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ANALYSIS OF THE AMPLITUDE STATISTICS OF RANDOM SIGNALS

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Introduction

This paper describes the background to measuring random signals and the development of a computerised system to measure and analyse random signals.

One of the most important measurements of a signal is its mean square value. An important parameter in evaluating the mean square value of a signal is the averaging time, T, that is used. If T could be made infinite then the true mean square value of the random signal could be evaluated :

$$\mu = \overline{x^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t')\,dt'$$

In practice a finite value of T must be used, and the measurement of the mean square value for a random signal may be in error from the true value. For many random signals the mean square values will follow a Gaussian probability density function. If this is the case then it has been shown that the variance of the values is inversely proportional to the averaging time used and the bandwidth of the random signal [1]. Establishing this allows an averaging time to be specified such that the measured mean square value will, for example, be within 10% of the true value with a confidence level of 95%.

The probability density function (PDF) of the random signal, for a specified averaging time is found from :

$$y(t) = x^2(t) = \frac{1}{T}\int_t^{t+T} x^2(t')\,dt'$$

and evaluating the probability, p(y), that y(t) lies in the range y to y+dy. If the random signal has a Gaussian PDF then :

$$p(y) = (2\pi\sigma^2)^{-0.5} \exp -[(y-\mu)^2/2\sigma^2]$$

The investigation has concentrated on measuring the statistics of the mean square values of random signals digitally, to see whether the above theory can be applied.

System Development

The software and hardware to analyse random signals digitally was developed around an Atari 1040ST computer. The GEM environment was incorporated in the program which allows the software to be controlled by a mouse and keeps keyboard entry of information to a minimum. The hardware consists of a 12-bit A-to-D converter circuit interfaced through the computers cartridge port, and through software a sampling rate of 51.2 kHz was achieved allowing an audio bandwidth signal to be digitised.

The main problem with analysing a random signal is that a long sample of the signal is required so that the statistics of a large number of mean square values can be found. Typically several minutes of the random signal will need to be analysed and obviously this length of signal cannot be stored directly in the computers memory at the sampling rate being used. However, because the investigation was interested in calculating mean square values the fully digitised signal does not need to be stored. Instead the digitised signal can be squared and each successive 512 values accumulated together.

Each accumulated value represents 0.01 seconds of the squared signal. Doing this means the minimum averaging time that can be used is T=0.01 seconds, however any multiple of this value can be used up to the total length of the signal. The mean square values are found by adding 0.01 second segments together until the total averaging time is reached, this value is then divided by the total number of A-to-D readings in the summation. (512 per 0.01 second segment) The next mean square value is found by starting the summation from the next 0.01 second segment along from the start of the previous summation.

The software allows a length of signal to be digitised and stored in the way just described. It then allows an averaging time to be applied to the signal to calculate the mean square value of the signal as a function of time. From this the PDF is calculated and if it is a Gaussian then :

$$p(y) = (2\pi\sigma^2)^{-0.5} \exp -[(y-\mu)^2/2\sigma^2]$$

By taking logs : $$\ln p(y) = -1/2\sigma^2 (y-\mu)^2 - \ln (2\pi\sigma^2)^{-0.5}$$

So that if ln p(y) is plotted against $(y-\mu)^2$ and a straight line can be fitted through the points then the PDF is Gaussian, and the variance can be found from the gradient of the line fitted. By repeating this procedure for a number of different averaging times the theory that the variance is inversely proportional to the averaging time can then be examined.

The next step in the analysis procedure is to filter and digitise the random signal so that the bandwidth and the frequency content of the random signal can be controlled. This signal is then processed as before. If this is repeated for a number of filtered signals then the relationship between the variance and the bandwidth of the signal for a given averaging time can be investigated. It should be noted that the variance has to be normalised to the mean square value of the signal to obtain a relationship that is inversely proportional to the bandwidth of the signal.

The software allows the digitised signal and the results calculated from it to be stored to disk. Graphical and tabulated presentation of the results is possible and from them an averaging time can be calculated to measure a random signal to a specified accuracy.

Analysis of a Simulated Random Signal

To test the software a random signal was derived from a sequence of pseudo-random numbers generated by the computer. The frequency content of the signal was checked by performing a 1024 point FFT on samples of the signal on the Atari and averaging the results. This showed that the averaged spectrum was constant over the frequency range 0-20 khz, i.e a white noise signal. To obtain a bandlimited signal the number sequence was passed through a digital filter. Four octave band filters were configured using the digital filter and signal lengths of 200 seconds were generated. The frequency response of the digital filter was measured by using the FFT software to calculate the Fourier transform of the impulse response of the digital filter.

These signals were then read into the software for analysis and a number of averaging times were applied to each. This allowed the relationships between the variance and the averaging time, and the normalised variance and the bandwidth of the signal to be examined. The results for these signals are shown in figures 1-4.

Results

The PDF for the 1KHz octave band filtered signal is shown in figure 1, the shape appears that it might possibly be Gaussian. The software, as mentioned, allows a test to be performed for this to be established. Figure 2 displays the result, the natural log of the PDF for a mean square value is plotted against the square of the difference of the mean square value and the true mean square value. Negative values of $(y-\mu)^2$ are rotated about the point where the line crosses the y-axis, this allows a more accurate line to be fitted. Only values of p(y) that are within 90% of the maximum value of p(y) are plotted, this is done because the discrete nature of digital analysis does not allow values of the PDF to extend to infinity.

By using a range of averaging times on this signal a set of results are built up. Averaging times between 0.1 to 2.0 seconds were applied to this signal and the relationship between the normalised variance and the averaging time can be seen in figure 3, where a graph of normalised variance against 1/T is plotted. The points lie on a resonable straight line, so for this signal the theory has been shown to be correct.

Figure 4 shows the result for the second part of the theory; that the normalised variance is inversely proportional to the bandwidth of the signal. By looking at the results of the four octave band filtered signals for a fixed averaging time, T=0.1 seconds in this example, and plotting the normalised variance against the reciprocal of the bandwidth, a straight line is found again, proving the theory is applicable to this signal.

Future Signals to be Investigated

The simulated signal was rather an ideal signal to analyse and so some 'real' signals from traffic and a building site have been recorded and filtered into octave, third octave bands and percentage bandwidths about a frequency, to be analysed by the software to see whether the theory holds true for these signals.

References

[1] BENDAT J S. and PIERSOL A G, Random Data : Analysis and Measurement Procedures (1986)

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Figure 1

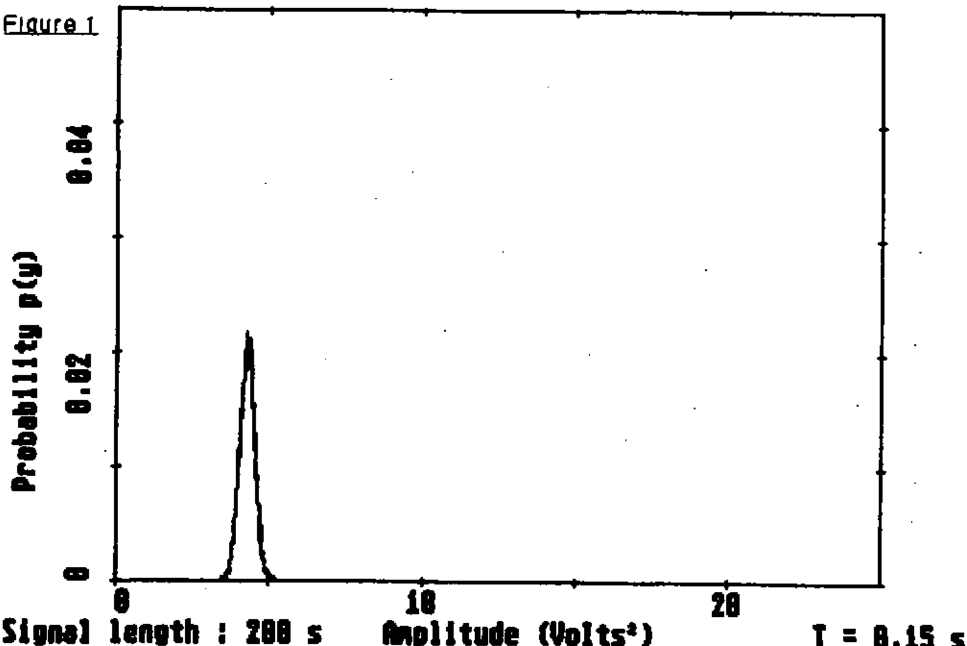


Figure 2

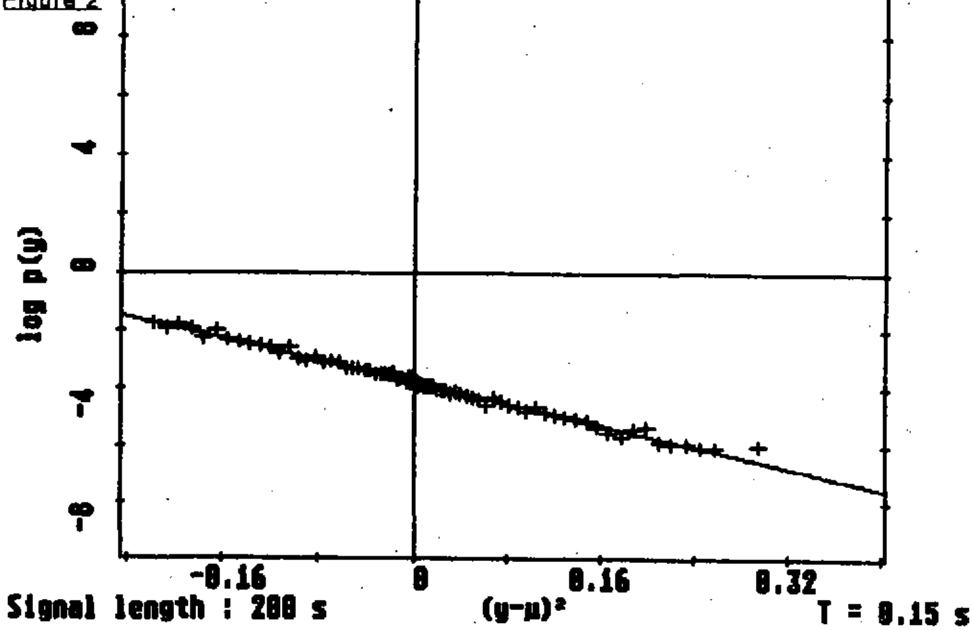


Figure 3

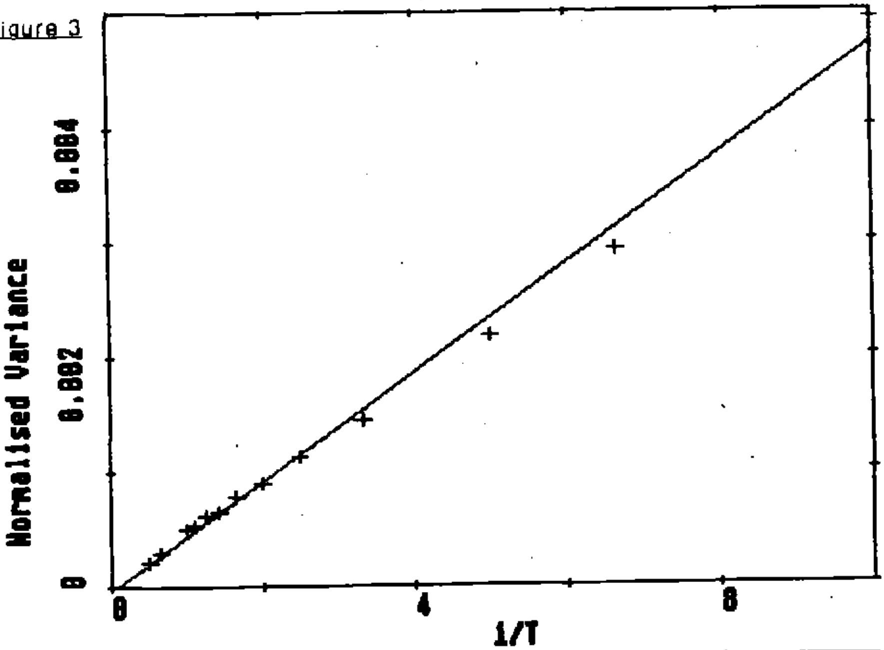
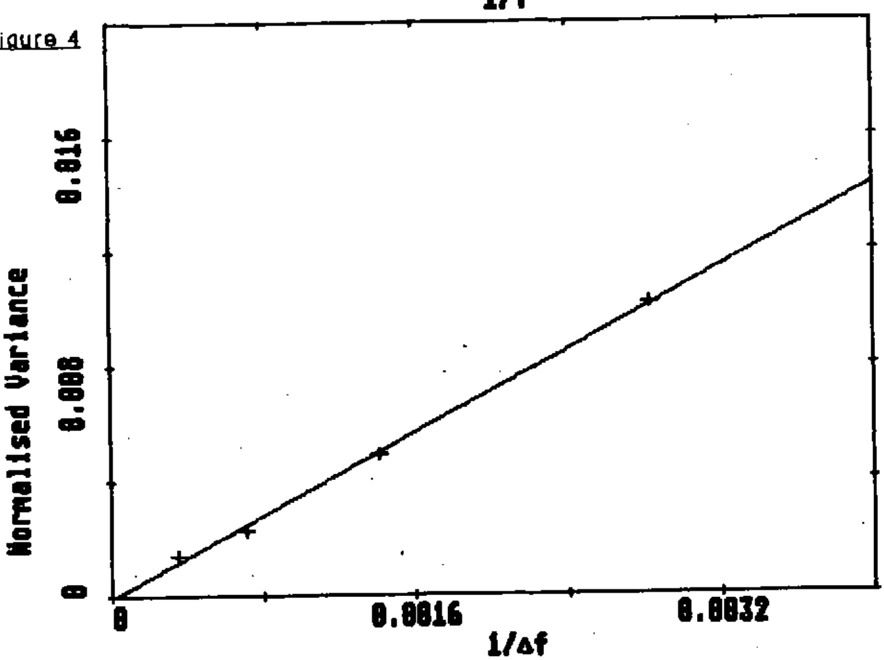


Figure 4



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A HEMISPHERICAL CAP ARRAY FOR NOISE SOURCE LOCATION

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With the advancement of underwater acoustics signal processing, noise and its reduction has gained increasing importance. Clearly to reduce noise in an efficient manner it is imperative that the location intensity, and frequency content of the sources are identified. This paper concerns an array used to measure noise sources induced by fluid flow past an axi-symmetric body, in the frequency range of 1 kHz to 50 kHz in air. Theory using the Rayleigh Integral, indicated that an appropriate design was a hemispherical spiral array, in which the spacing between each turn of the active element decreases towards the centre of the array. This shading effect enabled good directivity to be obtained with a limited length of piezoelectric cable over a wide frequency range. The theoretical calculations were compared with experimental results taken in a large water tank using a spark source.