where c_0=343 m/s.

For example, in Fig. 1 this condition is satisfied at the points of crossing of two straight dashed lines, corresponding to the angles of incidence $\theta = 5^\circ$ and $\theta = 15^\circ$, with the dispersion curves. For $\theta = 5^\circ$ two crossings are shown at $f \approx 2.85$ kHz and $f \approx 3.40$ kHz.

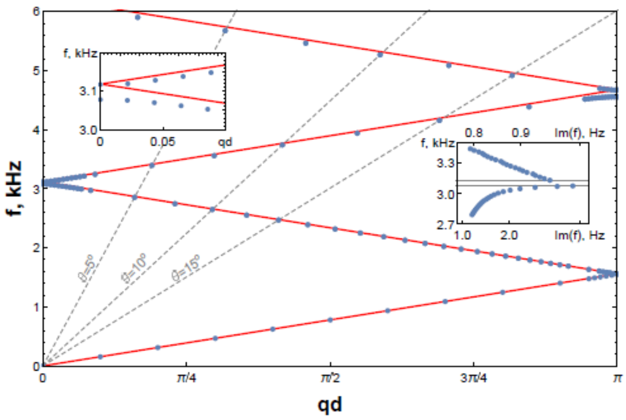

Figure 1.- Band structure of an infinite periodic chain of shells imbedded in inviscid air. Solid straight (red) lines show linear dispersion in air. Dots (blue) are the real parts of eigenfrequencies calculated.

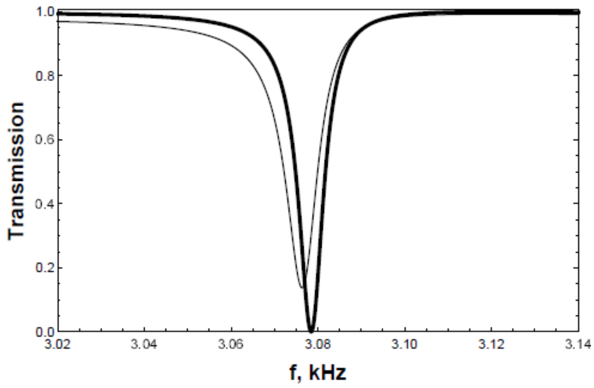

Figure 2: Transmission spectrum of an infinite chain of perforated cylindrical shells at normal incidence. Thick and thin lines are for inviscid and viscous air respectively.

3. Results for a finite chain

The spectrum of eigenmodes is established for sufficiently long chain. Otherwise, the collective behavior of the eigenmodes is weakly manifested which leads to a very shallow minimum in the transmission. For a chain of 15 cylinders, the minimum is hardly seen, even if the viscosity is neglected. The resonant minimum becomes deeper and sharper with the number of scatterers. The position of the minimum is blue-shifted with the number of scatterers approaching 3078 Hz, which is the first optical frequency in the 1D band structure of the infinite periodic chain of shells imbedded in inviscid air. It is interesting to note that the deepness of the minimum is less sensitive to the number of scatterers than the position of the resonance. This can be concluded from the fact that the transmission at minimum approaches zero faster than the resonant frequency approaches its limiting value of 3078 Hz.

Due to different dispersion, the scattering of incident waves by a chain of perforated cylinders exhibits an interesting anomaly. For both eigenmodes the direction of their phase velocities coincides with the direction of the wave vector **q** = q^y. However, the group velocity for the lower band is opposite to q that leads to scattering of the incoming wave in the ”wrong” direction. Figure 3 shows the splitting of a bi-frequency signal when it hits the chain at the angle of 10° in ideal (a) and viscous (b) air. The frequencies of the monochromatic components, 2625 Hz and 3715 Hz, are obtained from the band diagram and correspond to the minima in the transmission spectrum for oblique incidence. Since the chain contains only 25 shells and the angle of incidence is not very small, essential part of acoustic energy propagates directly through the chain. Nevertheless, a clear pattern of split fringes is observed even for viscous air. The efficiency of splitting is quite good for a pure mechanical splitter, for ideal (viscous) air 8% (5%) of energy is converted into the low-frequency component, propagating down (anomalous scattering) and 10% (7%) of energy is converted into the higher-frequency component, propagating up (normal scattering).

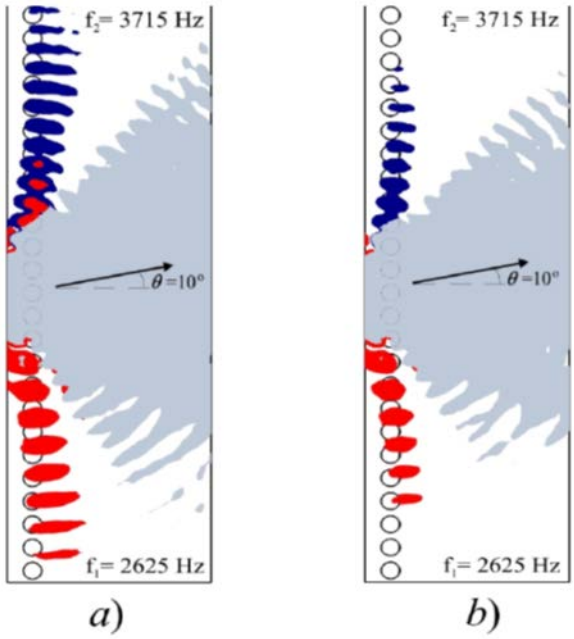

Figure 3. Distribution of intensity of bi-frequency signal (with f_1 = 2625 Hz and f_2 = 3715 Hz) transmitted through a chain of 21 perforated shells. The background air is inviscid (a) and viscous (b). The central part of the diffraction pattern (shown in grey) is a mixture of two sound waves with frequencies f_1 and f_2. The red and blue fringes are the split monochromatic components.

In summary, we have demonstrated redirection and splitting of sound waves impinging a periodic chain of thin perforated cylindrical shells. These conclusions have been obtained using an analytical approach which has been developed here. Further experimental work should be performed in order to support our theoretical predictions, which foresee useful devices for the filtering and splitting of sound waves at selected frequencies.

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