

# VARIATIONS WITHIN SIMPLE MODELS FOR STRUCTURE-SOIL INTERACTION

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The dynamic response of blocks sitting on a half-space is considered. Buildings and other structures placed on or within the ground influence the transmission of seismic waves. Hence, the presence of a building will have an impact on the dynamic response of neighbouring buildings. Furthermore, obstacles such as concrete blocks lead to wave scattering that may be beneficial or unfavourable for the response of a building close to, for example, a railway. To account for this dynamic cross coupling via the soil, a model must be accurate enough to provide the correct overall behaviour of the scattered wave field. However, simplicity is also important when a model should be used for design purposes, especially in the early stages of design and feasibility studies. The paper addresses two models in 2D and 3D based on different methodologies. Results are discussed regarding their capability to quantify vibration reduction when a periodic combination of masses are added on the ground to mitigate waves.

Keywords: Vibration reduction, lumped-parameter, Greens function, Elastic half-space.

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## 1. Introduction

With increasing densification of the cities, it becomes increasingly important to assess ground vibration resulting from construction work, heavy traffic and factory machinery. Most of the energy of such vibration is concentrated in the low frequency range, and the complex nature of wave propagation in the ground does not permit direct analogies to acoustic screening. Instead, many methods for analysis of dynamic soil–structure interaction (SSI) have been proposed, and various techniques have been suggested to mitigate ground vibration.

Vibration mitigation techniques aim to protect sensitive buildings by creating a “shadow zone” within the propagation path beyond an “isolation” element: an isolating screen, a trench or wave barrier, or a wave-impeding block (WIB). In this study, the emphasis is on WIBs and their location on the ground surface relative to a vibration source and a receiver. Due to its large mass, a WIB as a heavy rigid block imposes a rigid-like boundary condition over the ground surface at its footprint. This has the effect of impeding the Rayleigh wave transmission but has, in itself, limited efficiency due to other ground waves that have the possibility to “diffract” around the rigid boundary. However, a second effect—introduced by the ground—is the resultant mass-stiffness system which has the ability to store and release energy similar to a single-degree-of-freedom (SDOF) system. Given certain site-specific characteristics, it could be possible to control the isolation efficiency.

The dynamic behaviour of the ground surface and its influence on soil coupling to structures has been considered by many authors. One of the earliest contributions was by Warburton et al. [1] who considered the interaction of two rigid circular foundations using a mixed integral equation approach. Peplow et al. [2] studied WIBs within layered ground and Krylov [3] studied the effect of blocking

masses such as concrete masses placed on the ground. More recently Dijckmans et al. [4] made a comprehensive study of an array of blocks placed alongside a railway track. The principle of this solution was to modify the wave propagation regime of the ground by introducing an inertial mass near the load.

In this paper, the transmission and reduction of vibration transmission to the far field of the surface of the ground is investigated using a Greens-function method in 3D and a simple lumped parameter model in 2D. Section 2 outlines the applied methodology, and Section 3 presents a study of the insertion loss provided by an array of blocks placed on the ground surface. Finally, a short summary and conclusions are given in Section 4.

## 2. Methodology

Andersen and Clausen [5] provided a formulation of the impedance associated with rigid surface footings on layered ground. As an extension to this, a formulation is here given for multiple rigid blocks on the surface of the ground or embedded in the soil. As proposed by Andersen [6] and Bucinskas et al. [7], structure–soil–structure interaction for systems consisting of  $N_F$  rigid bodies can be expressed via the impedance matrix  $\mathbf{Z}_F(\omega)$  relating the displacement and rotation components of all bodies to the forces and moments acting on all bodies:

$$\mathbf{Z}_F(\omega)\mathbf{D}_F(\omega) = \mathbf{F}_F(\omega), \quad (1a)$$

$$\mathbf{D}_F(\omega) = \{\mathbf{D}_{f1}^T(\omega) \ \mathbf{D}_{f2}^T(\omega) \ \cdots \ \mathbf{D}_{fN_F}^T(\omega)\}^T, \ \mathbf{D}_{fn}^T = \{U_{x,n} \ U_{y,n} \ U_{z,n} \ \Theta_{x,n} \ \Theta_{y,n} \ \Theta_{z,n}\}^T, \quad (1b)$$

$$\mathbf{F}_F(\omega) = \{\mathbf{F}_{f1}^T(\omega) \ \mathbf{F}_{f2}^T(\omega) \ \cdots \ \mathbf{F}_{fN_F}^T(\omega)\}^T, \ \mathbf{F}_{fn}^T = \{P_{x,n} \ P_{y,n} \ P_{z,n} \ M_{x,n} \ M_{y,n} \ M_{z,n}\}^T. \quad (1c)$$

As indicated, each three-dimensional rigid body has three translational and three rotational degrees of freedom. Each individual rigid body is discretized into a number of points, and the Green's function is utilized to evaluate the influence from all points to all points in the model, eventually leading to a flexibility matrix the inverse of which multiplied by the displacement of all points associated with each individual mode of rigid-body motion provides  $\mathbf{Z}_F(\omega)$ . The displacement response at any points, e.g. on the surface of the ground, can be found by post processing, using again the Green's function to obtain the influence from interaction forces provided by the rigid-body motion.

As proposed by Jones [8], the 2D model is in essence a lumped-parameter model provided by point-mass loads, and the interaction between masses is described by a transfer-mobility matrix approach. The effect of the blocks is incorporated onto a semi-analytical half-space model as a linear perturbation of the original model. To build a lumped-parameter model, a number of load-to-receiver models calculate elements of matrices and load vectors. Note that element entries of the mass-to-mass transfer mobility matrix for each excitation frequency,  $\omega > 0$ , are calculated independently of the load and receiver position. Considering the balance of forces at the masses, this leads to a set of linear simultaneous equations for the unknown coefficients  $\hat{R}_{ij}$ :

$$Y_F(x, x_0, \omega) = Y_0(x, x_0, \omega) + \sum_{i=1}^{N_F} \hat{R}_{ij}(x_i, x_0, \omega) \hat{Y}_j(x, x_j, \omega). \quad (2)$$

## 3. Results

The study concerns an array of blocks placed on the ground surface of a homogeneous half-space. Table 1 provides the material properties. The distance between the blocks, centre to centre, is 4.0 m, and the blocks are considered rigid with mass density 2400 kg/m<sup>3</sup>. The blocks are 2.0 m long in the longitudinal direction (along the array), and 2.0 m – 8.0 m wide in the transverse direction, and 2.0 m high. One to five blocks are present in the array, and no blocks are present in the reference case.

The first block is placed at a distance of 4.0 m (centre to centre) from a rigid massless plate being subjected to harmonic excitation. Example results are shown in Fig. 1 for a case in which five blocks are placed on the ground surface. For the 2D model, masses are added as mass loads uniformly distributed over 2.0 m strip-widths.

Table 1: Properties of the soil.

Soil	Shear modulus (MPa)	Poisson's ratio	Mass density (kg/m <sup>3</sup> )	Loss factor
Homogeneous half-space	20	1/3	2000	0.020

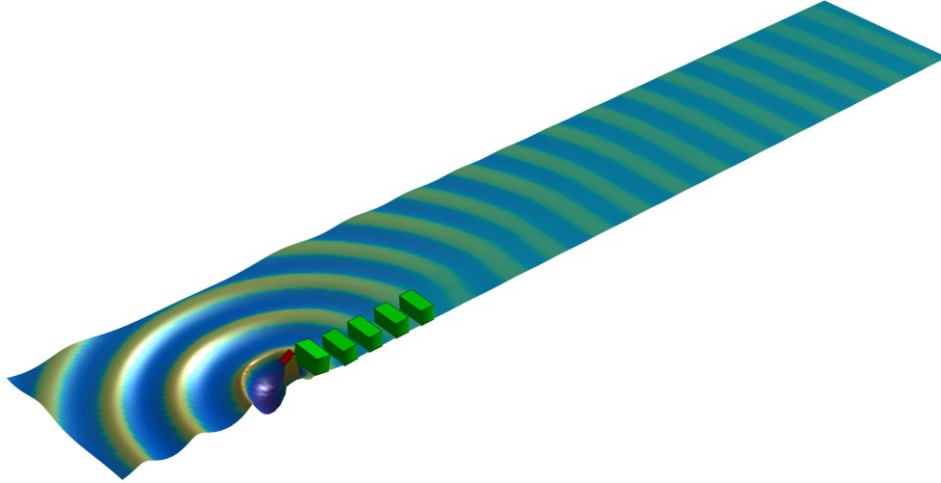


Figure 1: Example results for homogeneous 3D half-space subjected to a harmonic load applied at 25 Hz on a rigid plate and with five rigid (green) blocks situated on the soil surface.

To quantify the change in response at an observation point due to the presence of a number of blocks placed on the ground, the insertion loss  $IL_{dB}$  (in decibel),

$$IL_{dB} = 20 \log_{10} \frac{|U_0|}{U_{ref}} - 20 \log_{10} \frac{|U_k|}{U_{ref}} = 20 \log_{10} \frac{|U_0|}{|U_k|}, \quad U_{ref} = 10 \text{ pm}. \quad (3)$$

is calculated. Here  $|U_0|$  denotes the magnitude of the vertical displacement response at an observation point in the case with no blocks present, and  $|U_k|$  is the magnitude of displacement response at the same position when  $k$  blocks are present,  $k = 1, 2, \dots, 5$ . Results are presented for observation points placed at different positions along the array, up to a distance of 112 m away from the loaded plate, behind the WIBs. The frequency range, 0–80 Hz, relevant to whole-body vibration, is considered.

### 3.1 Influence of arrays of blocks placed on the ground surface: 3D model

Figure 2 shows the insertion loss (IL) achieved with different arrays of blocks placed on the ground surface. As a general trend, the WIB arrays provide positive IL behind the WIBs, and there is a clear effect of adding more WIBs to the array at all three receiver positions, 28.0 m, 56.0 m and 112.0 m. The introduction of rigid blocks, on the surface of the ground, effectively creates destructive interference between propagating and diffracting waves beyond the blocks. It is clear from the results, on the LHS of Fig.2, that increasing numbers of blocks results in a spatial uniform increase in vibration reduction at 16 Hz, 32 Hz and 64 Hz up to around 120 m.

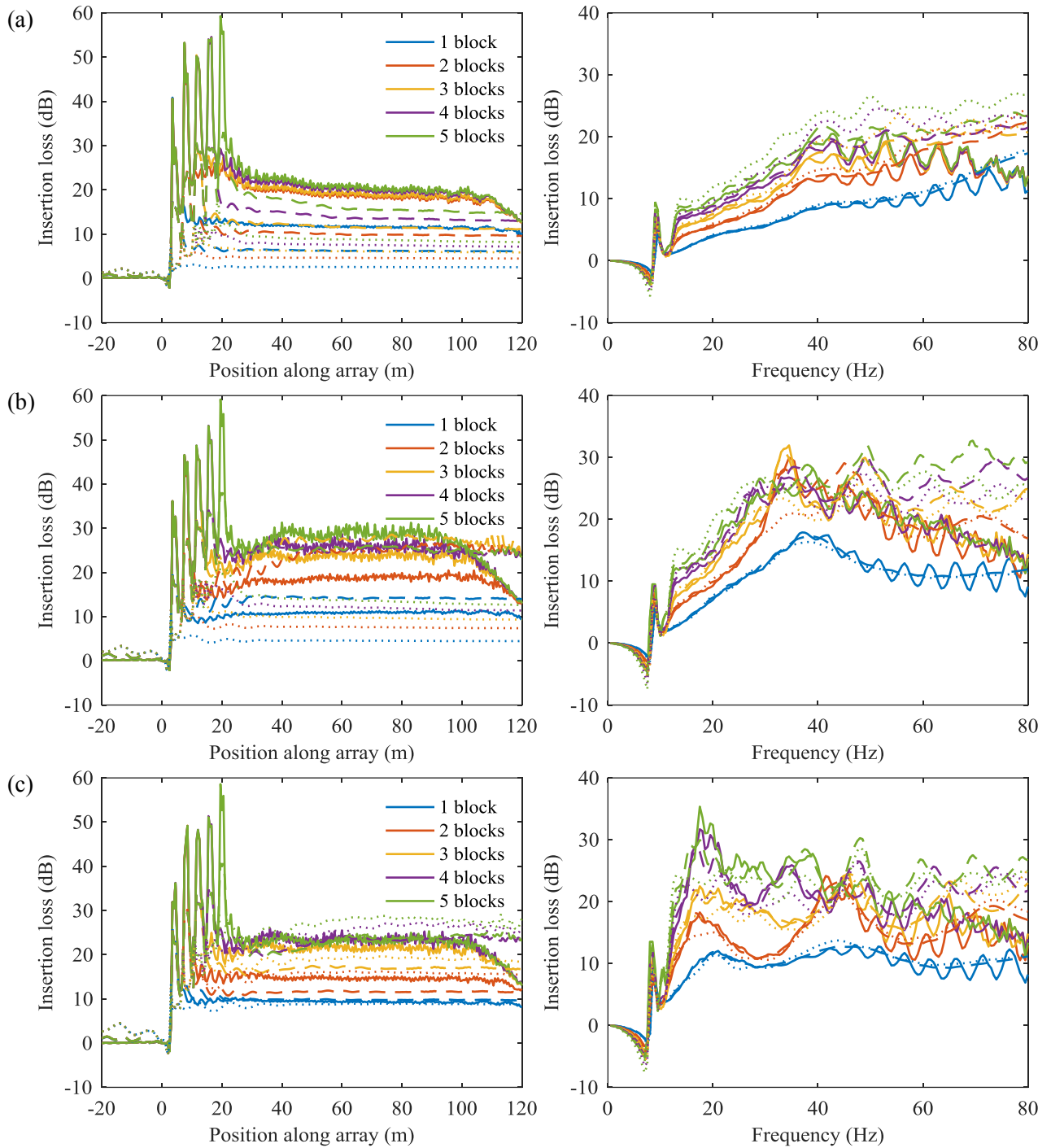


Figure 2: Rows indicated by (a) transverse width of blocks, 2.0 m; (b) transverse width of blocks, 4.0 m; (c) transverse width of blocks, 8.0 m. (LH Column) Full lines (—) indicate results at 64 Hz; dashed lines (— —) indicate results at 32 Hz; dotted lines (·····) indicate results at 16 Hz (these results have been found by averaging over 9 points with a distance of 0.5 m, i.e. a total length of 4 m. (RH Column) Full lines (—) indicate results at  $x = 112$  m; dashed lines (— —) indicate results at  $x = 56$  m; dotted lines (·····) indicate results at  $x = 28$  m (these results have been found by averaging over 9 points with a distance of 0.5 Hz, i.e. a total length of 4 Hz).

Beyond the mass-spring resonance frequency, around 8 Hz, according to Fig. 2 (RHS) the transverse width of blocks placed on the ground surface provide an IL of about 10–30 dB for different block widths. Clearly, the wider blocks from (a)–(c) have more mass, doubling each time, hence the insertion losses are expected to increase. The maximum IL occurs at 38 Hz for the 4 m wide WIBs, Fig. 2(b), but at 20 Hz for the 8 m wide WIBs, Fig. 2(c). No clear maximum is observed for the 2 m wide WIBs, Fig. 2(a). The waviness of the IL is due to the patterns of destructive interference. Notice

that an onset of stop-band behaviour can be observed at 45 Hz. Below the mass-resonance frequency around 10 Hz, the blocks on the ground surface have a poor effect. Obviously, this effect must be avoided, given the intention of mitigating rather than amplifying the vibration.

### 3.2 Influence of arrays of blocks placed on the ground surface: 2D model

Arrays comprising one to five wave impedance blocks each with a block mass of 9,600 kg (Fig. 3 LHS) and a distributed mass (Fig. 3 RHS) are considered in the 2D analysis. In the model, the blocks are modelled as point masses effectively distributed over a finite strip, 2.0 m in length along the array, over the ground surface. Using this methodology, it is possible to implement the 2D model as described in Section 2. For each case, the insertion loss (IL) is calculated at frequencies up to 80 Hz and for distances up to 112 m from the load centre, which covers practical ranges for human exposure to ground-borne vibration and receiver distances for vibration from rail and road traffic.

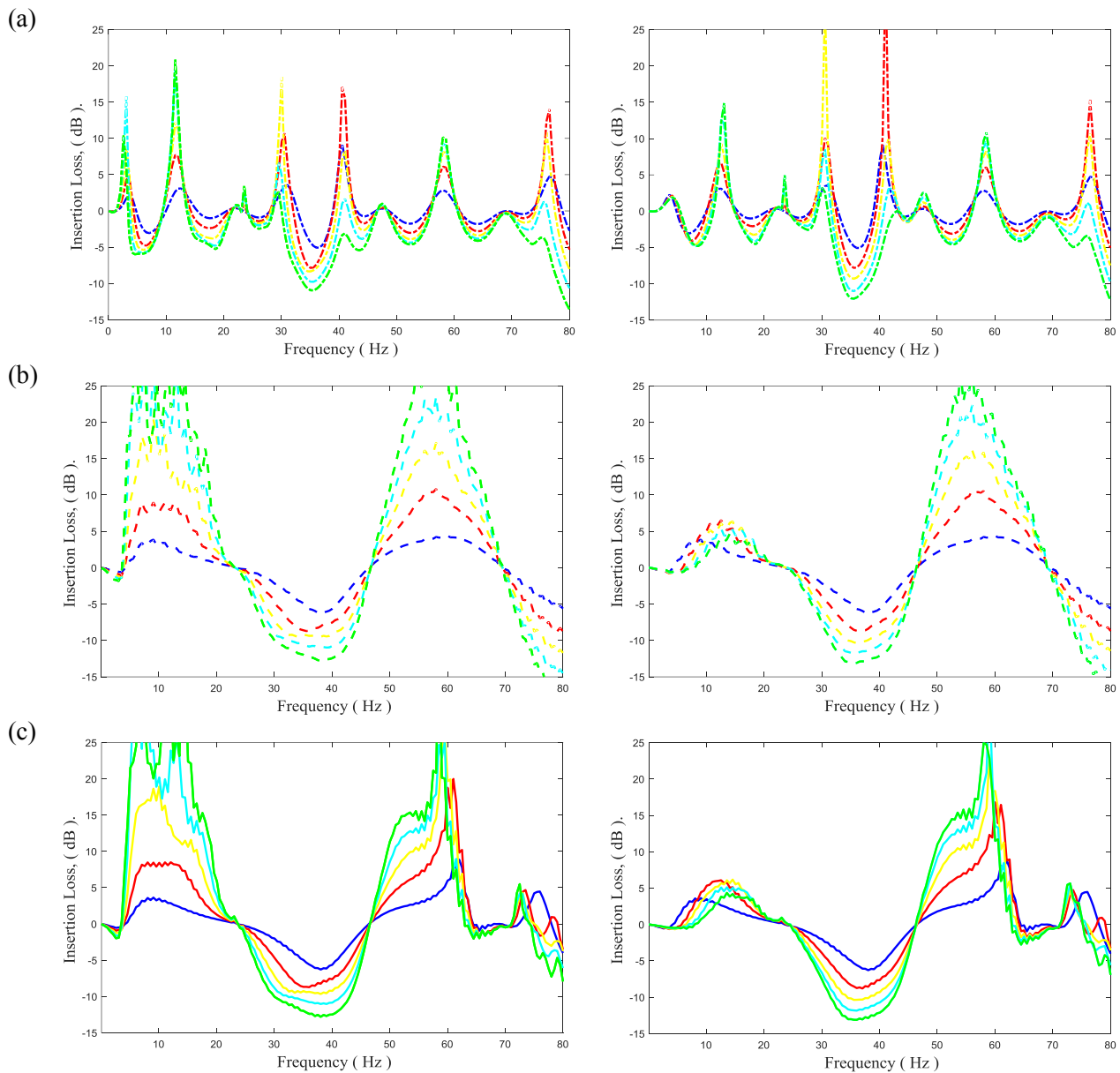


Figure 3: Rows indicated by: (a) Dotted lines (·····) indicate results at  $x = 28$  m; (b) dashed lines (— —) indicate results at  $x = 56$  m; (c) full lines (—) indicate results at  $x = 112$  m. Legend indicator: Blue = 1 block, Red = 2 blocks, Yellow = 3 blocks, Cyan = 4 blocks & Green = 5 blocks. (LH Column) Insertion loss at receiver position for block-mass = 9,600 kg. (RH Column) Insertion loss at receiver position for block mass = 9,600 kg distributed over all related masses.



Figure 3 illustrates the difference between locating an array of masses (9,600 kg each) with increasing number starting from a single location 4.0 m (blue-line) to five masses equally spaced from 4.0 m to 20.0 m to the same sequence of masses where the total mass (9,600 kg) is shared between the blocks. For example, located at 4.0 m, 8.0 m and 12.0 m, the three blocks are 3,200 kg each. Comparing insertion loss between the two illustrates the striking effect of a periodic array, especially Figure 4 (b), (c) at 48 m and 96 m. For five blocks spanning 4.0 m to 20.0 m the insertion loss at these receiver positions is independent of the total mass 48,000 kg versus 9,600 kg the insertion loss reaches 25 dB in all cases. However, at the lower end of the spectrum, 10 Hz in this case, the mass-resonance effect evidently defines the difference between the two cases.

## 4. Conclusions

In this paper, two analytical models that predict ground vibration from loads applied on rigid plates on the ground surface have been extended to estimate the impact of heavy masses placed on the ground surface on the vibration transmission beyond the load. It was found that masses rigidly attached to the ground surface can yield positive insertion losses for a broad frequency range and the interaction between periodically aligned masses can produce a beneficial effect. Further research, comparing the results of the proposed simple analytical models in 3D and 2D with each other and those from a model based on the finite-element method and/or the boundary-element method, would complement the results shown here. Also experiments should be conducted to validate the models for real-life scenarios.

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