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The use of a Pohlman cell in acoustic holography with reference to the reconstruction of complete sound fields within hollow cylinders.

by

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In an acoustical holographic system both the phase and amplitude of a sound field are preserved by recording the interference pattern produced by the diffracted field (U_o) and a reference sound field (U_R). Although most applications have been concerned with the diffracted field yet in fact <u>all</u> the sound field information is stored at the detector. Assume the latter is a 'square law' device then the intensity (J) of the received radiation will be given by $J = UU^* = U_R U_R^* + U_o U_o^* + U_R U_o^* + U_R^* U_o$ —— (1) where * denotes the complex conjugate. At the detector the sound interference pattern is visualized and the resulting optical distribution is recorded on film as a distribution of the optical transmission coefficient of the film. On illuminating the hologram transparency with coherent light then reconstructions of the original (true) wavefront together with its complex conjugate are produced through the terms $U_o^* U_R$ and $U_R U_o^*$ of (1), the other two terms of which correspond to the undiffracted part (U_c) of the reconstructing wave which is merely attenuated on passing through the transparency. Since phase and amplitude are recorded the complete sound field is reconstructed as a three-dimensional light distribution, so that the opportunity is presented of storing complete sound fields on a single transparency for detailed study in the reconstruction process. Since $\lambda_S >> \lambda_L$, where λ_S and λ_L are respective wavelengths of the sound and the reconstructing light waves, then there is a significant difference in the longitudinal (M_{Lo}) and the lateral (M_{LA}) magnifications produced in the holographic process, given by $M_{Lo} = \pm \frac{1}{\mu} M_{LA}^2$)2) where $\mu = \lambda_L/\lambda_S$. As a result in the reconstruction, sound fields of a few cm path length become several metres long. The scaling (factor m) of the hologram between detection and reconstruction is common in acoustic holography and besides reducing the <u>effective</u> value of μ it improves, at low acoustic frequencies, the diffracting efficiency of the hologram transparency. Considering the hologram as a complex Fresnel zone plate with a 'built in' object its focal length (f') is given approximately by $1/f' \cong \pm (1/Z - 1/Z_c)$ —(3), where Z and Z_c are respectively the axial distances of the reconstructed image point and the reconstructing source point from the plane of the hologram. Similarly for the recording process the 'effective' focal length (f) of the hologram is given by $1/f = 1/Z_o - 1/Z_R$ (4), Z_o and Z_R being the axial distance of an object point and of a reference source point, respectively, from the recording plane of the hologram.

The focal lengths are related by the expression $f' = m^2 f/\mu$ —(5) which for parallel illumination gives $f = \pm \mu Z/m^2$, since $f' = \pm Z$ [from (3)].

Now from the geometry of the recording system f may be found and with the knowledge of m and μ, the position (Z) of the image point is determinable. Consequently Z may be found as a function of Z_0

In this paper an account is given of the use of a specific acoustical visualization device, called a Pohlman cell, to study the pattern of the sound field inside a hollow metal tube across and along its axis. The cell depends for its action upon the couple exerted by an acoustic field on a thin disc freely suspended in a fluid (cf Rayleigh disc) so that the disc tends to set 'broadside-on' to the incident radiation, the magnitude of the effect being proportional to the square of the acoustic particle velocity. In the present experiment the particles consisted of a suspension of aluminium powder in water with the addition of a simple dispersing agent, the thin cell being provided with a good sound transmission window of Mylar film and an optical window. A sound wave incident on the membrane will produce an orientation of the reflected light which may be directly related to the intensity distribution of the sound field.

The sound field was produced in a water tank lined with dimpled matting of an aluminium-loaded butyl rubber compound and a conventional off-axis Fresnel holographic system was used. This ensured that in the final reconstruction there would be an angular separation of the true and conjugate images. A matched pair of PZT ceramic transducers constituted the acoustic sources operating at 0.963 MHz and mounted to reduce the side-lobes in their far-fields. The metal tube was suspended by light threads so as to be coaxial horizontally with one of the sources and located in its far-field, so that the plane sound waves may be assumed to be incident.

The hologram as recorded by the Pohlman cell was photographed using Kodak Tri-X film and the scaling factor (m) was about 0.05. The film was developed in Microdol-X to a high gamma (γ), so as to give sufficient contrast to produce a high diffraction efficiency and hence a brighter reconstructed image. A Spectra Physics He-Ne laser $(\lambda_L = 6328A^o)$ with an output of about 12 mW (although 1 mW gave reasonable reconstruction) was used as light source. The reconstructed length of the 30 cm metal tube was about 10 metres. At any point in the reconstructed image a photographic plate or film may be inserted and a permanent record made of the light distribution. The transmission distribution (T) on the film negative is related to the light intensity distribution (I) in the image by the formula $T = I^{\gamma/2}$. By choosing $\gamma = 1$, the transmission distribution assumes the form of the square root of the light intensity distribution, and in turn this is proportional to the sound particle velocity and hence to the acoustic pressure distribution within the tube. By scanning the film recording with a micro-densitometer and plotting the output on a pen recorder the distribution is obtained of the acoustic pressure across a diameter within the tube.

The method of calculating theoretically the acoustic distribution in the tube was as follows:- Firstly the exact three dimensional equations of linear elasticity were utilized to represent the dynamic configurations of the tube and the fluid. The solutions of the equations of motion then gave the following expression for the fluid pressure inside the tube:-

$$p_f = A_f \omega^2 \rho_f \, J_0\left[(1 - 1/c_p^2)^{1/2} \, \Omega \, (r/b)(b/a)\right] \exp\{-i(\omega t - \xi Z)\}$$

where a and b are the outer and inner radii of the tube, p_f and ρ_f are the pressure and volume density of fluid and ω and ξ are, respectively, the circular frequency and axial propagation constant, r is the radial co-ordinate and $J_0(\alpha)$ is the Bessel function of first kind and order zero. C_p and Ω are the normalized phase velocity and frequency parameter respectively and are given by the following expressions $C_p = (\omega / \xi C_f)$; $\Omega = (\omega a / C_f)$ where C_f is the sound velocity in the fluid.

The characteristic equation was obtained by satisfying the conditions of the continuity of stresses and displacements at the fluid-tube interface. For a particular frequency parameter, the characteristic equation gave only one real phase velocity less than the sound velocity in the fluid. The phase velocities higher than the sound velocities in the fluid were complex and were associated with the spatially attenuated modes. For some of the modes the attenuation was, however, very low. The details of the analysis of the characteristic equation are being published in a separate paper.

At a frequency of 0.963 MHz, the characteristic equation was solved for the first five modes which were then superimposed to obtain the resultant pressure distribution inside the tube. The pressure was normalized with respect to the pressure at the axis of the tube.

Figure 4 gives the experimental and the theoretical plots of the normalized pressure distribution at the middle plane of a 30" long brass tube with outer and inner diameters as 1.9 cms. and 1.7 cms. respectively.

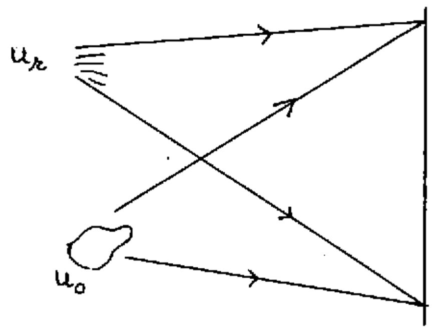

FIG. 1: HOLOGRAM FORMED AT DETECTOR

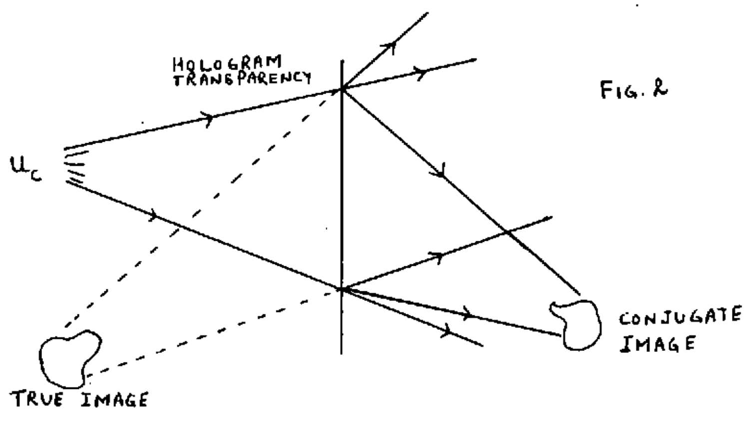

FIG. 2

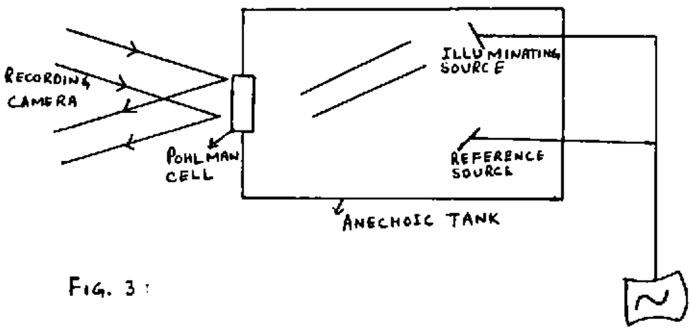

FIG. 3 :

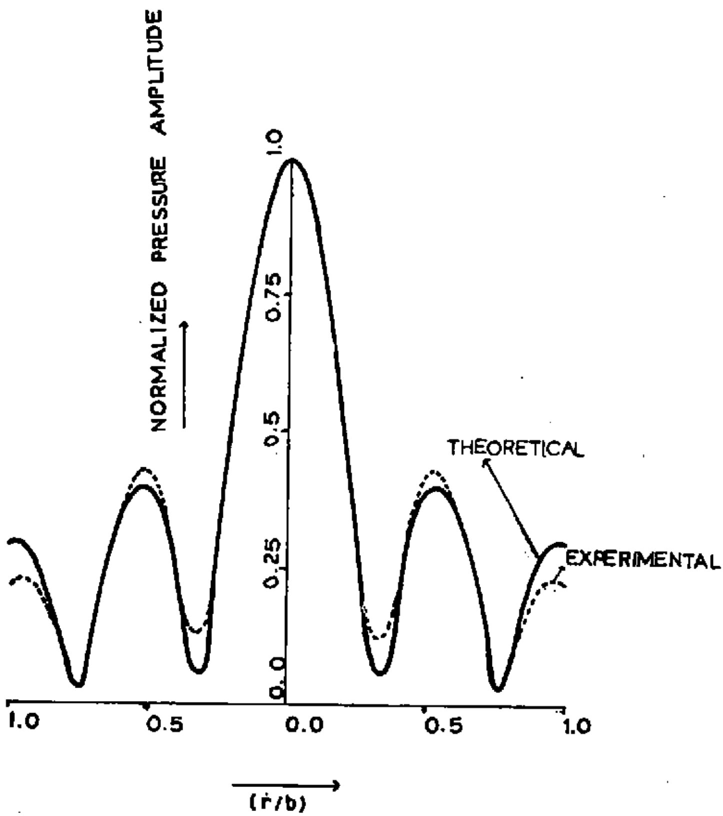

FIG. 4 NORMALIZED PRESSURE AMPLITUDE ALONG TUBE-RADIUS FOR (b/a)=0.895