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OPTIMIZATION OF PLANETARY GEAR SYSTEMS BY MEANS OF GENETIC ALGORITHM

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Effect of tooth profile modifications on dynamic behaviour of the planetary gear systems is analysed in this study. A nonlinear 2D lumped mass model of planetary gear system with time varying mesh stiffness, bearing compliance and non-smooth nonlinearity due to the backlash is taken into account.

Genetic Algorithm (GA) is used to find the optimum profile shape of each element of the planetary gear system and the goal is to optimize the total static transmission error of the gearbox. The comparison between vibrations of the optimized and pure involute gear sets is performed in order to evaluate the effectiveness of the proposed method for optimization of the planetary gear systems.
Keywords: (planetary gear, optimization, tooth profile modification, genetic algorithm)

1. Introduction

Static transmission error (STE) is the major source of noise and vibration in gears. In order to decrease effects of variation of STE, designers generally apply tip and root relief in order to improve gear performances and reduce the noise and gear vibrations.

Effect of gear tooth profile modification on reduction of the peak-peak value of STE, was evaluated previously by Parker and Bahk in 2013 [1]. In this work the interaction between multiple meshes impacted by mesh phasing and profile modifications for multiple meshes, was studied; Tooth profile modifications were applied using gap constraints for the elastic elements along the line of action and the calculations of mesh stiffnesses were done only for the gears without profile modification in this work [1]. The influence of shape deviations and profile errors on gear dynamic behavior was analyzed in 1996 by Velex and Maatar [2]. In 2008, the influence of manufacturing errors on the planetary gear stresses and planet load sharing was studied by Ligata, Kahraman and Singh [3]. In 2015, Velex et al. represented a formulation for the definition of profile modifications in high-contact-ratio (HCR) spur gears and in narrow-faced spur and helical gears, in order to select optimum relief minimizing the fluctuations of transmission error (TE) over a range of loads [4, 5]. Recently in 2016-2017, the effect of tooth profile errors, assembly errors and tooth profile modifications on dynamic response, on reduction of vibration and noise and on transmission error and mesh stiffness fluctuations of spur and helical gears has been studied [6-10].

The analysis of the literature shows that, few studies were focused on optimization of the planetary gear systems.

Dynamic optimization of spur gears was previously studied by Faggioni et al. in 2011[11]. In 2014, analysis of optimum profile modifications in spur and helical gears was performed by Bruyère and Velex [12]. An applied genetic algorithm (GA) had been developed by Barbieri et al., for optimizing spur gear pairs in terms of static transmission ratio [13, 14].

Recently in 2016, an optimum profile modification (PMs) in planetary gears with regard to dynamic mesh forces with ring-gear compliance was analyzed by Chapron et al. [15]. In this work the sensitivity of these optimum PMs to speed and load was analyzed.

The complex dynamics of a single-stage planetary gear system with time varying mesh stiffness and backlash was simulated previously by the authors [16] using a dynamic model with three equally spaced planets.

In this work the tooth profile modifications are applied to the planetary gear model simulated previously by the authors [16] using a dynamic model with three equally spaced planets (Figure 1(a)). and effect of tooth profile modifications on vibration reduction of sun gear is studied. The GA is used to find the optimal profile modifications on the sun and planets and the dynamic behavior of the entire model of the planetary gear system is compared for optimized and pure involute gear sets. Mesh stiffnesses are calculated for gear meshes before and after applying tooth profile modifications and the effect of tooth profile modifications on natural frequencies of the system is evaluated using static analysis and this effect is obvious in results of RMS of sun rotation.

2. Dynamic model

A nonlinear 2D lumped mass model of planetary gear system with time varying mesh stiffness, bearing compliance and nonsmooth nonlinearity due to the backlash and tooth profile modification is taken into account. The time varying meshing stiffnesses and error functions are evaluated by means of a nonlinear finite element model, through an accurate evaluation of global and local tooth deformation. For the sake of brevity, the reader is referred to Ref. [16] where the equilibrium equations and nonlinear dynamic behavior of the system are discussed in detail.

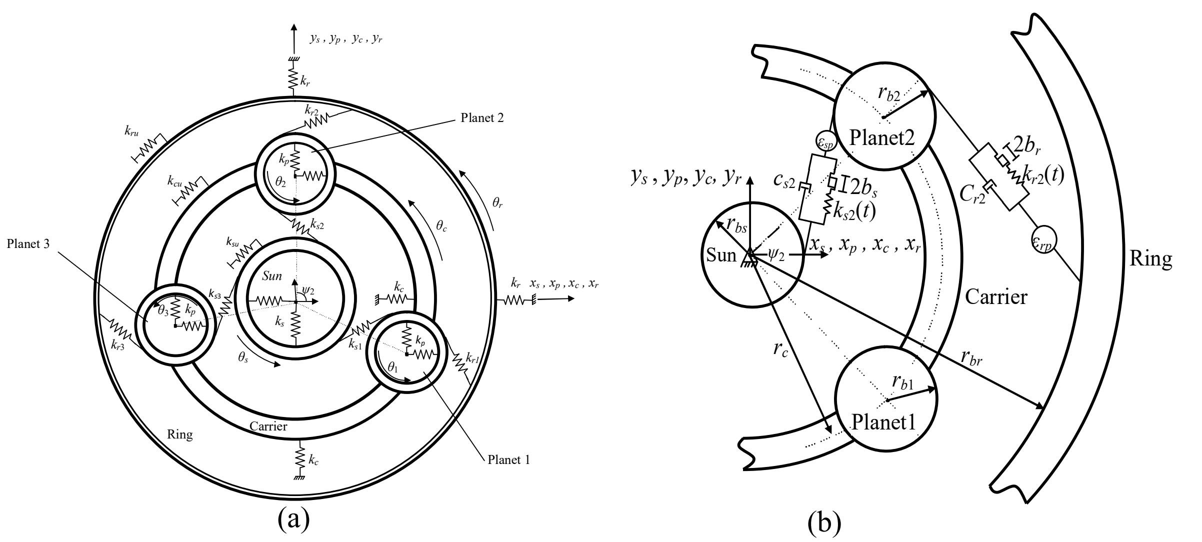


Figure 1: (a) Physical planar model of the single stage planetary gearbox, (b) Details of the meshing modeling with tooth profile modification.

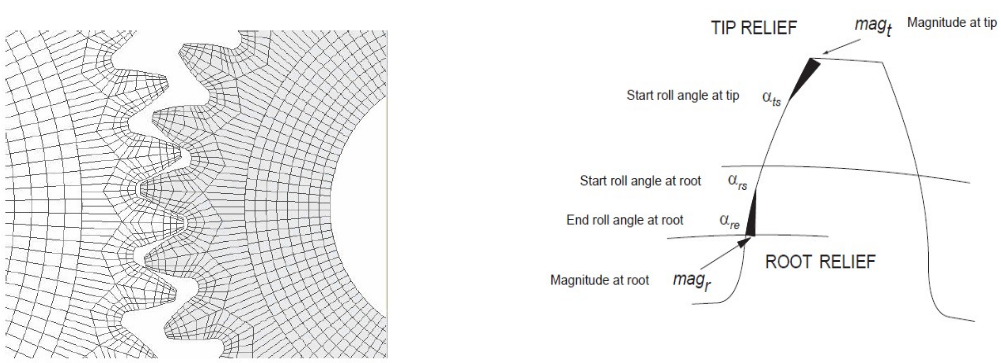
The basic dynamical equilibrium equations contain (3N+9) nonlinear ordinary differential equations, where N is the number of planets; e.g. when N=3 they will be 18 coupled equations. The effect of tooth profile modification on mesh stiffnesses and contact losses due to the profile modifications are taken into account as well as backlash.

The error functions ε_{sp} and ε_{rp} (Figure 1(b)) used for modeling the teeth profile modifications for sun-planet and planet-ring meshes are periodically time varying at the mesh frequency similarly to $k_{sn}(t)$ and $k_{rn}(t)$ [See Ref. 16, Eqs.2-6]; the mesh stiffnesses between sun-planet and ring-planet gears; and The excitation frequency ω_m is $\omega_m = \omega_s z_s z_r / (z_s + z_r)$ where ω_s is the angular velocity of the sun gear and z_s and z_r are the teeth numbers of the sun and the ring gears.

Mesh stiffnesses and the error functions are evaluated by means of '*HelicalPair*' software [17]. These functions are evaluated separately for external gears (sun-planets mesh) and internal gears (ring-planets mesh).

The gears are in contact only if the relative motion of the mating gears along the line of action exceeds the gap of a tooth pair which is equal to the sum of profile modification amounts at both nominally contacting point plus total backlash.

Figure 3(a) shows standard profile modifications on a spur gear tooth, which consist in a removal of material from the tip (tip relief) or the root (root relief), according to different manufacturing parameters. The ''start roll angle at tip'' α_{ts} and the ''magnitude at tip'' mag_t specify the point on the profile at which the relief starts and the amount of material removed at the tip radius; α_{rs}, mag_r and α_{re} have similar meaning, where the current roll angle a is given by $\alpha = \sqrt{\left(\left(d/dg1\right)^\wedge 2-1\right)}$, see Figure 3(a). Since the removal of material is measured along the direction normal to the profile, usual representations of the reliefs are given as deviation from the theoretical involute profile: Figure 3(b).



(a) Representation of profile modification parameters

(b) K-chart for linear modifications

Figure 3: a: Finite element model, b: Deviation from involute, tip and root relief.

Using this model and adding time-varying stiffnesses computed by FEM and error functions due to profile modifications, the STE of the system is evaluated. STE of the planetary system is defined as sun rotation in radians with locked carrier.

3. Validation with Calyx

The previously validated model for nonlinear dynamics of planetary gears using all degrees of freedom for sun, planets, carrier and ring gears [16] is investigated here by adding tooth profile modifications on different gear element of this case study, see Figure 4. (Gearbox data are available in Tables 1.) The model after applying the tooth profile modification is validated by means of a commercial software package CALYX (planetary 2D).

Table 1: Parameters of the case study planetary gear set

Parameter	Sun	Planet	Ring	Carrier
Number of teeth	27	35	99	-
Module [mm]	2.8677	2.8677	2.8677	-
Pressure angle [deg]	24.60	20.19		-
Working Center Distance [mm]		88.89		-
Root diameter [mm]	70.485	91.440	284.150	-
Outer diameter [mm]	84.074	105.004	304.800	-
Inner diameter [mm]	57.15	73.66	271.73	-
Base diameter [mm]	70.40	91.26	258.130	177.80
Translational bearing stiffness [N/m]	2.19e9	2.19e9	2.19e10	2.19e10
Rotational bearing stiffness [N.m/rad]	0	0	2.19e10	2.19e10
I/r^2 [kg]	1.56	2.46	-	24.80
Mass [kg]	1.64	1.33	-	21.82

The comparison between results of STE for proposed model and finite element analysis from CALYX/Planetary2D software are plotted here for different types of tooth profile modification on ring, sun and/or planet gears. The amount of 10 micron for tip and root relief for all gears tooth modifications is selected in this comparison. The results presented in Figure 4, showing good agreement with FEA results.

4. Optimization of a planetary gear system by means of GA

The goal of the proposed approach is to find an optimal set of profile modifications in order to reduce the overall vibrations in planetary gears. The objective function is the peak to peak of the static transmission error of the planetary gear; therefore optimization is performed on a static basis and for the nominal load. Optimization parameters are starting point and magnitude of tip and root profile modifications for sun and planet. Therefore eight parameters are to be found by means of GA. A description of the algorithm can be found on Ref. [13]. This is worthwhile noting that profile modifications on ring are not considered because it is more expensive to be manufactured, nevertheless the usage of both tip and root reliefs on planet are effective for both STE on the sun-planet and planet-ring mesh. (GA parameters and parameter ranges are available in Tables 2, 3.)

Table 2: Parameter of the genetic optimization

Parameter	
Number of strings in the population n_{pop}	50
Crossover probability p_c	0.6
Mutation rate p_m	0.033
Multiplier for the fitness scaling c_{mult}	1.5
Number of iterations niter	100
Type of modification	Linear
Binary code	Standard
Objective function	Peak to peak of STE

Table 3: Parameter ranges for the genetic optimization

Parameter	Start	End
α_{ts}	Roll angle at operating pitch diameter	Roll angle at tip diameter
mag_t	0	40 µm
α_{rs}	Roll angle at operating pitch diameter	Roll angle at initial point of contact diameter
mag_r	0	40 µm

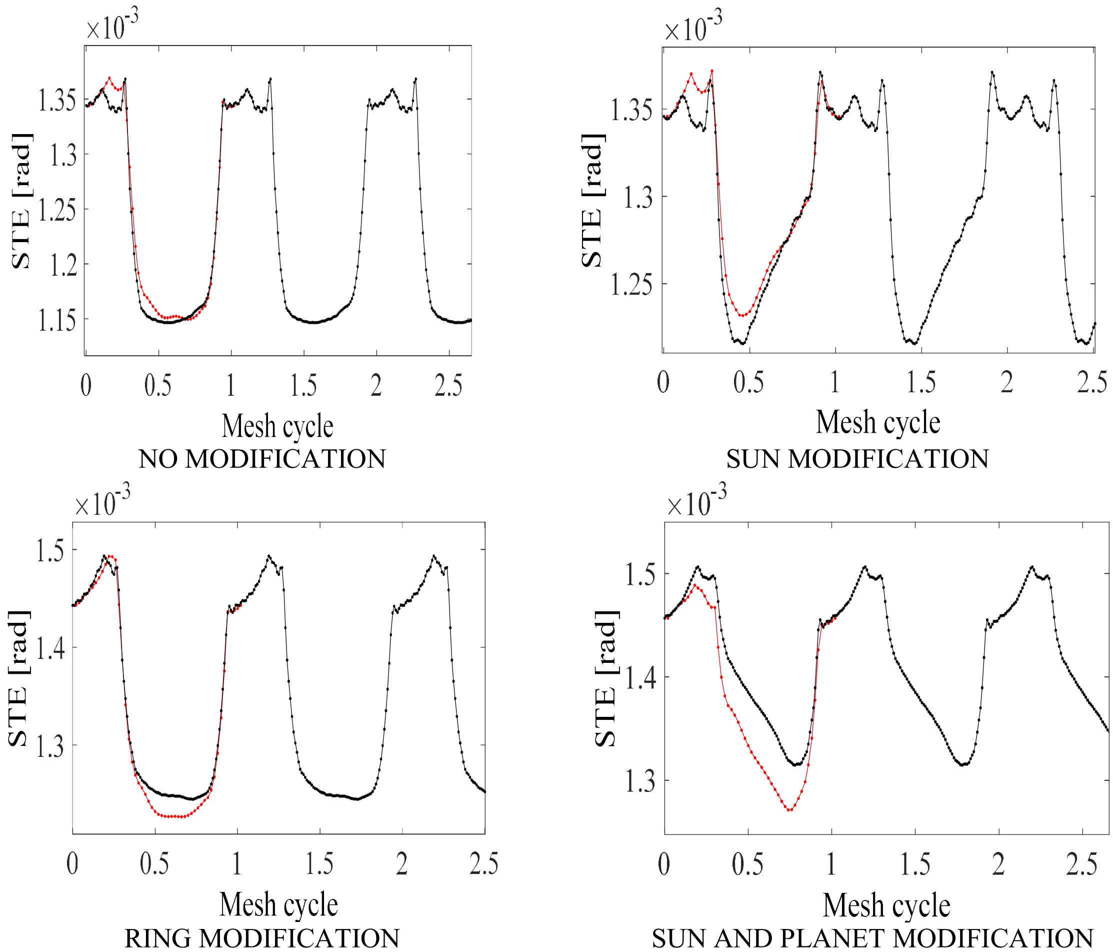


Figure 4: Sun rotation (comparison with CALYX/Planetary2D results) with tooth profile modification; red lines are the results of Planetary2D for a mesh period and black lines refer to peak-peak value of STE of the planetary gear after applying TPM.

Static and dynamic response of this planetary gear model is analyzed in details and compared in Figure 4, for the model with pure involute gears and optimized gear profile.

The optimal tip and root reliefs for sun and planets are given using K-charts along the profile as deviation from the theoretical involute profile in figure 5.

Static response of individual external and internal gear pairs evaluated and the results of STE are presented in Figure 6, for both sun-planet and planet-ring meshes before and after doing the optimization in order to see the effect of tooth profile modifications on the static response of planetary gear system.

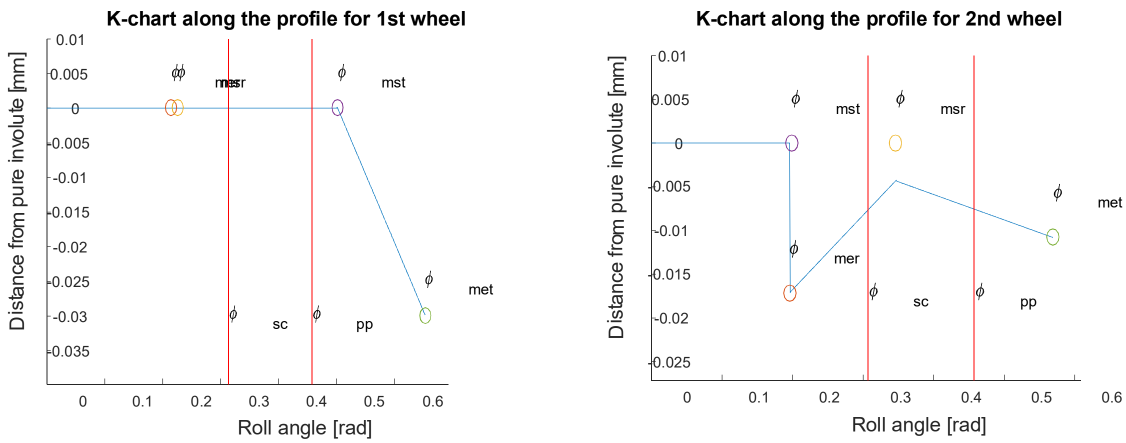
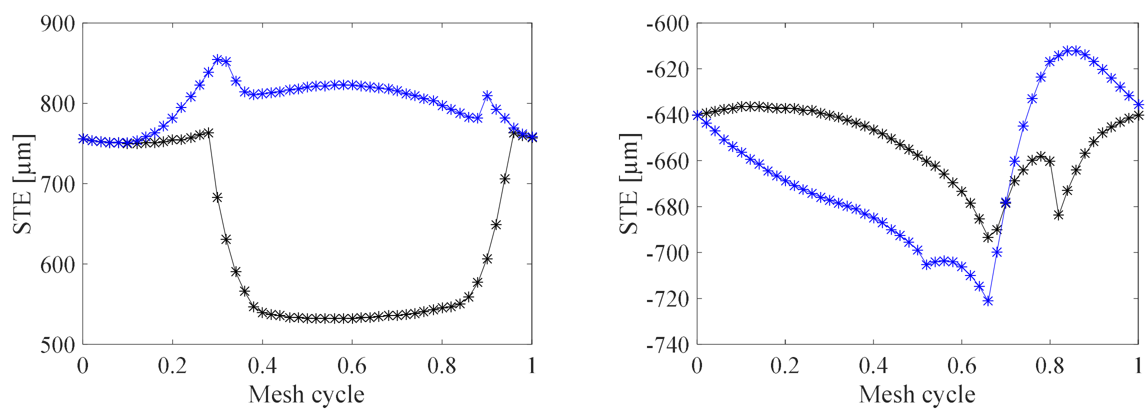


Figure 5: Start modification at tip and root for sun and planet gear for optimized gear set; 1st wheel refe to sun gear and 2nd wheel refers to planet gear.

The effectiveness of the optimized set in terms of broadband dynamic response is checked by means of the dynamic model.

Figure 7, represents the RMS (mean removed) of sun compared for planetary gear sets with pure involute and optimized gears.

The vibration levels obtained from optimized set are smaller than the vibration levels obtained using a pure involute. This confirms the good dynamic performance of the optimized gear set.



(a) (b)

Figure 6: Comparison of static transmission error before and after optimization a) sun-planet mesh, b) planet-ring mesh

Profile modifications are effective in the frequency ranges where no contact loss occurs; and in the case of contact loss, optimal profile modifications reduce the ranges. Tooth profile modification is also effective on meshing stiffnesses and natural frequencies of the system. The natural frequencies of both models (before and after optimization) are listed in Table 4. (Mean values for mesh stiffnesses for this case study before and after applying the tooth profile modifications are respectively equal to 6.14e+08 *N/m* and 5.59e+08 *N/m*).

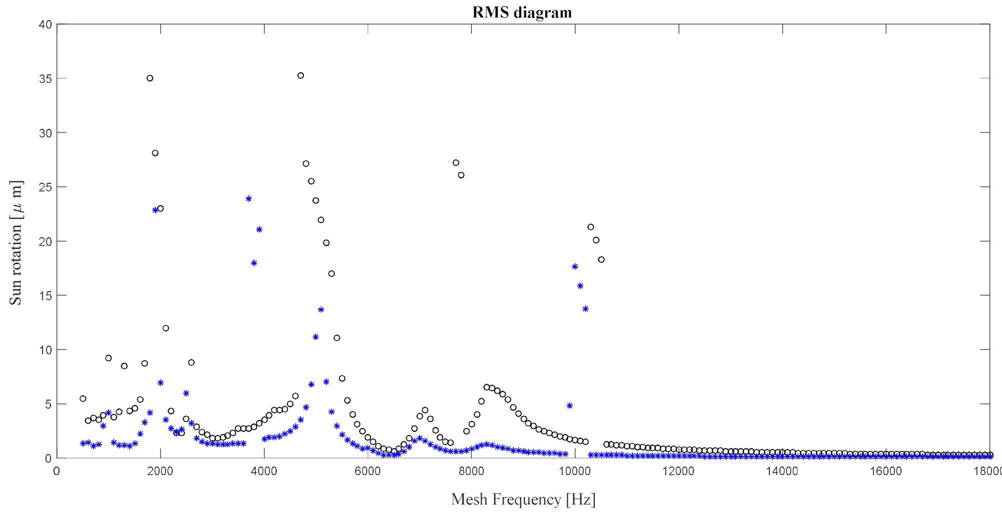


Figure7: RMS of sun rotation vs. mesh frequency; blue stars present the effect of optimization on planetary gear response, black circles are results for pure involute gears

Table 4: Comparison of natural frequencies for planetary gear model with and without TPM

Frequency mode number	Natural frequency [Hz] case 1: *pure involute gear model*	Natural frequency [Hz] case 2: *optimized model*
1	1749	1728
2	1749	1728
3	2017	1904
4	3253	3089
5	3253	3089
6	5188	5030
7	6373	6346
8	6373	6346
9	7096	6960
10	7334	7221
11	7334	7221
12	8451	8220
13	8507	8328
14	8507	8327
15	22295	22198
16	22295	22198
17	53264	53264
18	149096	149074

5. Conclusion

Dynamic behavior of the entire model of the planetary gear system is analyzed at different rotational velocities before and after applying tooth profile modifications. The optimized gear set is found by means of GA.

As expected, by optimizing the planetary gear system natural frequencies of the system decrease due to reduction of the mesh stiffnesses.

The results obtained suggest that an optimal set of profile modifications can reduce overall vibrations and peak-peak value of STE in planetary gearboxes.

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