

# NON-FRAGILE OBSERVER-BASED CONTROLLER WITH AN APPLICATION TO ACTIVE VIBRATION ATTENUATION

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This note tackles the active disturbance rejection problem in accordance with the robust, resilient observer-based regulation problem for a class of linear systems with structured uncertainty. The stability of the technique is guaranteed by employing the direct Lyapunov theorem. With aiming at addressing the fragility issue, two sets of time-dependent bounded uncertain terms are admitted for the controller gain and observer gain in the synthesis process. Then, the proposed control system is transformed into linear matrix equality/inequalities (LME/LMIs) framework. Next, the closed-loop system is implemented in real-time experiments on a vibrating system to evaluate its disturbance rejection performance. For this purpose, the mismatch disturbance scenario is investigated on a multi-input single output cantilever piezo-laminated beam. The nominal mathematical model of the smart structure is reduced in the frequency band with a limited number of states in the structure of parametric modeling. The classical state-feedback problem is replaced with state-observation technique to formulate an output-feedback controller. Finally, the proposed combination is implemented experimentally on the full-order system. The results confirm that the closed-loop configuration has a robust performance in terms of the active disturbance attenuation with the structured uncertainty and the presence of the nonlinear Lipschitz stable dynamics.

Keywords: vibration control, state observer, resilient controller, smart structure

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## 1. Introduction

The non-fragility of the control system can be studied by investigating the sensitivity of the controller (observer) with respect to feedback- (observer-) gain [1]. The uncertainty in the controller gain as a key design limitation is inspected by Corrado and Haddad [2]. The control design procedure is extended to address the proposal of a novel robust, resilient static output feedback regulator for systems with controller gain uncertainty. Wang *et al.*, [3] proposed a non-fragile observer design technique by use of the multiple Lyapunov function method and delta-operator theory. Lien *et al.*, [4] proposed a non-fragile observer based controller for linear systems. However, the numerical design example is limited to a deterministic mathematical state-space model. This issue is tackled in Oveisi *et al.*, [5] for structures with mismatch uncertainties in system elements.

The modern structural design procedure requires the properties such as self-adaptation and smartness in order to achieve a multi-functional, practical, robust, environmentally-oriented performance. This perception is handled by sensitive multi-domain transducers that can be instrumented together with the host structure and robustly react to the environmental stimuli. Such adaptive structural configurations are widely used in mechanical assemblies to control the stress, strain, and sound [6-7].

These active structures require sensor/actuator elements that can then be used to withstand their sensitivity to unwanted disturbances [8]. Piezoelectric actuators are widely utilized due to their capability of pairing mechanical tension and electric field [9, 10]. The development of finite element method (FEM) and analytical methods in modeling these transducers progressed at the same rapid step as active structural control methods [11]. Applications of non-fragile controllers are widely studied. For instance, Du *et al.* tackled the application of non-fragile  $H_\infty$  robust controller on an uncertain four degree of freedom building model by an appropriate use of Bounded Real Lemma (BRL) [12]. Yazici *et al.*, [13] proposed a robust delay-dependent controller by using the Lyapunov–Krasovskii functional (LKF) and BRL in order to reach to a minimization algorithm that gives a sub-optimal solution for the disturbance rejection controller in the presence of structured uncertainty. They implemented the designed controller on a building model subjected to seismic excitations. Ramakrishnan and Ray presented a delay-dependent non-fragile  $H_\infty$  controller for a nonlinear system with time-varying delay using LKF [14].

The main goal of this paper is to propose a controller that can handle the fragility problem of both the controller and observer elements in the presence of structured uncertainty, unmodelled dynamics of the stable linear or nonlinear type under external excitation. The dynamics of the controller is presented in section 2. The process of performance evaluation for the control system is carried out by implementation on a vibrating system. In section 3, the experimental setup is introduced, and in section 4, the mathematical procedure of extracting the nominal reduced-order dynamics of the plant is discussed. Finally, in section 5, the simulation and experimental results are explained in more details.

## 2. General style parameters

### 2.1 System Definition

Consider the linear time-invariant (LTI) MIMO system in Eq. (1) in the state space form.

$$\dot{x} = (A + \Delta A)x + Bu + Hw + f, \quad y = Cx + Du, \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$ ,  $w \in \mathbb{R}^r$ ,  $f \in \mathbb{R}^n$ , and  $y \in \mathbb{R}^q$  are the state, input, square-integrable external disturbance, nonlinearity/un-modelled dynamics, and output vectors, respectively. In addition,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$ ,  $H \in \mathbb{R}^{n \times r}$ ,  $C \in \mathbb{R}^{q \times n}$ , and  $D \in \mathbb{R}^{q \times (m+r)}$  are state, input, disturbance, output, and feedthrough constant matrices. It is assumed that the system in Eq. (1) is stabilizable and detectable. Moreover,  $\Delta A$  is the associated unknown structured perturbation matrix which is time-dependent. In order to avoid non-convex problem, the uncertainty due to the input matrix ( $\Delta B$ ) is not investigated in this study. It should be mentioned that, from the vibration control point of view, the unmodelled/nonlinear dynamics satisfy the Lipschitz uncertainty condition [15, 16] and the uncertainty of system matrix should satisfy the mismatch condition [17, 18]. Dynamics of the proposed non-fragile observer is defined as Eq. (2)

$$\dot{\hat{x}} = (A + \Delta A)\hat{x} + Bu - (L + \Delta L)[y - \hat{y}], \quad \hat{y} = C\hat{x}, \quad (2)$$

where  $\hat{x}$  and  $\hat{y}$  present the estimation of the states and of the output, respectively and  $L, \Delta L \in \mathbb{R}^{n \times q}$  are the observer gain and observer perturbation associated with fragility of the observer. In addition, the non-fragile observer-based control effort is considered to be  $u = (K + \Delta K)\hat{x}$ , where  $K, \Delta K \in \mathbb{R}^{m \times n}$  are the feedback gain and feedback perturbation matrices, respectively. It is assumed that the perturbation matrices are independent of  $L$  and  $K$  and have the structure  $\Delta K = M_K F_K N_K$ , and  $\Delta L = M_L F_L N_L$ , in which  $M_K, N_K, M_L$ , and  $N_L$  are known matrices with appropriate dimension. However, the time-varying unknown matrices with Lebesgue-measurable elements [17],  $F_K(t)$  and  $F_L(t)$ , should satisfy  $F_K^T F_K \leq I$ , and  $F_L^T F_L \leq I$ , correspondingly. Moreover, it is assumed that, there exist positive scalars,  $a$  and  $g$ , such that  $\|\Delta A\| \leq a$ , and  $\|f\| \leq g\|x\|$ . By defining the estimation error as  $e = x - \hat{x}$ , using (1) and (2), the dynamics of the states and the estimation error are presented as Eq. (3)

$$\begin{aligned} \dot{x} &= ((A + \Delta A) + B(K + \Delta K))x - B(K + \Delta K)e + Hw + f, \\ \dot{e} &= (A + \Delta A + (L + \Delta L)C)e + Hw + f, \end{aligned} \quad (3)$$

## 2.2 Main Results

**Theorem 1.** *The uncertain system (1) together with the observer system (2) is quadratically stable and satisfies the  $H_\infty$  norm constraint  $\|T_{yw}\|_\infty < \gamma_0$ , if there exist two positive definite symmetric matrices  $P, R \in \mathbb{R}^{n \times n}$ , positive scalars  $\varepsilon_i, i = 1, 2, \dots, 5$ , and matrices  $\hat{P} \in \mathbb{R}^{m \times m}, \hat{L} \in \mathbb{R}^{n \times q}$ , and  $\hat{K} \in \mathbb{R}^{m \times n}$  such that the following LMI/LME system is minimized*

$$\begin{aligned} & \min \gamma \\ & \text{Subject to} \\ & \Omega = \begin{bmatrix} \chi & \psi & \phi \\ & \omega_1 & 0 \\ * & & \omega_2 \end{bmatrix} < 0, \\ & PB - B\hat{P} = 0, \\ & \chi = \begin{bmatrix} Z_{11} & -B\hat{K} & PH & \varepsilon_5 N_K^T \\ & Z_{22} & RH & -\varepsilon_5 N_K^T \\ & & -\gamma I & 0 \\ * & & & -\varepsilon_5 I \end{bmatrix}, \quad \psi = \begin{bmatrix} aP & gP & PBM_K \\ & 0 & \end{bmatrix}, \\ & \phi = \begin{bmatrix} & 0 & & \\ aR & gR & RM_L & \varepsilon_5 C^T N_L^T \\ & 0 & & \end{bmatrix}, \quad \omega_1 = \text{diag}(-\varepsilon_1, -\varepsilon_3, -\varepsilon_5), \\ & \omega_2 = \text{diag}(-\varepsilon_2, -\varepsilon_4, -\varepsilon_5, -\varepsilon_5). \end{aligned} \quad (4)$$

where

$$\begin{aligned} Z_{11} &= A^T P + PA + \hat{K}^T B^T + B\hat{K} + C^T C + \varepsilon_1 I + \varepsilon_3 I + \varepsilon_4 I, \\ Z_{22} &= A^T R + RA + C^T \hat{L}^T + \hat{L}C + \varepsilon_2 I. \end{aligned}$$

Moreover, 0 represents the zero matrix with appropriate dimension. The robust non-fragile observer and controller gain are given by  $K = \hat{P}^{-1}\hat{K}, L = R^{-1}\hat{L}$ .

## 2.3 Preliminary Results and Proof

Here, some preliminary Lemmas are introduced that will be used in the proof of Theorem 1.

**Lemma 1.** [19] *For two arbitrary vectors such as  $p, q$  the following inequality is valid for  $\epsilon > 0$*

$$p^T q + q^T p \leq \epsilon p^T p + \epsilon^{-1} q^T q. \quad (5)$$

**Lemma 2.** [20] *For real matrices  $\Sigma, \Psi, X$  and symmetric matrix  $M$ , the first statement can be guaranteed iff the second one holds for a positive scalar  $\zeta$  and  $\Psi^T \Psi \leq I$ ,*

- 1)  $M + \Sigma \Psi X + X^T \Psi^T \Sigma^T < 0$ ,
- 2)  $M + \zeta^{-1} \Sigma \Sigma^T + \zeta^{-1} (\zeta X^T)(\zeta X) < 0$ .

**Lemma 3.** [20] *The linear system  $\dot{x} = Ax + Hw, y = Cx$ , satisfies the  $H_\infty$  norm constraint  $\|T_{yw}\|_\infty < \alpha$  with Lyapunov function  $V(x) = x^T Mx, M > 0$  if for  $t > 0$ ,*

$$\dot{V} + y^T y - \alpha^2 w^T w < 0. \quad (7)$$

**Lemma 4.** [5, 20] *For a given matrix  $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$ , with symmetric  $S_{11}$  and symmetric negative definite  $S_{22}$  the following two statements are equivalent*

- 1)  $S < 0$ ,
- 2)  $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$ .

**Proof.** Consider the following Lyapunov function

$$V = x^T P x + e^T R e, \quad (9)$$

by derivation of Eq. (9) with respect to time along the system trajectories, one can obtain

$$\begin{aligned}
 \dot{V} = & x^T A^T P x + x^T \Delta A^T P x + x^T K^T B^T P x + x^T \Delta K^T B^T P x - e^T K^T B^T P x - e^T \Delta K^T B^T P x \\
 & + w^T H^T P x + f^T P x + e^T A^T R e + e^T \Delta A^T R e + e^T C^T L^T R e + e^T C^T \Delta L^T R e \\
 & + w^T H^T R e + f^T R e + x^T P A x + x^T P \Delta A x + x^T P B K x + x^T P B \Delta K x \\
 & - x^T P B K e - x^T P B \Delta K e + x^T P H w + x^T P f + e^T R A e + e^T R \Delta A e \\
 & + e^T R L C e + e^T R \Delta L C e + e^T R H w + e^T R f.
 \end{aligned} \tag{10}$$

Then, by utilizing Eq. (5) on the terms with uncertainty in system matrix and unmodelled dynamics, one can obtain

$$\begin{aligned}
 x^T \Delta A^T P x + x^T P \Delta A x & \leq x^T (\varepsilon_1 I + \varepsilon_1^{-1} a^2 P^2) x, \\
 e^T \Delta A^T R e + e^T R \Delta A e & \leq e^T (\varepsilon_2 I + \varepsilon_2^{-1} a^2 R^2) e, \\
 f^T P x + x^T P f & \leq x^T (\varepsilon_3 I + \varepsilon_3^{-1} g^2 P^2) x, \\
 f^T R e + e^T R f & \leq x^T (\varepsilon_4 I) x + e^T (\varepsilon_4^{-1} g^2 R^2) e.
 \end{aligned} \tag{11}$$

By using Lemma 2 and assuming  $\mathcal{L}^T = [x^T \quad e^T \quad w^T]$ , the uncertain terms due to the fragility of controller and observer gain can be rewritten as

$$\mathcal{L}^T \begin{bmatrix} \Delta K^T B^T P + P B \Delta K & -P B \Delta K & 0 \\ * & C^T \Delta L^T R + R \Delta L C & 0 \\ * & * & 0 \end{bmatrix} \mathcal{L} \leq \mathcal{L}^T \Omega_1 \mathcal{L}, \tag{12}$$

where  $*$  presents the lower left triangle terms which are the transpose of the upper right ones and  $\Omega_1$  is presented as

$$\begin{aligned}
 \Omega_1 = & \begin{bmatrix} Y_{111} & -\varepsilon_5^{-1} (\varepsilon_5^2 N_K^T N_K) & 0 \\ * & Y_{122} & 0 \\ * & * & 0 \end{bmatrix}, \\
 Y_{111} = & \varepsilon_5^{-1} P B M_K M_K^T B^T P + \varepsilon_5^{-1} (\varepsilon_5^2 N_K^T N_K), \\
 Y_{122} = & \varepsilon_5^{-1} R M_L M_L^T R + \varepsilon_5^{-1} (\varepsilon_5^2 C^T N_L^T N_L C) + \varepsilon_5^{-1} (\varepsilon_5^2 N_K^T N_K).
 \end{aligned} \tag{13}$$

Via Lemma 3 and Eqs. (10-13) one can obtain Eq. (14) as

$$\dot{V} - \gamma_0^2 w^T w + y^T y = \mathcal{L}^T \Omega \mathcal{L}, \tag{14}$$

where,

$$\begin{aligned}
 \Omega = & \Omega_1 + \Omega_2 + \Omega_3, \\
 \Omega_2 = & \begin{bmatrix} Y_{211} & -P B K & P H \\ * & Y_{222} & R H \\ * & * & -\gamma_0^2 I \end{bmatrix}, \quad \Omega_3 = \begin{bmatrix} Y_{311} & 0 & 0 \\ * & Y_{322} & 0 \\ * & * & 0 \end{bmatrix}, \\
 Y_{211} = & A^T P + P A + K^T B^T P + P B K + C^T C, \\
 Y_{222} = & A^T R + R A + C^T L^T R + R L C, \\
 Y_{311} = & \varepsilon_1 I + \varepsilon_1^{-1} a^2 P^2 + \varepsilon_3 I + \varepsilon_3^{-1} g^2 P^2 + \varepsilon_4 I, \\
 Y_{322} = & \varepsilon_2 I + \varepsilon_2^{-1} a^2 R^2 + \varepsilon_4^{-1} g^2 R^2.
 \end{aligned} \tag{15}$$

Consequently, by assuming  $PB = B\hat{P}$ ,  $\hat{P}K = \hat{K}$ ,  $RL = \hat{L}$ , and  $\gamma = \gamma_0^2$ , and successively using the Schur complement on Eqs. (14) and (15), the LMI (4) can be obtained. That completes the proof. ■

**Remark 1.** The LMI in Eq. (4) is affined on the subject to the defined matrices since it is guaranteed by the use of the LME conversion, which simply transforms a non-convex problem to a convex one. This non-convexity is due to the introduction of the observation error in the Lyapunov equation.

### 3. Experimental setup

The proposed controller has been tested on the clamped-free smart beam made of an aluminium core layer with two piezo-actuators (DuraAct™ P-876.A15) that are attached on one side. The geometrical constants of the vibrating system under study can be seen in Fig. 1. The velocity of the lateral vibration of the free end of the beam is measured as the output signal obtained by a laser Doppler vibrometer VH-1000-D (see Fig. 2). The ADC, DAQ, and DAC in Fig. 2 are representing the analog to digital converter (DS2004), dSPACE digital data acquisition board (DS1005), and digital to analog

converter (DS2102). In addition, due to the working voltage range for piezo-patches, the control signal is amplified by the PI E-500 Amplifier. In order to implement the proposed control system, SIMULINK platform is used to compile the control algorithm and subsequently to upload it to dSPACE board DS1005. The real time signals are accessible through the software ControlDesk.

**Remark 2.** In the proposed system, the control input is applied through the piezo-actuator channels and the mechanical disturbance acts through a mismatch disturbance channel which may be realized as the signal generated through the shaker (Bruel & Kjaer Type-4809).

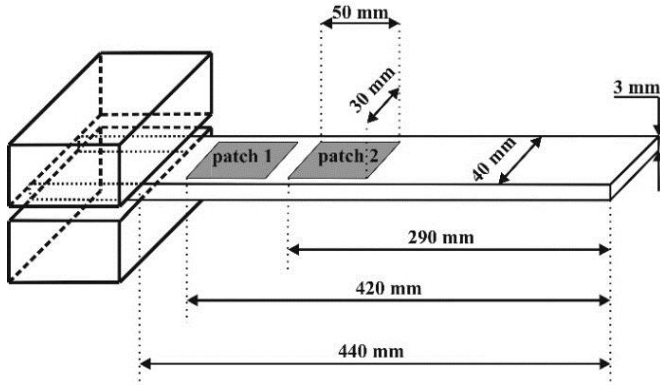


Figure 1: Geometry of the smart beam.

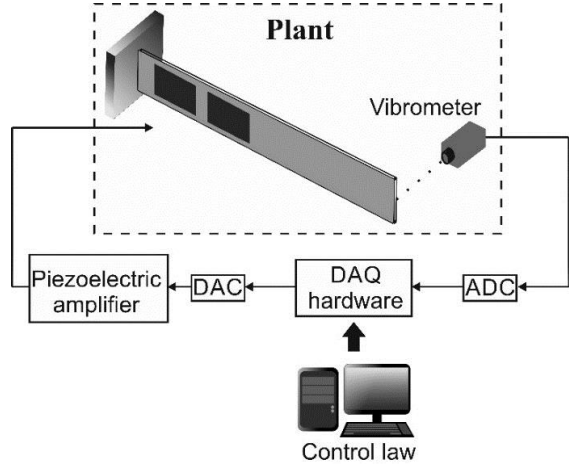


Figure 2: Experimental rig for validating the performance of the control system.

## 4. MODELLING THE MECHANICAL SYSTEM

The dynamic equation of motion is obtained by FEM analysis in the linear piezo-elasticity domain. First, the partial differential equation of motion in spatial- and the time-domain is transformed to system of ODEs by using the orthogonality for a single mode-shape. It is proven that for the system under study, the dominant mode of the vibration is the first mode-shape [21, 22]. The detailed procedure of extracting the dynamics of the coupled electro-mechanical system including the piezo-actuators is explained in [23]. Final representation of the system in the state-space form of Eq. (1) is obtained with the following system matrices

$$A = \begin{bmatrix} 0 & \Lambda \\ -\Lambda & 2Y\Lambda \end{bmatrix}, B = \begin{bmatrix} 0 \\ B_m \end{bmatrix}, H = \begin{bmatrix} H_{mq} \\ H_{mv} \end{bmatrix}, C = [C_{mq} \quad C_{mv}], D = 0, \quad (16)$$

where  $\Lambda = \text{diag}(\omega_i)$ , and  $Y = \text{diag}(\xi_i)$ ,  $i = 1, \dots, n$  with  $\omega_i$  and  $\xi_i$  being the natural frequency and the structural damping ratio of  $i$ -th mode-shape and  $B_m$ ,  $H_{mq}$ ,  $H_{mv}$ ,  $C_{mq}$ , and  $C_{mv}$  representing the modal realization of the electro-mechanical effect of the piezo-patches, external disturbance, displacement, and the velocity measurement of the system after applying the orthogonality of mode-shapes [24].

## 5. Simulation and experimental evaluation

In this section, the implementation of the proposed controller in Eq. (4) on the smart cantilever beam in Eq. (16) is studied. It is assumed that the system matrix  $A$  has 10 % structured uncertainty in its elements in the control design procedure. Since the verification of the controller is performed on the experimental continuous system, the nonlinear terms or un-modelled dynamics  $f$  will be the higher order mode-shapes which are considered as norm-bounded terms in the designing process. The closed-loop system is implemented on the real-time data acquisition platform of the dSPACE with a sampling rate of 10 kHz. The predefined task of the controller is to guarantee the robust stability and performance in conjugation with real-time vibration amplitude suppression in frequency ranges close



to resonance eigenvalues. Therefore, studies are carried out in the time-domain using the experimental rig shown in Fig. 3. Comparison of the response of the closed-loop system with the proposed controller and the response of the open loop system under a harmonic excitation with the frequency close to the first natural frequency (14 Hz) is shown in Fig. 4.



Figure 3: Experimental setup for verifying the performance of the controller.

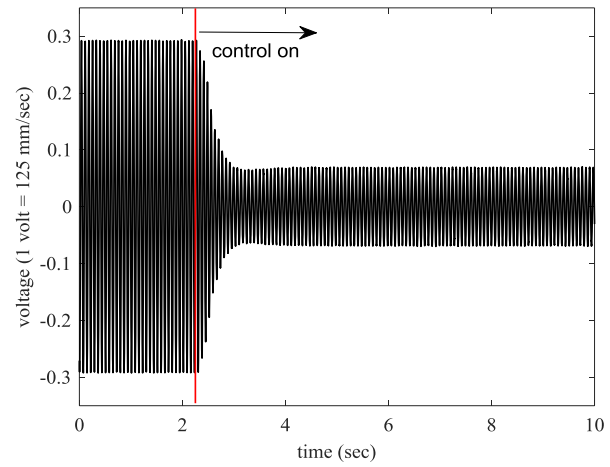


Figure 4: Measured velocity output before and after activation of the control system.

In this figure, the feedback channel is activated after 2.24 seconds, in order to give distinguishable results from the performance of the closed-loop system. Fig. 4 shows that the performance of the controller in suppressing the vibration is acceptable in the presence of the modelling uncertainties. Fig. 5 shows the control effort that is acting on the two piezo-actuators of the smart beam. The maximal applied voltage is limited to 100 Volts, and any higher voltage is saturated. Also, the estimated error of the observer is depicted in Fig. 6. In Figs. (4-6), as expected the vibration continues to exist, since the disturbance continues to excite the system throughout the entire duration of the experiment.

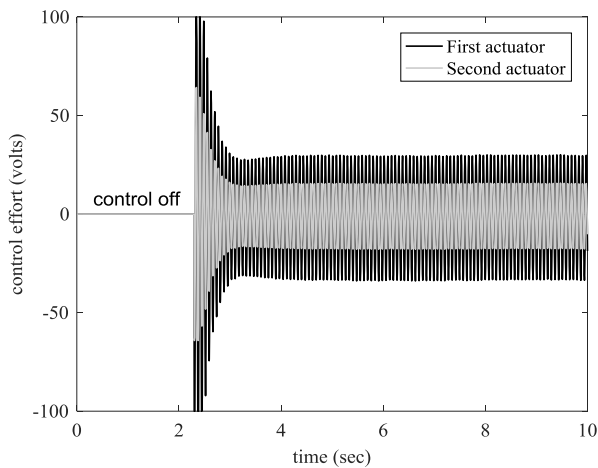


Figure 5: Control effort acting on each of the piezo-actuator patches.

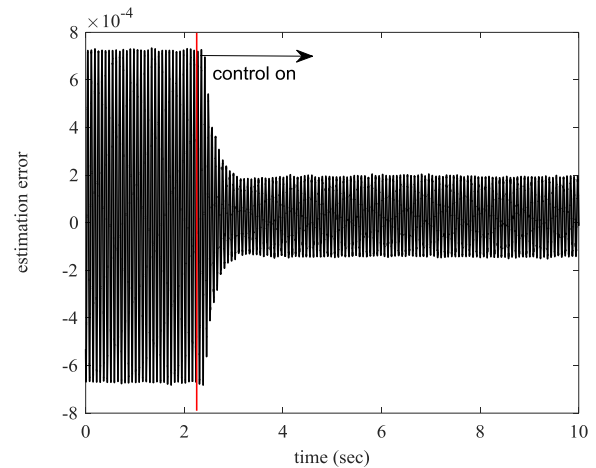


Figure 6: Estimation error of the non-fragile observer.

Finally, to evaluate the robustness of the system to the higher unmodelled dynamics, the structure is excited by a sine signal with the frequency 77 Hz which covers the second mode-shape of the system. Fig. 7 shows the robust performance of the control system under consideration of the uncertainty modeling and excitation of unmodelled dynamics in the presence of the external disturbances.

In addition Figs. (8) and (9) are representing the control input and the estimation error, respectively.

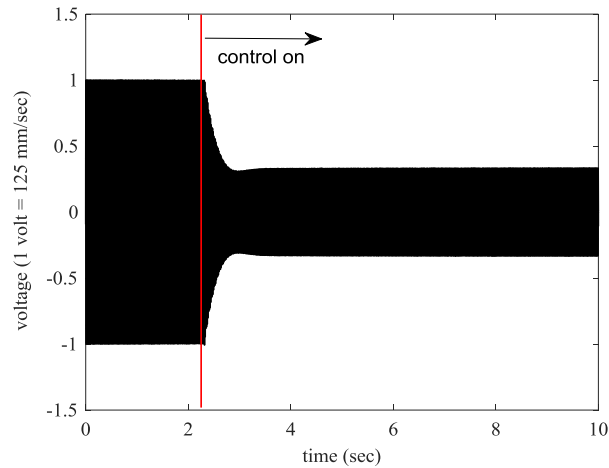


Figure 7: Measured velocity under the excitation of unmodelled dynamics.

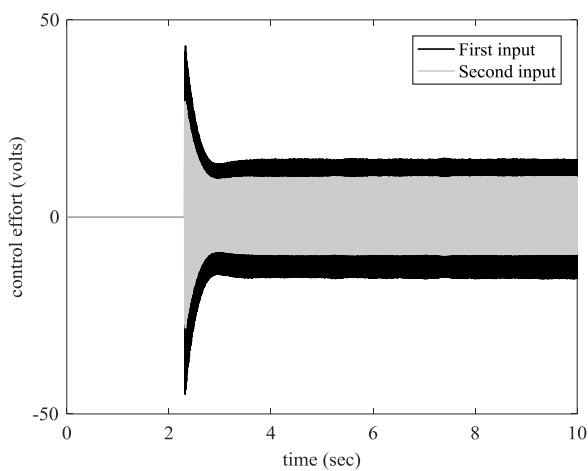


Figure 8: Control effort acting on each of the piezo-actuator patches.

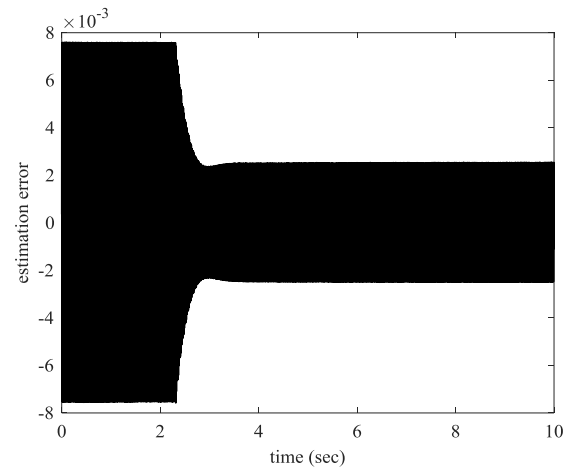


Figure 9: Estimation error of the non-fragile observer.

## 6. Conclusion

In this paper, the non-fragile observer-based control problem of uncertain reduced-order linear system is studied in an LMI/ME framework. The direct Lyapunov theorem is employed for coupled observer/controller stability analysis and synthesis. The controller is implemented on a real full-order mechanical vibrating system in the presence of the undesired external excitation. The robust performance of the closed-loop system in real-time is investigated experimentally.

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